CHAPTER 14

EXPERIMENTAL INVESTIGATIONS OF PERIODIC WAVES NEAR BREAKING

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ABSTRACT

The results presented are from a series of experiments of periodic waves approaching breaking on a gently sloping beach.

The recorded surface profiles are compared with a cnoidal theory taking into account the effect of the bottom slope. In both theory and experiments a skewness of the profile is observed. A fair agreement is found within certain limits of the wave height to water depth ratio and when the slope is sufficiently gentle. The change in water depth over a wave length must be sufficiently small.

The detailed analysis of the surface profiles including the skewness element shows good agreement with the cnoidal theory in cases where the wave height variation is as predicted by cnoidal wave shoaling.

The recorded surface profiles together with the recorded phase velocities are further used in computing the wave energy flux based on the theoretical relations from linear and cnoidal theories. Reasonable constancy is observed over the whole length covered by the experiments.

Finally the recorded wave set-down is compared with the results of linear and cnoidal theories. When approaching breaking neither of the theories can apparently predict the recorded water level changes.

1. INTRODUCTION

An extensive series of experiments with periodic waves approaching breaking on a 1:34.26 plane sloping beach were carried out from 1975 through 1978.

In these experiments were studied the wave transformation on a gentle slope up to the point of breaking, and the measurements supplied information about height, speed of propagation, and variation of mean water level, as well as the shape and deformation of the surface profiles. In particular the wave profiles differ significantly from those reported in literature because the waves generated were nearly free of free second harmonic components, which so seriously have hampered other investigations.

Much of the raw data from 17 different wave conditions is published in Buhr Hansen and Svendsen (1979). This report further contains a detailed description of the experimental set-up and data acquisition system.

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Some of the results extracted from the measurements have already been published in journals and at conferences:

Svendsen and Buhr Hansen (1977 and 1976 a) compared the observed wave height variation over the sloping bottom with the theoretical shoaling of the waves as predicted by sinusoidal and cnoidal wave theories. A good agreement between experiments and theory was observed for waves of relatively small deep water steepnesses (Svendsen and Buhr Hansen, 1976 a, Fig. 4).

In case of rather steep waves, deviations between experiments and theory become pronounced (Svendsen and Buhr Hansen, 1976 a, Fig. 5). However, the shoaling assumption requires that the relative change in water depth, h, over a wave length, L, is small. This indicates that, in case of very long waves, other deviations between experiments and theory might be observed. This is in Svendsen and Buhr Hansen (1977) expressed in terms of a bottom slope parameter $S \equiv h_x L/h$, where h_x is the bottom slope. To fulfill the shoaling assumption, linear theory requires that S is of the same order of magnitude (or smaller) as the wave steepness, H/L, and Svendsen (1974) showed that for (first order) cnoidal waves S must be small compared to $(h/L)^2$. Further, H/h must be sufficiently small ($\ll 1$).

In this paper a detailed analysis of wave surface profiles will be presented, and the results will be compared with a cnoidal theory covering cases from very long waves with low steepness to shorter waves with a considerable steepness.

The analysis of the surface profiles includes a comparison with a cnoidal theory taking into account the distortion (skewness) of the profile due to the sloping bottom as given in Svendsen and Buhr Hansen (1978). A preliminary comparison between some measured wave profiles and the above mentioned theory is included in Svendsen and Buhr Hansen (1976 a, 1978).

Further, the wave energy flux, as calculated from the experimental data, will be analysed for the influence of profile skewness and phase velocity. Svendsen and Buhr Hansen (1976 a) contains some preliminary comparison between the measured and the theoretical phase velocities.

Finally this paper will include a comparison of wave set-down near breaking between the experiments and different theoretical approaches. Svendsen and Buhr Hansen (1976 b) contains some cnoidal wave approximations for the wave set-down and a comparison with two experiments.

2. THE EXPERIMENTAL SET-UP

The waves are generated by a piston type wave generator in a flume 32 m long, 60 cm wide, with a plane beach sloping 1:34.26 (see Fig. 1). The motion of the wave generator is controlled by a PDP 8 mini-computer, which generates a command signal of the form

$$\xi = e_1 \sin \omega t + e_2 \sin(2\omega t + \beta) \tag{1}$$

(Buhr Hansen and Svendsen, 1974). The recording system is described in details in Buhr Hansen and Svendsen (1979).

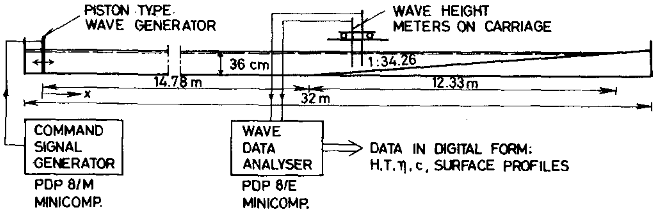

Fig. 1 Experimental facility

The resulting wave profiles on the horizontal bottom are in Svendsen and Buhr Hansen (1977, 1978) compared with a Stokes' second order wave for rather short waves ($U = HL^2/h^3$ is about 2 and 7, respectively), indicating the generation of waves with almost no free second harmonic components.

It appears that even for waves with U as large as 40 (which is far up in the cnoidal region), the waves generated remain of clean and constant form. In Fig. 2 the recorded surface profiles of three wave conditions are compared with theoretical cnoidal profiles, and a good agreement is observed.

