CHAPTER 40

A COMBINED FE-BIE METHOD FOR WATER WAVES

by

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ABSTRACT

The finite element method and boundary integral equation method are general approximation processes applicable to a wide variety of engineering problems. After a brief description of the combined method, several examples are given for water waves problems : tides, harbour oscillations and waves diffraction and refraction.

INTRODUCTION

Most of the studies in coastal engineering are in relation with water waves coming from deep sea (tides, long and short waves). The difficulty is to take into account the influence of deep sea (infinite or supposed to be so) as exciting and receiving domain.

COMBINED FE-BIE METHOD

The assumption of simple harmonic linear waves yields elliptic partial defferential equations. These equations may be solved by :

(i) the boundary integral equation method (BIE) based on Green's identity which leads, under certain conditions, to an integral equation relating the solution and its normal derivative on the boundary.

(ii) the finite element method (FE) in which the region is divided into a number of elements.

The first one is usefull for simple geometric domains because integral equations can be simplificated before integration. The second method is fitted for complex geometric domains owing to elements.

Many anthors (Zienkiewicz, Berkhoff, C.C.Mei...) have already described the combined method. So the purpose of this paper will be to present new results showing its advantages.

In few words, the infinite or semi-infinite domain is divided into two regions where each method is applied.

In the external region, where the water depth is assumed to be a constant, the solution will be a superposition of the incident wave and an outgoing wave which is due to the presence of an harbour or any obstacle. This outgoing wave will be represented by a superposition of waves coming from

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point sources at the boundary Γ between the sea and the aera of interest and must satisfy the conditions at the other boundaries : full reflection on rigid walls and radiation condition of Sommerfield at sea. So the solution in the external region can be written in the general form :

$$\alpha\ (f(P) - \psi(P)) = \int_\Gamma \frac{\partial G}{\partial n}(P, M)\ (f(M) - \psi(M))\ dM - \int_\Gamma G\ (P, M)\ \frac{\partial}{\partial n}\ (f(M) - \psi(M))\ dM \qquad (1)$$

where
f is the total potential
ψ is the incident potential
G is the Green function of the problem.

The expression is also correct on the boundary Γ, $(M \in \Gamma)$ by taking the limit (only α changes).
The solution f must be continuous through the boundary with respect to wave height and phase.
So equation (1) can be interpreted as a boundary condition for the internal region.
In this one, the finite element procedure is possible by direct coupling of the solution f and its normal derivative on the common boundary Γ.
The system is solved only in the internal region but the influence of the external one is taken into account thanks to the boundary integral condition.

APPLICATIONS

Some examples of results from the combined FE-BIE method are briefly described here.
The tides in a semi-enclosed sea opened to a semi-infinite ocean are simulated using a linearized, vertically integrated, dissipative form of the Laplace Tidal Equation. A linear bottom friction is used. The tide is modelled by setting the tide-generating force terms to zero and specifying the free surface elevation to infinity as a Kelvin wave. So the equations can be written as follows :

$$- i\omega\eta + \frac{\partial}{\partial x}(h\mu) + \frac{\partial}{\partial y}(h\nu) = 0$$

$$- i\omega\mu + g\frac{\partial\eta}{\partial x} - \Omega\nu + \rho\mu = 0$$

$$- i\omega\nu + g\frac{\partial\eta}{\partial y} + \Omega\mu + \rho\nu = 0$$

where η, μ, ν are the complex amplitude of surface elevation and current components.

h is the mean water depth
g is the gravity acceleration
Ω is the Coriolis number
ρ is the bottom friction coefficient
ω is the tide pulsation.
These three equations can be transformed in one :

$$\frac{\partial}{\partial x}\left[h\ (A\ \frac{\partial \eta}{\partial x} + B\ \frac{\partial \eta}{\partial y})\right] + \frac{\partial}{\partial y}\left[h\ (A\ \frac{\partial \eta}{\partial y} - B\ \frac{\partial}{\partial x})\right] + \eta = 0 \qquad (2)$$

