CHAPTER 10

INVESTIGATIONS ON IRREGULAR WAVES IN HYDRAULIC MODELS

by

Karl-Friedrich Daemrich, Wolf-Dietrich Eggert
Sören Kohlhase
Franzius-Institut, University of Hannover, Germany

ABSTRACT

The first part of the paper deals with some aspects of wave generation by mechanical wave-generators, especially with hydraulic transfer functions for pusher movement of the paddle, and the influences of signal characteristics and analysis methods. In the second part, results of measurements of orbital velocities and pressures in irregular waves are presented, together with some results from theoretical simulation methods in the frequency and time domain, based on linear wave theory.

SCOPE OF INVESTIGATIONS AND INSTRUMENTATION

The possibility of the generation of predictable and reproducible irregular waves with servo-controlled wave-machines extends considerably the validity of hydraulic models and also allows an effective basic research for the improvement of dimensioning methods.

To get an impression of the transformation process of paddle-generated waves, hydraulic transfer functions between pusher movement of the board and generated wave trains are measured in different water depths, using as well noise spectra as natural wave trains as a basis for control signal calculations.

The investigations on orbital velocities and pressures in irregular waves are presented mainly under the aspect of the applicability of special analysis and simulation methods within investigations with irregular waves.

The investigations were performed in a wave basin of the "SONDERFORSCHUNGSBEREICH 79" and in a wave channel at the FRANZIUS-INSTITUTE of the University of Hannover (Table 1).
### Table 1: Dimensions of the hydraulic models

<table>
<thead>
<tr>
<th></th>
<th>wave basin</th>
<th>wave channel</th>
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</thead>
<tbody>
<tr>
<td>length</td>
<td>45 m</td>
<td>118 m</td>
</tr>
<tr>
<td>width</td>
<td>18 m</td>
<td>2.2 m</td>
</tr>
<tr>
<td>max. water depth</td>
<td>0.6 m</td>
<td>1.5 m</td>
</tr>
<tr>
<td>max. wave height</td>
<td>0.4 m</td>
<td>0.5 m</td>
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Both facilities are equipped with servo-controlled hydraulically driven wave generators and allow predefined partial flap/partial piston motions of the wave board.

Control signals were generated with a "WAVE SYNTHESIZER" as developed at the "HYDRAULIC RESEARCH STATION WALLINGFORD" and with a "WAVE SIGNAL GENERATOR" consisting of a tape-reader, a Digital to Analog Converter and some more electronic equipment, as produced by the "DANISH HYDRAULIC INSTITUTE".

The "WAVE SYNTHESIZER" generates pseudo-random signals with selectable power spectra and different sequent lengths. However, the generated signals do not consider special wave characteristics like flat troughs and peaked crests or a definite wave grouping. The "WAVE SIGNAL GENERATOR" reads digitized data from punch tape and converts them to analog signals. With this instrument it is possible to generate control signals with arbitrary characteristics. The input signal may be converted with a built-in approximate transfer function for pushertype motion. This allows to generate wave-trains - which are numerically described by the punch tape - at the wave board without any further calculations.

### INVESTIGATIONS ON WAVE GENERATION

To generate a suitable spectrum in a basin or a channel the following transfer functions have to be considered:

- the mechanical transfer function of the wave-generator
- the hydraulic transfer function between paddle movement and generated wave at the paddle or near the paddle
- the transfer function for the transformation of the waves with increasing distance from the wave-generator.

The characteristics of the mechanical transfer function of the wave generator depend on the type of the generator and the settings of the servo-amplifiers and shall not be discussed here.
To prove the validity of the theoretical hydraulic transfer functions, several test series were performed. Fig. 1 shows a comparison of measured and theoretical transfer functions for a deepwater wave spectrum with noise characteristics.

![Fig. 1](image)

**Fig. 1** Measured and theoretical hydraulic transfer functions for different wave steepnesses in deepwater \( \frac{d}{(g \cdot T_m^2)} = 0.058 \)

- a) \( \frac{H_{1/3}}{(g \cdot T_m^2)} = 0.0051 \) (nonbreaking waves)
- b) \( \frac{H_{1/3}}{(g \cdot T_m^2)} = 0.0077 \) (slight breaking of high waves)

The wave probe was located in a distance of 5 m from the wave board. The transfer function was calculated from one sequence length of the pseudo-random signal (1024 data) without any averaging in this case, while the spectrum was smoothed by a moving average (11 data) in the frequency domain.