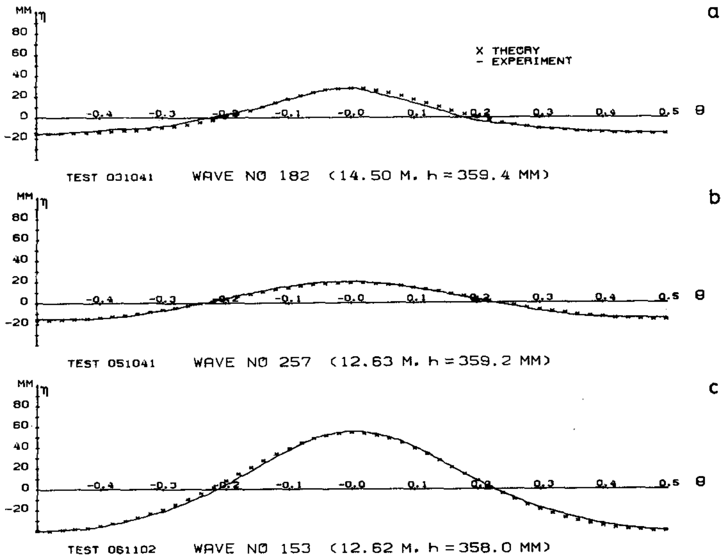

Fig. 2 Surface profiles on horizontal bottom. Measurements compared with first order cnoidal theory. a) T = 1/.3 s, U = 32.2; b) T = 1/.5 s, U = 10.7; c) T = 1/.6 s, U = 17.5

For the longest of these waves the wave height variation over the sloping bottom is clearly not predicted by theoretical cnoidal wave shoaling (see Fig. 3), indicating that the bottom is not 'sufficiently gently sloping' in this case.

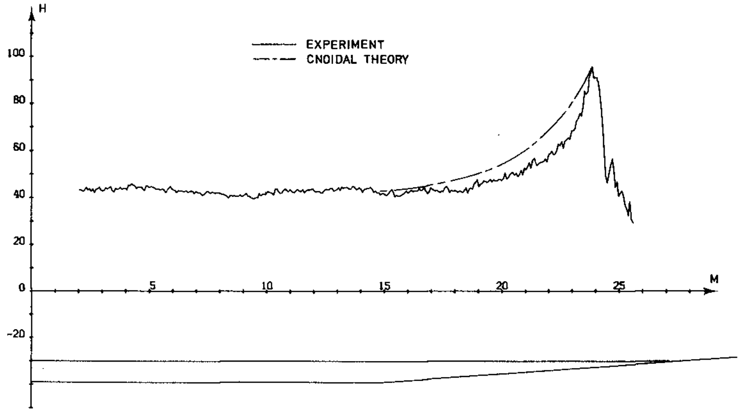

Fig. 3 Variation of wave height for 'long' waves with low steepness $T = 1/.3$ s, $H_0/L_0 = .0019$

Table 1 includes the basic dimensionless parameters for the wave conditions analysed in this paper. In the following the waves will be characterized by their period, T, and deep water steepness H_0/L_0.

Period T(s)	H_0/L_0	Generated height H(mm)	At h ≈ 360 mm U	S	$T\sqrt{g/h}$	At breaking ($\sim (H/h)_{max}$) h(mm)	U	S	$T\sqrt{g/h}$
1/.3	.0019	40	32.2	.50	17.4	89	1800	1.30	35.0
1/.4	.0038	40	17.5	.37	13.1	85	1000	.93	26.9
1/.4	.0064	71	30.6	.37	13.1	129	650	.74	21.8
1/.5	.0060	38	10.7	.28	10.4	69	890	.92	23.8
1/.5	.0102	67	18.7	.28	10.4	112	520	.66	18.7
1/.6	.0093	38	7.0	.22	8.7	64	630	.73	20.6
1/.6	.0167	69	12.3	.23	8.7	107	300	.54	16.0
1/.6	.0241	99	17.5	.23	8.7	149	220	.45	13.5
1	.0279	40	1.8	.12	5.2	56	205	.45	13.2

Table 1 Basic data for analysed waves

3. SURFACE PROFILES ON SLOPING BOTTOM

The deformation of waves passing over a sloping bottom is evaluated theoretically by Svendsen and Buhr Hansen (1978). Their theoretical result is based on a two-scale expansion of the cnoidal theory in which it is essential that the bottom slope is so small that the shoaling condition is fulfilled, $S << (h/L)^2$, but still so steep that the deformation (skewness) due to the slope is an order of magnitude greater than the second order terms of the constant depth solution, $S >> (h/L)^4$. Simultaneously H/h must be sufficiently small. Under these assumptions the surface profile is given by

$$\eta = \eta^{(0)} + \eta^{(1)} \tag{2}$$

where $\eta^{(0)}$ is the constant depth, first order cnoidal solution

$$\eta^{(0)} = \eta_{min} + H\, cn^2(\theta,m) \tag{3}$$

and $\eta^{(1)}$ is the skewness term given by

$$\eta^{(1)}/H = 3\,(L/h)^2\, h_x\, T\sqrt{g/h}\; G(\theta,U) \tag{4}$$

where g is the acceleration of gravity, and the function $G(\theta,U)$ is given in Fig. 4.