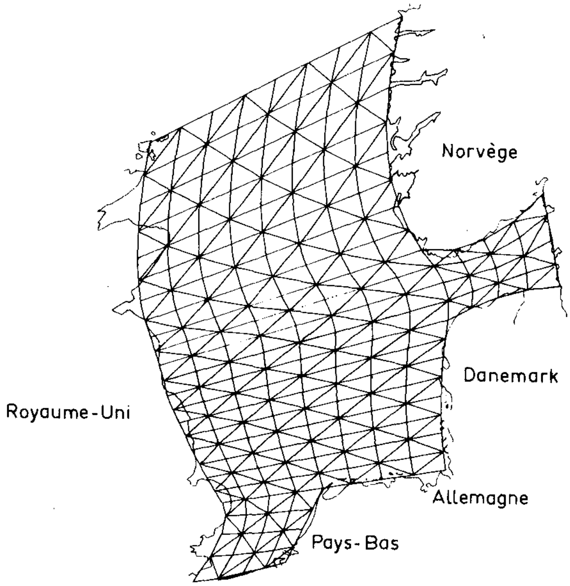
where A and B are complex functions of ω, Ω, g and ρ.
In the external region, the water depth is assumed to be a constant, so equation (2) is simplified (the B factors disappear) and it can be shown than the Green function of the problem is an Hankel function.
In the internal region, the finite element procedure is easily applied to equation (2).
The M_2 constituent of the tide in the North sea has been calculated by this way. The numerical results (fig. 1) are in good qualitative agreement with the observations, particularly in the reproduction of the amphydromic points. An important thing to point out, is that there is only two parameters to calibrate to solve this problem :
- the wave direction in the ocean, but its influence is very weak
- the bottom friction coefficient which fixes the position of the amphidromic points.

The repartition of amplitude and phase on the common boundary is obtained from the model.

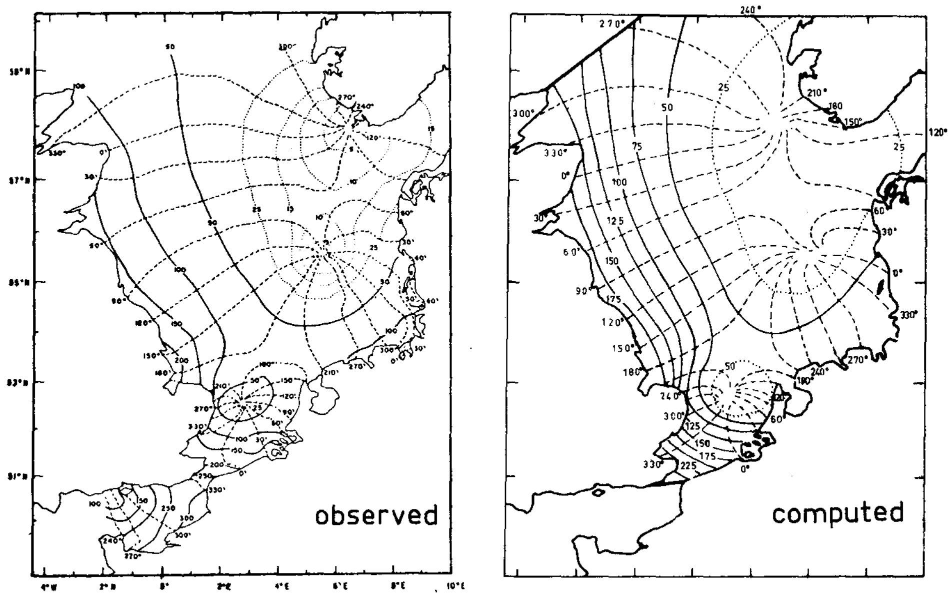
The response of harbours to long waves of differents frequencies coming from deep sea can be obtained by the same way. The oscillations are simulated using the same Laplace equation but Coriolis effects and bottom friction can be neglected. In this case, the B factor disappears in equation (2).
The method yiels eigen frequencies and correspondant amplification factors of the harbour opened to the sea (see fig. 2 in the case of Marseille Harbour). This is particularly usefull to study seiches in harbours.
In decreasing the wave length, the last example is the computation of wave diffraction and refraction. In this case the governing equation is

$$\frac{\partial}{\partial x}\ (n\ \frac{\partial f}{\partial x}) + \frac{\partial}{\partial y}\ (n\ \frac{\partial f}{\partial y}) + nk_o^2\ f = 0 \qquad (3)$$