There is a rather good agreement between theory and measured values, even for slight breaking conditions, and it can be seen that it may be possible to measure transfer functions very effective also with such an unrealistic wave spectrum.

As a next example results from waves in more transitional water are used to demonstrate the influence of the control signal characteristics.
From former tests in smaller waterdepths it could be expected, that transformation processes influence the quality of the hydraulic transfer functions.

Transformation process is called the changing of the wave train from a simple irregular water level fluctuation without any typical characteristics - for instance generated by a band-limited noise signal - to a wave train with distinct characteristics (peaked crests and flat troughs) depending on the relative water depth. We therefore selected a natural wave train as a basis for control signal generation of this example (Fig. 2).

Fig. 2 Measured and theoretical hydraulic transfer functions for different signal characteristics in transitional water depths \( \left( \frac{H_{1/3}}{g \cdot T_m^2} = 0.0056; \frac{d}{g \cdot T_m^2} = 0.025 \right) \)

- a) natural wave train
- b) random wave train with same amplitude spectrum, but random phase spectrum

Fig. 2a shows the measured transfer function using a wave train which was generated according to the natural wave train. This wave train includes all characteristics typical for the water depth and the period-range.

Contrarily, Fig. 2b shows the transfer function measured with an artificial wave train with the amplitude spectrum corresponding to the natural wave train, but with random
The distinct minor scatter of the first transfer function may be seen as an indicator for the optimization of the control signal i.e., the transformation of the wave train from random-noise characteristics to the suitable and necessary characteristics for this waterdepth is indicated clearly by an increased scatter in the transfer function.

The next example shows hydraulic transfer functions in different water depths for wave spectra, generated with pseudo random control signals (Fig. 3).

Fig. 3 Measured and theoretical hydraulic transfer functions for different water depths d (pseudo random control signals)

To make clearer the increasing losses, the transfer functions were smoothed in the frequency domain for this example.

The differences between measured and theoretical transfer functions are increasing with decreasing water depths. This may be explained by the stronger transformation processes.

These examples lead to the conclusion that for deepwater and to some extent also for transitional water depths the theoretical transfer functions are sufficient to calculate control signals for spectrum generation, but that corrections are necessary in small relative water depths, especially if bandlimited random noise signals are used as control signals.

Finally, the problem of generating special wave trains in a definite distance from the wave board shall be mentioned.
Some tests were performed, simulating a self-correcting system with the DHI-WAVE SIGNAL GENERATOR. The desired wave train was used as an input signal and corrected with the measured transfer function. This correction was repeated several times. It was found that with a well characterized wave train as a first input signal the first correction leads to rather good results (Fig. 4). However, this work is only at the beginning and demands more experience.

![Graph showing wave measurements](image)

Fig. 4 Simulation of a self-correcting system with the DHI-WAVE SIGNAL GENERATOR

SOME REMARKS ON THE CALCULATION OF TRANSFER FUNCTIONS

As shown for demonstration before, the transfer functions were generally calculated from wave trains with pre-defined repetition periods and always the complete sequenth-length was analyzed as one data block. Sometimes a frequency smoothing was performed.

Calculation in that way was done, because the distance between the wave board and the wave probe leads to unrealistic values of the measured transfer functions, if the time series is shared into small time cycles, and the final results were calculated from the averaged auto-power and cross-power spectra. The following example may demonstrate this (Fig. 5).
**Fig. 5** Influence of the calculation method on the magnitude and phase of a transfer function

a) averaged auto- and cross-spectrum

b) without averaging but with equivalent frequency smoothing of the magnitude

Fig. 5a shows a transfer function calculated from eight parts of the time series using the averaged auto- and cross-power spectra; Fig. 5b shows the transfer function calculated from the same time series without averaging, but with an equivalent frequency-smoothing.