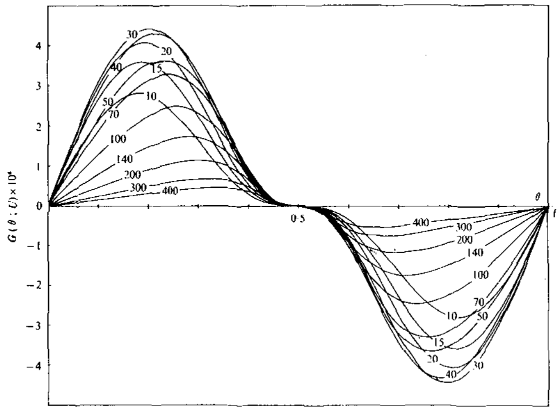

Fig. 4
$G(\theta,U)$ as function of phase angle. Each curve marked with the value of $U = HL^2/h^3$
(From Svendsen and Buhr Hansen, 1978)

Recorded surface profiles: In analysing the experimental results, the total recorded η is assumed to be a sum of two contributions as given by (2). Knowing the basic features of each of the two terms, $\eta^{(0)}$ and $\eta^{(1)}$ may be separated through

$$\eta_\theta^{(0)} = \frac{1}{2}(\eta_\theta + \eta_{-\theta}) \quad \text{and} \quad \eta_\theta^{(1)} = \frac{1}{2}(\eta_\theta - \eta_{-\theta}) \qquad (5) \text{ and } (6)$$

with $\theta = 0$ at the wave crest.

A direct comparison between the measured and the theoretical profiles as well as the $\eta^{(0)}$ and $\eta^{(1)}$ are given in Fig. 5. The theoretical profiles are computed using the recorded horizontal-bottom-wave height $H^{(0)}$, determined from the $\eta^{(0)}$ profile, and the actual h as input values to the computations.

The four profiles in Fig. 5 are for two different wave conditions, and for each of these are shown the recorded profiles halfway between the toe of the slope and the point of breaking and the last profile recorded before the point of breaking. The recorded $\eta^{(0)}$-profiles are

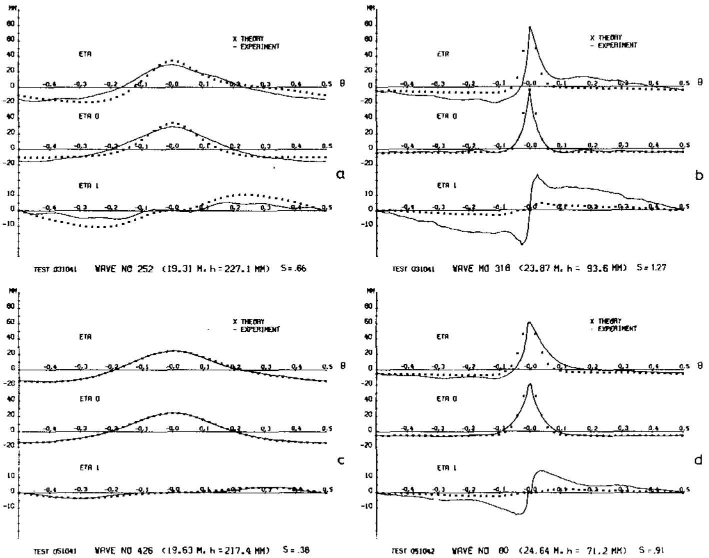

Fig. 5 Surface profiles on sloping bottom. Measurements compared with cnoidal theory. a) and b) T = 1/.3 s, H_0/L_0 = .0019; c) and d) T = 1/.5 s, H_0/L_0 = .0060

seen to agree fairly well with the theoretical profiles even for relatively high H/h values. On the other hand, the observed skewness of the profiles close to breaking is obviously not predicted by the theory. This might, of course, be expected since neither the slope parameter S nor the H/h ratio are within the limits of the theoretical approximation.

Analysis of observed wave skewness: The figures 6 through 9 show the variation of the maximum relative skewness $\eta^{(1)}_{max}/H^{(0)}$ (abbreviated to MRS) with $H^{(0)}/h$ for six different wave conditions. The characteristic feature of the theoretical variation with H/h showing a maximum MRS is clearly reproduced in the tests with waves of low deep water steepness, Figs. 6,7,8 and 9 a. Only in case of the shortest of the waves (Fig. 9 a) is the maximum MRS observed at the same H/h as predicted by the theory. When the wave period is increased (Figs. 8,7 and 6) the maximum MRS should according to the theory be found at decreasing H/h ratios, but this is not observed in the experiments.

Inspection of the figures further shows that the development of the skewness from the toe of the slope is as predicted by the theory in Fig. 8, and with a reasonable accuracy in Figs. 7 and 9 a. In Fig. 6 the skewness is obviously developing gradually over a considerable range of $H^{(0)}/h$ from the toe of the slope, indicating - presumably - that the slope is too steep for this long wave to adjust itself to the local water depth instantaneously as assumed in the theory. This further supports the observation in Fig. 3 indicating that the wave height