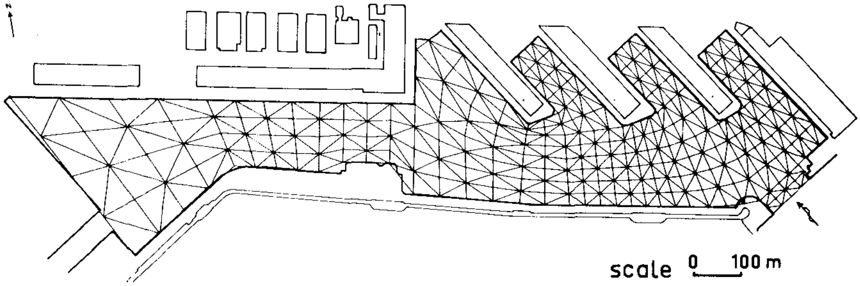
where k_o is the wave number



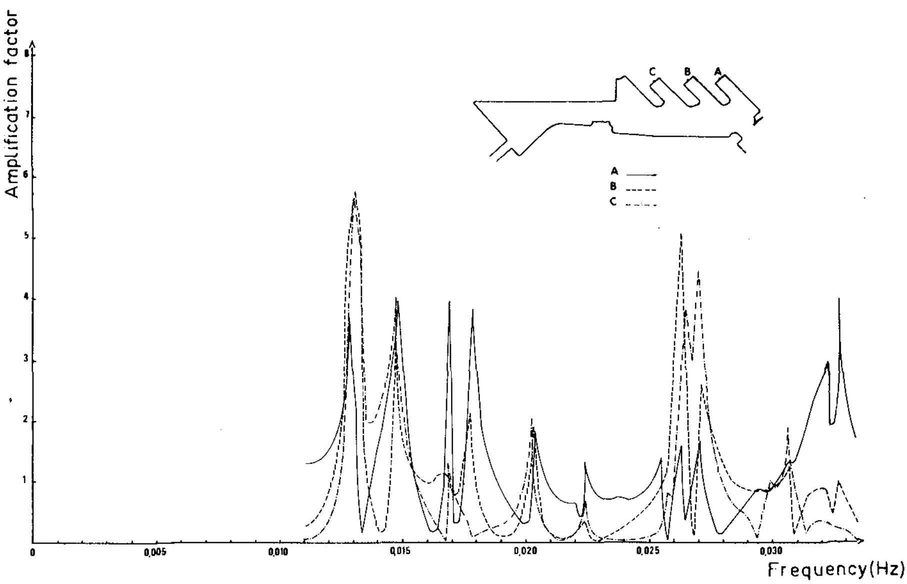
TIDES IN NORTH SEA
CONFIGURATION OF ELEMENTS



M_2 TIDAL CHART
Fig.1



SEICHES IN MARSEILLE HARBOUR
CONFIGURATION OF ELEMENTS



HARBOUR OSCILLATION CURVES

Fig.2

n is the shoaling number
f is the potential.

Equation (3) is similar to equation (2). When the depth and consequently the shoaling number are constant the Green function is also an Hankel function. The same procedure can be applied.

The main restriction of the model is the numerical requirement of about five computing points over one wave length to compute the surface elevation with a sufficient accuracy. Figure 3 shows the solution of wave diffraction and refraction for the case of an island on a flat or parabolic shoal. The numerical results are in good qualitative agreement with the analytical solution. Amplitudes are generally underestimated mainly for short wave lengths (probably in relation with the number of computing points).