The tendency can be seen clearly. The averaged transfer function (Fig. 5a) does not describe the change in the wave spectrum sufficiently.

**DEVELOPMENT OF THE WAVE SPECTRUM WITH INCREASING DISTANCE**

Fig. 6 shows two examples of measured transfer functions between 5 and 12.5 m from the wave board (Fig. 6a) and between 15 and 22.5 m (Fig. 6b).

The change in the spectrum, expressed by the magnitude of these transfer functions, is not very important.

In addition, the development of some wave parameters from 2.5 to 30 m from the wave board in intervals of 2.5 m was calculated for this example (Table 2).
Fig. 6 Changes of the wave spectra with increasing distance and transfer functions

a) 5.0 m to 12.5 m from the wave board
b) 15.0 m to 22.5 m from the wave board

<table>
<thead>
<tr>
<th>distance [m]</th>
<th>H_s [cm]</th>
<th>H_1/3 [cm]</th>
<th>H_1/10 [cm]</th>
<th>H_{max} [cm]</th>
<th>T_m0,1 [sec]</th>
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<th>Q_p</th>
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<td>1.57</td>
<td>1.61</td>
<td>3.91</td>
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Table 2: Wave parameters in different distances from the wave board
It can be seen that characteristic wave parameters, e.g. significant wave heights or mean periods, derived from the spectrum or from the time series did not change with a clear tendency over this distance.

Only the peakedness factors $Q$ show a slight trend from about 3.4 at the first wave gauge to 3.9 in a distance of 30 m.

**ORBITAL VELOCITIES IN IRREGULAR WAVES**

Velocities and pressures, the actual values causing the load of a structure, are used as an input for many dimensioning methods dealing with wave forces on structures.

It was the purpose of the investigations to check simulation methods with respect to the prediction of velocities and pressures, especially the maximum positive and negative values in every single irregular wave.

Inductive type and impeller type probes were used to measure horizontal and vertical velocity components. However, transfer functions were only calculated from signals of the inductive type probe.

The spectra were mostly JONSWAP-spectra, generated with the "WAVE SYNTHESIZER", but additionally a natural wave train and a random noise spectrum were investigated for comparison.

The significant wave heights ranged from 10 to 25 cm, the water depth was 1.0 m in most of the tests; the velocities were measured in different depths ($-0.25$ m, $-0.50$ m, $-0.85$ m).

The first step of the analysis was the calculation of the transfer function between wave spectrum and velocity spectrum and the comparison with the theoretical transfer function according to linear wave theory.

Fig. 7 shows exemplarily some typical results for velocity components measured in a depth of 25 cm below "still water level".

It can be stated, that there are no significant differences between the transfer functions of the different wave spectra, and even or just the spectrum in Fig. 7b yields to a good agreement with the theory.

However, it was the purpose of the investigation, to predict maximum positive and negative velocities in every wave of a given wave train. Therefore, three simulation methods on the
Fig. 7 Transfer functions and coherence functions of orbital velocities
a) natural wave train
b) random noise control signal
basis of linear wave theory were applied, one in the frequency domain and two in the time domain.

For the superposition method with theoretical transfer functions the FOURIER-transformation of the measured wave train was calculated, each frequency component was multiplied with the pertinent value of the theoretical transfer function according to linear wave theory. The time series of the velocities was then recalculated or superimposed from this FOURIER-components. The maxima of the measured velocity time series were now compared with the maxima of the simulated velocities (Fig. 8).

Fig. 8 -Comparison of measured and with the theoretical transfer function predicted horizontal velocities (JONSWAP spectrum, $\gamma = 3.3$, $U = 13.8$ m/sec, scale 1 : 20)

a) positive maxima

b) negative maxima

The scatter of the data is relatively small, but the tendency is generally, that the measured horizontal velocities are lower than the predicted ones under a wave crest, and higher than the predicted ones under a wave trough.

The next two simulation methods are based on analysis methods in the time domain. Such methods have to be used, because it is not always possible to calculate a FOURIER-transformation, for instance when measurements are available only on recording chart, when signals are distorted or if there is no suitable computer equipment available. For these methods regular wave parameters must be defined for every irregular wave of the wave train.
A common method to define a wave parameter in the time domain is the zero-crossing method. For these examples the zero-down-crossing parameters were used to calculate velocities with the linear wave theory.