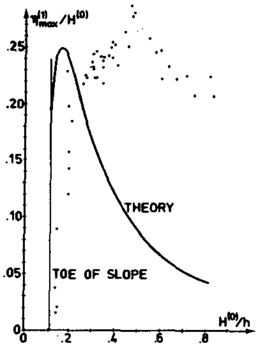

Fig. 6 Max. relative skewness T = 1/.3 s, H_0/L_0 = .0019

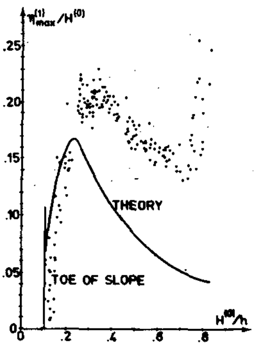

Fig. 7 Max. relative skewness T = 1/.4 s, H_0/L_0 = .0038

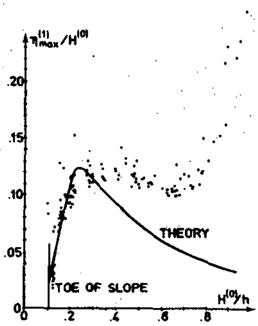

Fig. 8 Max. relative skewness T = 1/.5 s, H_0/L_0 = .0060

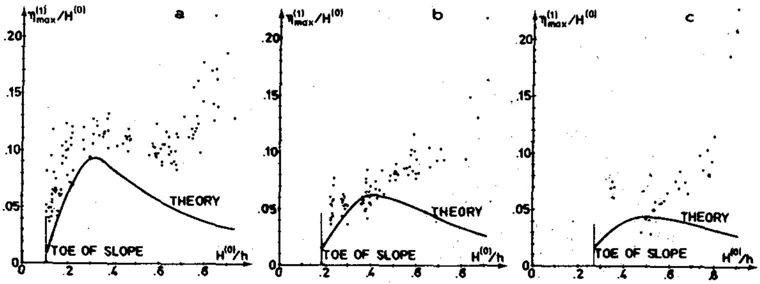

Fig. 9 Maximum relative skewness for waves with T = 1/.6 s
a) H_0/L_0 = .0093, b) H_0/L_0 = .0167, c) H_0/L_0 = .0241

does not vary as predicted by cnoidal wave shoaling. In the case of relatively short waves with greater steepness, Fig. 9 b and c, the characteristic feature of the theoretical MRS variation with a maximum and subsequent decrease of MRS for increasing $H^{(0)}/h$ is not observed in the tests. However, if not only the maximum recorded $\eta^{(1)}$ - which may obviously be greatly influenced by small disturbances of the waves recorded - but the entire $\eta^{(1)}$ profile is included in this analysis, the maximum and subsequent decrease of relative skewness is hardly observed in any of the tests. In Fig. 10 is shown the variations of $\mathrm{RMS}\left(\eta^{(1)}\right)/\mathrm{RMS}\left(\eta^{(0)}\right)$ for the same tests as reported in Figs. 6, 8 and 9 a. For the not too long waves the skewness development up to the theoretical maximum is still seen to be reproduced in the experiments, but the subsequent decrease is hardly noticeable. This may indicate that the shape of the recorded $\eta^{(1)}$ profile deviates more from the theoretical shape than the observed maximum value alone, which may be due to the presence of small reflected waves and parasitic waves. For two tests one measure of the shape of the $\eta^{(1)}$ profile is compared with the theory. In Fig. 11 is plotted $\mathrm{MS}\{\eta^{(1)}/\eta^{(1)}_{max}\}$ versus $U\,(= H^{(0)}L^2/h^3)$.

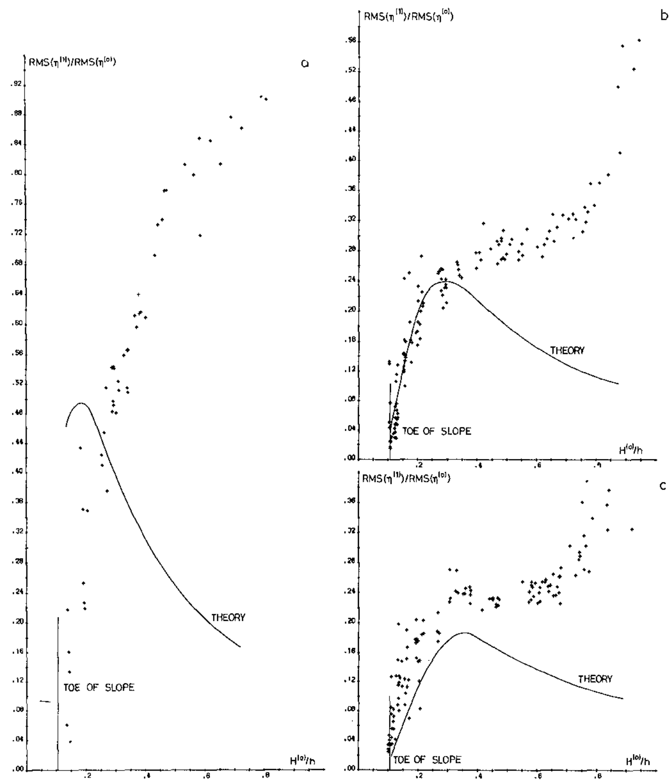

Fig. 10 Relative skewness in terms of $\left[\overline{\eta^{(1)2}}/\overline{\eta^{(0)2}}\right]^{1/2}$
a) $T = 1/.3$, $H_0/L_0 = .0019$; b) $T = 1/.5$, $H_0/L_0 = .0060$
c) $T = 1/.6$, $H_0/L_0 = .0093$

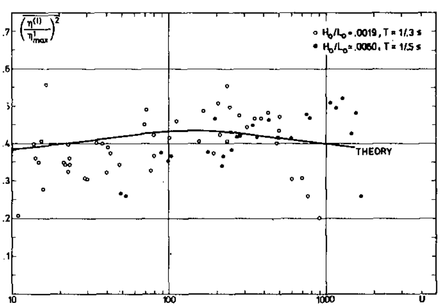

Fig. 11
Shape of $\eta^{(1)}$ profile
compared with theory

The measurements show a considerable scatter around the theoretical curve calculated from the $G(U,\theta)$ function given in Fig. 4, but the general trend of the theory is reproduced in the tests.

Having determined the $\eta^{(1)}$ profile from the experiments, it is from eq. (4) possible to compare an experimentally determined G function with the theoretical function. In Fig. 12 this comparison is shown in terms of RMS (G), indicating that the general trend is the same in theory and experiments, but also that for $U > 100$ all experiments show an experimental value above the theoretical one, and with a constantly increasing ratio between the measured and the theoretical values for increasing U. It is further observed that for $U > 100$ the experiments yield higher values for higher deep water wave steepness. Both these observations indicate the increasing importance of $H^{(0)}/h$, which in this region cannot really be regarded as small.

