STATE OF THE ART IN TIDE PREDICTIONS

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ABSTRACT

1. INTRODUCTION

By the end of the nineteenth century, scientists had succeeded in achieving the ability to make reasonably accurate tide predictions by the harmonic method (Schureman, 1941). Except for the building of larger and more sophisticated mechanical tide prediction machines using the harmonic technique, tidal mathematicians more or less rested on their laurels during the first half of the twentieth century; indeed, many scientists assumed there was no need for further tidal research. It is ironic that one of the very few geophysical sciences that already had acceptable methods of prediction should become the subject of significant improvement during the last half of this century. These improvements include: 1) least square analysis for all tidal constituents simultaneously, 2) response analysis and prediction, 3) extended harmonic analysis, 4) tidal measurements in deep water on the ocean floor, and 5) global numerical models of tides.

2. AIMS AND OBJECTIVES

The harmonic method of tide prediction is purely empirical, using predetermined tidal frequencies related to known astronomical and meteorological periods. It separates an observed time series of water level heights into a set of cosine curves, and then predicts by synthesizing the same curves for a future period. There is little geophysical enlightenment implicit in the procedure. Extended harmonic analysis increases the number of frequencies being considered; the previous limit was related to the use of a mechanical prediction machine, no longer a consideration since the advent of modern electronic computers. Although response analysis improves the accuracy of tidal predictions only slightly, it greatly enhances geophysical interpretation of the results, for the first time separating astronomical and radiational (meteorological) contributions. Pelagic (sea floor) tide data are necessary for global interpretations of tides, particularly for numerical modelers who combine sea-floor bathymetry with Laplace's tidal equations to produce cotidal and co-range charts for particular frequencies.

3. TECHNIQUES

To a large extent, most of the progress noted here has been made possible by the availability of fast electronic computers with large memory capacities.

3.1 Least square analysis.

Analysis of tidal data for n tidal frequencies requires a matrix solution of $(2n+1)$ normal equations, an impossible task 25 years ago, almost trivial today. This has become the usual harmonic method for a long series at many tidal institutions, (Zetler *et al.*, 1965).

3.2 Response analysis.

This uses as input functions the time-variable spherical harmonics of the gravitational potential and of radiant flux on the earth's surface; it computes complex weights for an observed series that are used for future predictions and for computing admittances, (Munk and Cartwright, 1966). At this time, it is primarily a research tool used to optimize geophysical interpretations.

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3.3 Extended harmonic analysis.

This is an extension of (3.1) using many more tidal frequencies for extremely nonlinear tides, (Zetler and Cummings, 1967). If the matrices become too large, it is satisfactory to group constituents by species rather than solving for all simultaneously.

3.4 Pelagic tides.

The ability to accurately measure tides in deep water on the sea floor is a development of the last two decades. A recently published global compendium of tidal harmonic constants for pelagic stations contains information for more than a hundred places, (Cartwright *et al.*, 1979).

3.5 Numerical modeling of tides.

Solutions of Laplace's tidal equations now include energy dissipation, ocean shelf-attraction and the solid earth yielding to the weight of the oceanic tidal column. Some solutions for the principal tidal frequencies are reasonably accurate, (Hendershott, 1973).

4. RESULTS

The methods described in the previous section are to a large extent completed research and the application of these improvements depends largely on economic considerations.

5. RECOMMENDATIONS

The significant improvements described follow a half-century of depending on a methodology achieved by the end of the nineteenth century. Inasmuch as the various traditional tide manuals have not been updated to encompass these developments, this paper may be a useful interim guide to the changes.

INTRODUCTION

In populated areas where tides are important, local predictions have been published in newspapers for as long as living people remember; their availability is taken as much for granted as times of sunrise and sunset. For the most part, their accuracy is also unquestioned, although the public is aware that during storms and hurricanes "tides" much higher, and occasionally lower, than normal (i.e., predicted) may occur.

Although we know that tides result primarily from the tide-producing forces of the moon and sun acting upon the rotating earth, our knowledge of the dynamics of ocean and estuarine tides needs significant improvement; empirical procedures developed in the nineteenth century make it possible to prepare reasonably good tide predictions for any location provided that tide observations have obtained at that place in the past and the local sea-floor topography remains unchanged. It is a tribute to many scientists, in particular Sir William Thomson (later Lord Kelvin) and George Howard Darwin in England and William Ferrel and Rollin A. Harris in the United States, that the state of the art in tide predictions reached so high a plateau in the early twentieth century that significant improvements were not achieved for a half-century.