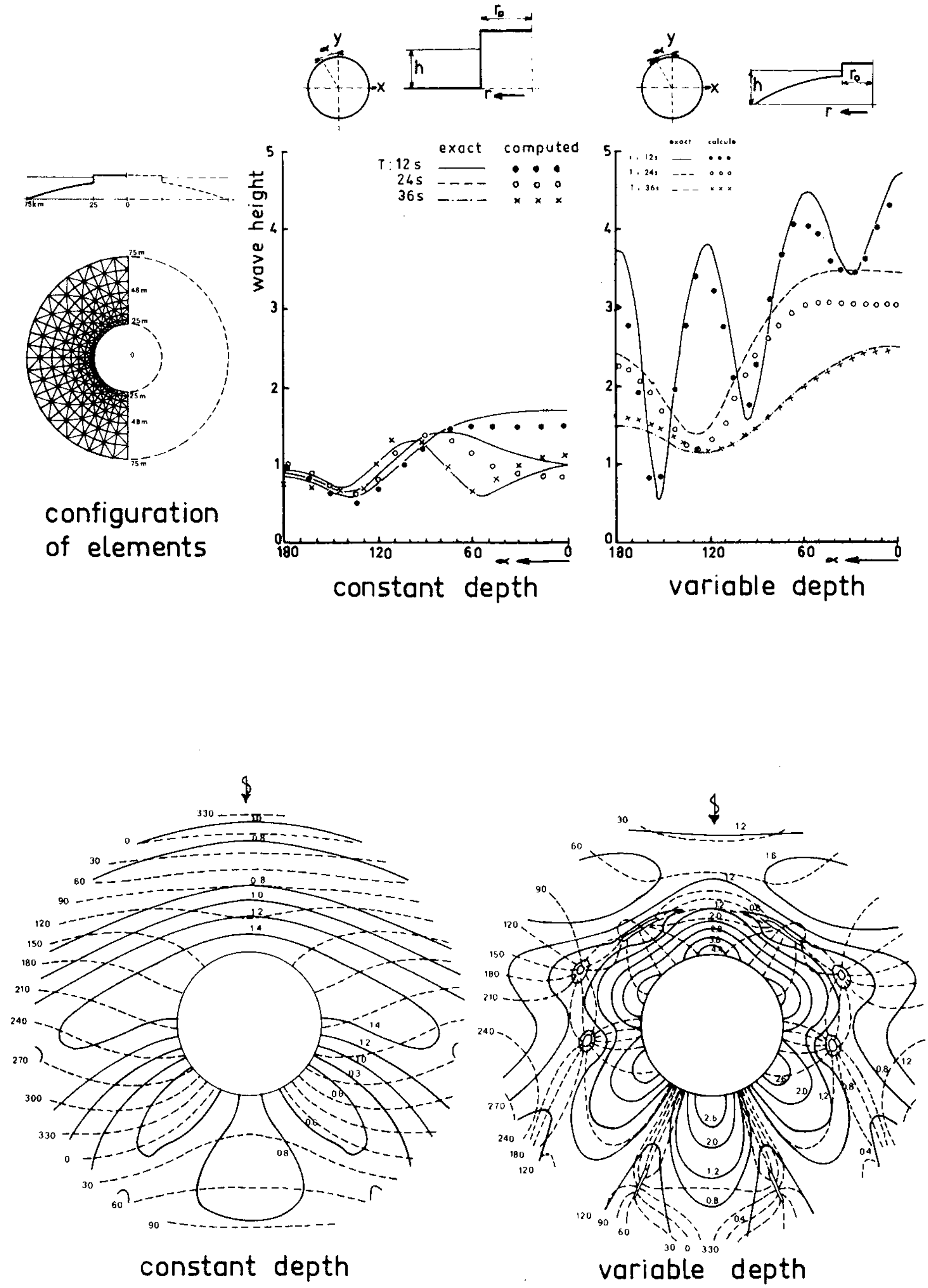
CONCLUSIONS

The mathematical model for linear simple harmonic waves described in this paper can be of great help to give some quantitative or qualitative information about waves coming from deep sea (tides and waves). The main restriction of the model is the numerical requirement with respect of large area compared with the mean wave length (especially for short waves) but the rapidity of the resolution, the development of the finite element method and the reduced number of parameters to calibrate the model (one or two for tides) give quite good informations very quickly.

REFERENCES

Zienkiewics, O.C., Kelly,D.W. and Bettess, P., The coupling of the finite element method and boundary solution procedures. Int. J. Num. Meth. Engng. Vol. 11, n° 2, 1977, pp 355-375.

Berkhoff, J.C.W. Linear wave propagation problems and the finite element method. Ch. 13, Finite element methods in fluid mechanics (Ed. R.H. Gallagher, J.T. Oden, C. Taylor and O.C. Zienkiewicz). Wiley, 1975.



WAVE REFRACTION DIFFRACTION
AROUND A CIRCULAR ISLAND
Fig. 3