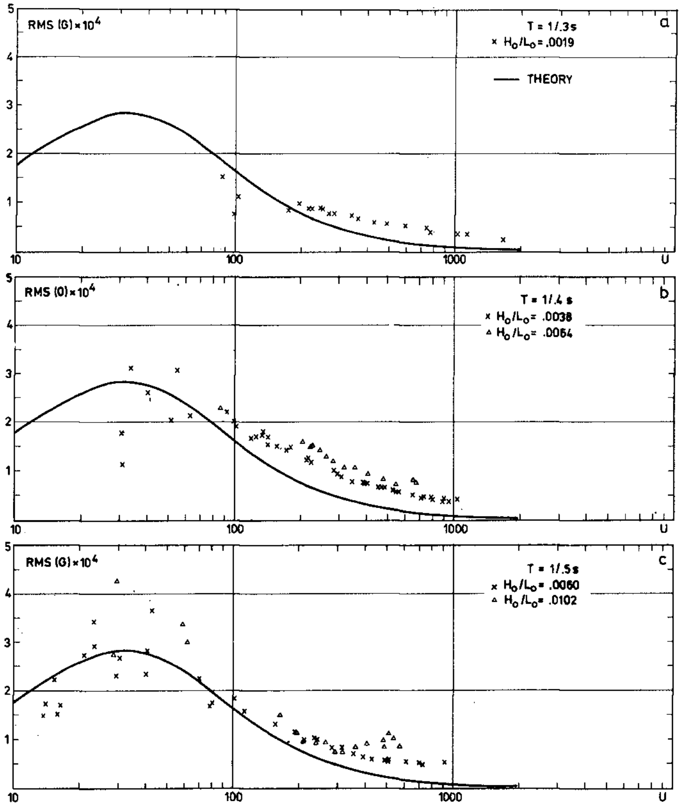

Fig. 12 Skewness function G. Comparison between theory and experiments

It is interesting to observe that the experimental G-values continue to decrease when the waves approach breaking, indicating the possibility of establishing a higher order theory valid for $H < h$ rather than the present theory requiring $H << h$.

Waves close to breaking: The skewness is seen to increase rapidly just prior to breaking (see Figs. 6 through 9). This might indicate the commencement of the irreversible deformation of the waves eventually leading to the actual breaking. For those tests, where the MRS shows a clear maximum and subsequent minimum for increasing $H^{(0)}/h$ (e.g. Figs. 6,7,8 and 9 a), the minimum MRS is observed at an $H^{(0)}/h$ which is very close to the theoretical H/h for the highest stable wave at horizontal bottom according to Ridler (1979), see Fig. 13. This may be taken as support of the idea that this indicates the commencement of the breaking process. Fig. 13 also includes the total wave height to water depth ratio, H/h, at the observed minimum MRS and the $(H/h)_b$ at the breaking point defined as max (H/h). Close to the breaking point

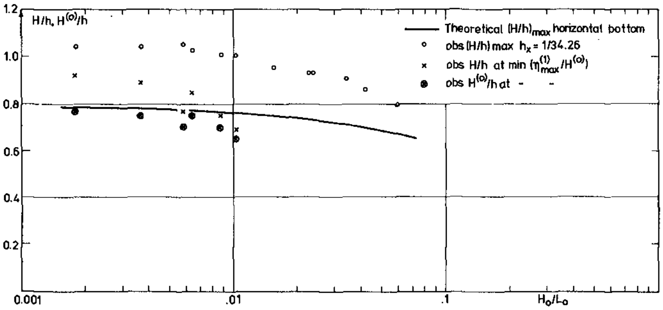

Fig. 13 Theoretical and observed $(H/h)_{max}$

the skewness yields a significant change in the wave height recorded, which is not accounted for in the theory. For waves with very low deep water steepness - which in the tests are at the same time 'long' waves - the total wave height may be more than 20% higher than the horizontal bottom wave height, $H^{(0)}$. For the 'shorter' waves with higher deep water steepness the total wave height is only 5 - 10% higher than $H^{(0)}$. This may again support the previous observations indicating incomplete shoaling conditions for the long waves of periods 1/.3 and 1/.4 s, since the $H^{(0)}$ variation if plotted in Fig. 3 would indicate greater discrepancy from the theory than the plotted H variation.

4. ENERGY FLUX FOR WAVES ON SLOPING BOTTOM

One of the basic assumption in wave shoaling is that the wave energy flux through a vertical section remains constant when the water depth is changing.

The basic expression for the energy flux, E_f, as function of time, t, is

$$E_f(t) = \int_{-h}^{\eta} (p^+ + \frac{1}{2} \rho (u^2 + W^2)) \ u \ dz \qquad (7)$$

where p^+ is the excess pressure due to the wave motion, u and w are the horizontal and vertical particle velocities, and ρ the density of the water.