A typical result from the same test as Fig. 8 is shown by Fig. 9.

![Fig. 9](image)

**Fig. 9** Comparison of measured and predicted horizontal velocities according to zero-down crossing definition

a) positive maxima

b) negative maxima

The scatter of the data is remarkably wider in comparison to the superposition method.

A better definition for the physical efficient part of the wave was found by comparing the measured wave train and the pertinent velocity time series visually (Fig. 10).

The similarity of the both time series is evident. It seems that, for instance, the velocity in the positive region has the same trend as the positive part of the wave. This effect, which is independent on the shape of the preceding wave trough leads to the following definition of the wave parameters for this special problem.

It was proved to take no longer the wave height and period of the whole wave, according to zero-crossing-definition, but to take the height and period of the pertinent half wave to calculate the maximum velocities with linear wave theory.
Fig. 10 Example of time series of waves and pertinent measured horizontal velocities

Fig. 11 shows a sketch of this definition, which is called "complementary method" in the following.

This - in the physical sense - efficient part of the wave was completed to a full sine wave for calculation.

Comparative results from the simulation of horizontal velocities with the complementary method are shown in Fig. 12, again for the same test as shown by Fig. 8 and 9.

The scatter of the results is of the same order as the scatter of the results from the much more comprehensive superposition method and remarkably smaller than the results according to zero-crossing method.

In the following, two more examples of measurements of positive horizontal velocities are given.
Fig. 12 Comparison of measured and predicted horizontal velocities according to complementary method
a) positive maxima
b) negative maxima

Fig. 13 shows results derived from measurements in a natural wave train with wave heights and periods similar to the preceding example.

For all simulation methods the scatter is of the same order as in the example before.

Finally, Fig. 14 shows results from a test with a wave time-series similar to the first example, but with amplitudes of only 50%.

The scatter is very small in comparison to the examples before, but still the same tendency can be seen.

The velocities under the wave crest are generally smaller than the predicted values and, under the wave trough, they tend to be higher or equal to the predicted values.

The three simulation methods were also used to determine vertical velocities theoretically. For the complementary method it was proved to take the pertinent wave height as sketched in Fig. 15.
Fig. 13 Comparison of measured and predicted positive horizontal velocities according to different simulation methods (natural wave train)

For calculations of the positive maximum of the vertical velocity the wave height between the preceding trough and the crest was used and the wave height between the crest and the following trough for the calculation of the negative maximum of the vertical velocity respectively. The corresponding wave periods were defined as shown in Fig. 15.

Some results from the measurements of vertical positive and negative velocities are presented in Fig. 16.

The scattering of the data using the complementary method and superposition method (with theoretical transfer function) is of the same order but, contrarily to the results from horizontal velocity components, there is no remarkable difference between the positive and negative velocities.
Fig. 14 Comparison of measured and predicted positive horizontal velocities according to different simulation methods (JONSWAP spectrum, $\gamma = 3.3$, $U = 13.8$ m/s, scale 1 : 20, halved amplitude)

Fig. 15 Definition sketch for determination of wave parameters according to complementary method (vertical velocities)
Fig. 16 Comparison of measured and predicted vertical velocities according to different simulation methods (JONSWAP spectrum, $\gamma = 3.3$, $U = 13.8$ m/sec, scale 1:20)

a) positive maxima

b) negative maxima
In this case the zero-down crossing method leads to good results for the positive values too, because nearly the same part of the wave is used to calculate the velocity components. However, for the negative velocities the scatter is remarkably wide.

The investigations on the pressure in irregular waves can only be mentioned in the scope of this paper. Theoretical simulations were done similarly to the simulations of the velocities. Results from the superposition method with theoretical transfer functions were still better than for the velocities. The scatter of the results simulated by the complementary method was less than the scatter from zero-crossing simulation. There is the same tendency too, that the predicted pressure under the wave crest is higher than the measured pressure and the predicted negative pressure is lower than the measured pressure in the wave trough.

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