When (7) is evaluated to the second order in η/h and averaged over a wave period, it reduces to

$$E_f = \rho\, g\, h^2\, c \left\{ \overline{\left(\frac{\eta}{h}\right)^2} + \frac{1}{2}\, \overline{\left(\frac{\eta}{h}\right)\left(\frac{\eta}{h+\eta}\right)^2} + \frac{1}{3}\, \overline{\eta\, \eta_{xx}} \right\} \tag{8}$$

Using the KdV equation, the last term may be expressed in terms of η

$$\frac{1}{3}\, \overline{\eta\, \eta_{xx}} = 2\left(\frac{c}{c_0} - 1\right) \overline{\left(\frac{\eta}{h}\right)^2} - \frac{3}{2}\, \overline{\left(\frac{\eta}{h}\right)^3} \tag{9}$$

where $c_0 = \sqrt{gh}$, and c is the cnoidal approximation to the phase velocity

$$c = c_0 \sqrt{1 + A\, H/h} \tag{10}$$

A = A(U) is given by the cnoidal theory, see e.g. Skovgaard et al. (1974). When further η is assumed to be a sum of the horizontal bottom profile, $\eta^{(0)}$, and a small skewness component, $\eta^{(1)}$, (8) may be written

$$E_f = \rho\, g\, H^2\, c\, B \tag{11}$$

with

$$B = \overline{\left(\frac{\eta^{(0)}}{H}\right)^2} + \overline{\left(\frac{\eta^{(1)}}{H}\right)^2} + 2\left(\frac{c}{c_0} - 1\right) \overline{\left(\frac{\eta^{(0)}}{H}\right)^2} - \overline{\left(\frac{\eta^{(0)}}{H}\right)^3}\, \frac{H}{h} \tag{12}$$

In B the first term is the first order horizontal bottom component

$$B_{01} = \overline{\left(\frac{\eta^{(0)}}{H^{(0)}}\right)^2} \tag{13}$$

which is a function of U only. Of the remaining three terms the first one is the skewness term, while the last two terms are the second order horizontal bottom terms. The direct influence of the skewness on the B value can be seen in Fig. 10 which shows directly the square root of the skewness term relative to the B_{01} term.

While the skewness term is obviously positive, the second order terms of the horizontal bottom profile will be negative for U < ∿ 390. This is seen when (10) is introduced in the last two terms in (12) yielding an overall negative result for

$$A < \frac{\overline{\eta^{(0)3}}}{\overline{\eta^{(0)2}}\, H} \tag{14}$$

(A is positive for U > 47 and a negligible influence of the second order horizontal bottom terms is observed for U > ∿ 250).

Experimental results for B_{01}: In Fig. 14 is shown the comparison between the theoretical B_{01} and the value obtained from the recorded $\eta^{(0)}$ profiles. (The theoretical wave length is used in computing U).

While the experiments with not too long waves and a low deep water steepness plotted in Fig. 14 b and one test in Fig. 14 c show a good agreement with theory up to U ∿ 700 (corresponding to $H^{(0)}/h \gtrsim .75$) the same agreement is not found neither in Fig. 14 a nor in the other two

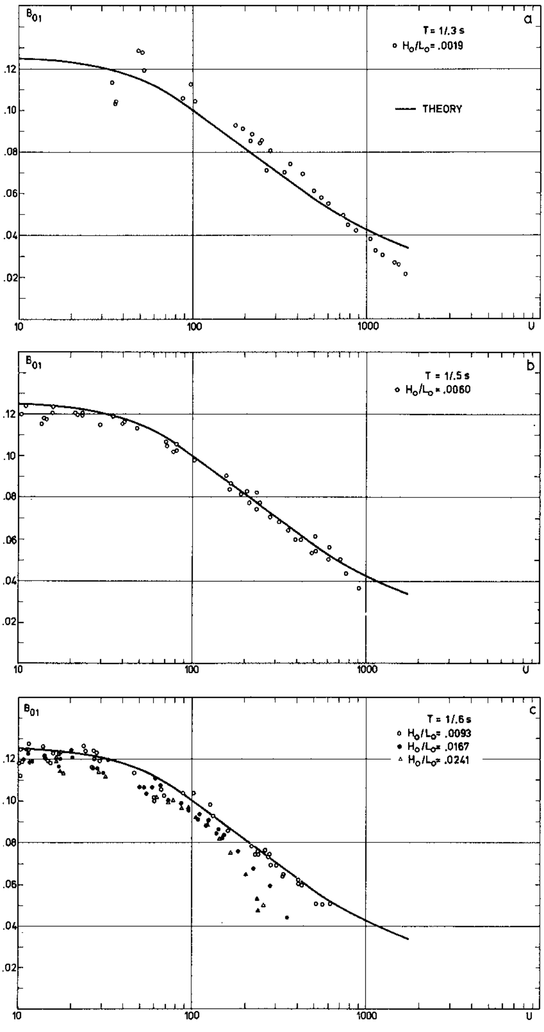

Fig. 14 Experimental values of $B_{01} = \overline{\eta^{(0)2}}/H^{(0)2}$ compared with cnoidal theory

tests included in Fig. 14 c. Fig. 14 a shows experimental values greater than the theoretical values, which is yet another indication of the incomplete shoaling of these long waves. Within $50 < U < 300$ the experimental values would agree with the theory if they had been recorded at a 15% greater water depth, indicating that the shape of the profile corresponds to a greater - earlier - water depth.

For waves with greater deep water steepness (Fig. 14 c) the experimental values are all below the theoretical curves. In this case only the change in wave height can explain the deviation from theory, indicating the importance of the $H^{(0)}/h$ ratio when this is not really small. For increasing wave steepness, a constant U will reflect increasing H/h ratios seen in Fig. 14 c to cause increasing discrepance between theory and experiments.

Experimental results for B: The combined influence of the skewness and the second order horizontal bottom terms on the energy flux may be analysed through analysis of B given by (12) compared with B_{01} given by (13).

The theoretical B is from (12), (3) and (4) seen to be a function of U, but at the same time dependent on $h_x T\sqrt{gh}$. Theoretically this causes B to be generally greater than B_{01} for long waves of low steepness, and smaller than B_{01} for shorter waves of considerable steepness.

In Fig. 15 is compared the experimental values for B with the theory for three experiments. For all experimental values are plotted two different B values based on both the theoretical and the recorded phase velocities.

The general trend of all three tests is that the combined skewness and second order influence on B is more pronounced in the experiments than predicted by the theory. This is especially pronounced in Fig. 15 a for the long waves with low steepness, and in this case the difference between measured and theoretical phase velocities is seen to be of minor importance.

As seen in previous comparisons, the test in Fig. 15 b shows a very close agreement with theory over the whole range of U values with measured and theoretical phase velocities plotting on either side of the theoretical curve. For the short, steep waves plotted in Fig. 15 c the trend is the same as in the comparison of B_{01} term in Fig. 14 c.

A comparison between measured and theoretical phase velocities is included in Svendsen and Buhr Hansen (1976 a) indicating a fair agreement with cnoidal theory. However, a closer analysis shows that for high values of H/h, the measured phase velocities are in general 5 -10% below the theoretical values.

Energy flux: Having determined B, the total energy flux may be computed from (11), now taking into account the combined effect of wave height, phase velocity and the shape of the surface profile. It should be noticed that (11) is valid in linear wave theory as well with

$$B = \frac{1}{16}(1 + G) \tag{15}$$

and

$$G = \frac{2\,kh}{\sinh\,2kh} \tag{16}$$

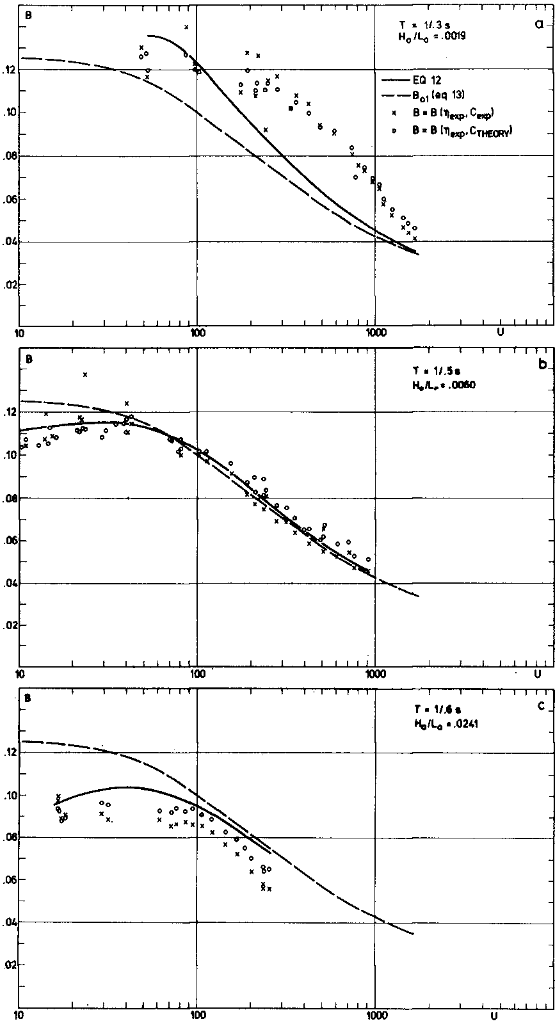

Fig. 15 Experimental values of B compared with cnoidal theory

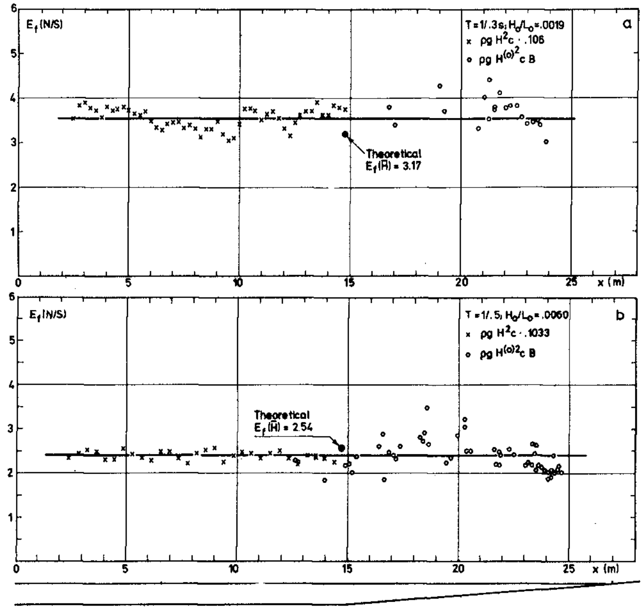

Fig. 16 E_f calculated from experimental results

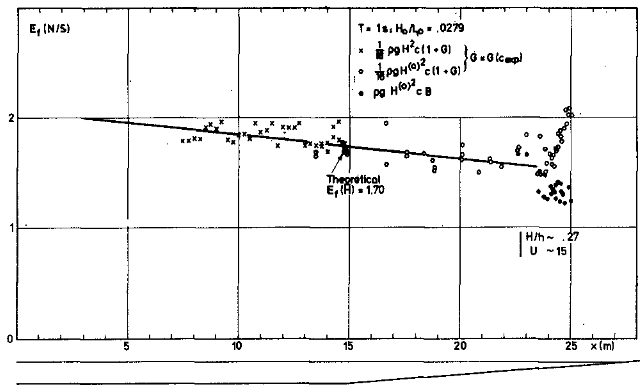

Fig. 17 E_f for short waves calculated from experimental results

The figures 16 and 17 show for three tests the energy flux determined from (11) using recorded data.

In front of the sloping bottom the recorded H and c values are applied together with an average B determined from the profile recordings available in this region. For the tests in Fig. 16 with rather long waves, B is determined from (12) with no skewness component. For the short waves dealt with in Fig. 17, B is determined from the linear theory (eqs. (15) and (16)).

Over the sloping bottom, on the other hand, $H^{(0)}$, the height of the horizontal bottom part of the profiles is used in determining E_f both in case of a cnoidal computation, Fig. 16, and in case of linear theory computations, Fig. 17.

The scatter about the - by eye placed - regression line is in both figures considerable, mainly due to variation in the recorded phase velocities. However, the two tests in Fig. 16 show some evidence of the presence of a small reflected wave, but with an overall constant energy flux. In Fig. 17 both cnoidal and linear computations of E_f are performed for $U > \sim 15$. As expected, the linear theory yields too high E_f values in this region, but the cnoidal theory on the other hand seems to underestimate E_f. The energy flux is in this case clearly decreasing by the distance from the wave generator (at x = 0). The rate of decrease corresponds to a rate of decrease in wave height of $\Delta H/H$ = .0054 pr. m, which agrees well with the direct measured attenuation of the wave heights, see Fig. 18. The same agreement is obviously not observed for the tests analysed in Fig. 16.

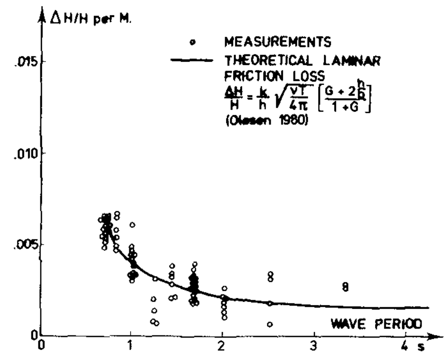

Fig. 18
Recorded attenuation of wave heights. Water depth 36 cm, flume width 60 cm
(Buhr Hansen and Svendsen, 1979)

5. WAVE SET-DOWN NEAR BREAKING

The last topic to be discussed in this paper is a comparison between recorded and theoretical wave set-down.

From linear theory the wave set-down is given by the well-known expression

$$b = -\frac{1}{16}\frac{H^2}{h}\frac{2\,kh}{\sinh\,2kh} \tag{17}$$

Close to breaking, however, the H/h ratio is not small, and usually the L/h is so large that (17) is a poor approximation, as indicated in both examples given in Fig. 19, where (17) is evaluated using the recorded $H^{(0)}$ and L (= cT) values.

The first approximation to b in cnoidal theory is (Svendsen, 1974)

$$b = -\frac{\overline{\eta^2}}{2\,h} \tag{18}$$

In Svendsen and Buhr Hansen (1976 b), (18) is shown to be as poor an approximation to the experimental results as the linear theory. This is confirmed in Fig. 19 where (18) is evaluated using the recorded η values.

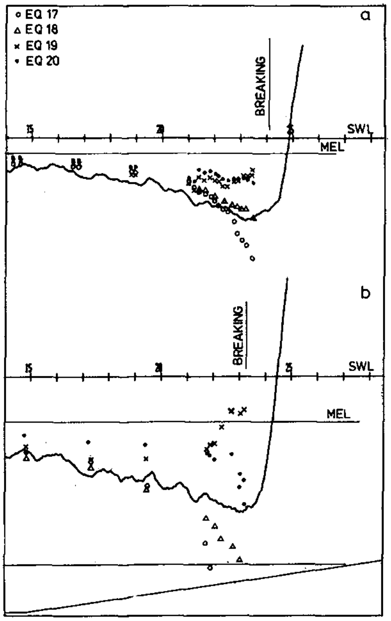

Fig. 19
Wave set-down
a) T = 1/.4 s, H_0/L_0 = .0038
b) T = 1/.5 s, H_0/L_0 = .0102

When taking into account the non-linearity of the bottom particle velocity and the vertical accelerations, Svendsen and Buhr Hansen (1976 b) obtain the following approximation to b in cnoidal theory

$$b = -\frac{c^2}{2\,g}\left[\left(2\,\frac{c}{\sqrt{gh}} - 1\right)\frac{\overline{\eta^2}}{h^2} - \frac{\eta}{2}\,\frac{\overline{\eta^3}}{h^3}\right] \tag{19}$$

When (19) is evaluated from the recorded η and c values, this is in Fig. 19 seen to underestimate the set-down close to breaking where (17) and (18) overestimate b.

Svendsen and Buhr Hansen (1976 b) develop (19) further through certain approximations. (The approximation op. cit. eq. (11) stated to be 'correct within a few per cent for $0 < \eta/h < 1$' is, however, only correct within 7% for $\eta/h > .4$. For $\eta/h = .1$ it is incorrect by 35% yielding a severe underestimate of the wave set-down at the toe of the slope in the present experiments, where (17) and (19) yield almost identical results.) The final equation given by Svendsen and Buhr Hansen (1976 b, eq. (14)) based on the above mentioned approximation is

$$b = -\frac{c^2}{2g}\left[\left(\frac{c^2}{gh} - .35\right)\overline{\frac{\eta^2}{h^2}} - 1.9\,\overline{\frac{\eta^3}{h^3}}\right] \tag{20}$$

In Fig. 19 this is to some extend seen to reflect the shape of the measured set-down but at a different elevation, when (20) is evaluated from recorded η and c values.

Compared to the observations in the previous chapters of this paper it is remarkable to observe that, while the surface profiles and the energy flux seem to be reasonably well predicted by the cnoidal theory, this is certainly not the case for the set-down.

6. REFERENCES

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