Although the Coast and Geodetic Survey (now called the National Ocean Survey) celebrated its centenary of published tide predictions in 1968, the Coast Survey had been supplying navigators with information from which they could prepare their own predictions as far back as 1844. This was done by including mean high-water lunitidal intervals (the interval between the moon's upper or lower transit over the local meridian and the following high water) and tidal ranges on the Survey's nautical charts. A published method for improving the crude predictions (presumably derived from John Lubbock's research in England in the 1830's) provided corrections for the effects of the phase of the moon and the declination and parallax of the moon and sun. However, the correlation between some tidal and lunar parameters were known and used long before the time of Newton.

Later in the nineteenth century, Thomson and Darwin developed the harmonic method of analyzing and predicting tides. The method uses sinusoids (tidal constituents) whose frequencies, derived from astronomic observations, are sums and differences of six basic frequencies in the motions of the earth, moon, and sun: the day, month, year, 8.9 years (lunar perigee), 18.6 years (lunar nodes), and 21,000 years (solar perigee). The amplitude and phase lag behind a theoretical (equilibrium) tide for each constituent are obtained by analysis; a prediction synthesizes the same curves for same future period. The distortion of tides as they move into shallow water is simulated by analyzing for frequencies that are harmonics of the fundamental frequencies or combinations of these.

Sir William Thomson made the first tide-predicting machine in 1873 under the auspices of the British Association for the Advancement of Science; it summed 10 constituents and automatically traced a curve showing the predicted heights. This country's first mechanical tide prediction machine (19 constituents) was designed by William Ferrel of the Coast and Geodetic Survey. The machine, introduced in 1885, provided for times and heights of high and low tides to be read directly from the dial indicators but did not produce a plotted prediction. An improved mechanical predictor for 37 constituents was completed by R.A. Harris and E.G. Fischer of the Coast and Geodetic Survey in 1910. This machine was the first to compute simultaneously the height of the tide and the times of high and low waters, the machine automatically stopping to allow the information to be read from dials. It also produced a continuous plot with time marks for each hour, maximum and minimum. Although the 37 incommensurable curves summed by the machine range from one cycle per year to eight cycles per lunar day (24.8 h), no constituent accumulated a phase error greater than 2 degrees in a year's predictions, a remarkable achievement in designing and fabricating the gear system.

FIGURE 1. First U.S. tide-predicting machine, used 1885-1911.

FIGURE 2. Second U.S. tide-predicting machine, used 1912-1965.

For many North Sea locations, combinations of the customary shallow- water constituents cannot adequately reproduce the shape of the observed tides. Horn in Germany and Doodson in England developed nonharmonic procedures for empirically correcting predictions to improve the accuracy of published tables.

RECENT CHANGES

Rapid proliferation and improvements in electronic computers in the middle of this century finally ended the long status quo in the state of the art for tide predictions. Although the Coast and Geodetic Survey tide predictor withstood the first assault when predictions on an IBM 650 were found to take longer than on the half-century-old machine, its victory was short lived, and by 1965 it had become a museum piece. A similar change took place at about the same time in England, but Kelvin mechanical tide-predictors may still be in use in some countries.

There were advantages other than speed and efficiency in shifting to electronic computers for tide prediction. As long as the fixed gearing of the mechanical predictor was limited to a finite set of particular constituents, research demonstrating the importance of other constituents had a low priority. Although there were sentimental regrets at replacing such a fine instrument, it was a distinct relief no longer to be dependent on the one machine; it was known to be wearing out, but, even worse, there was a fear of sabotage, particularly during war years.

In addition to the direct application of electronic computers to tide predictions, computer availability made possible more detailed analyses (for example, studies of 50 years of hourly heights, roughly a half million values) and digital recording (thus enhancing the state of the art for tide gauges as well as making possible more tide observations by reducing the manpower needs in data processing). Soon thereafter, dramatic advances were achieved in numerical modeling of tides, permitting for the first time research into the physics of tide generation in the world's oceans. Some of these advances have already contributed to improved predictions, and others will do so in the future.

It seems logical to group these recent improvements as follows:

A. *Data Analysis*

1. Change in harmonic analysis procedure. The traditional analysis in the Coast and Geodetic Survey (now the National Ocean Survey) solves for each constituent separately by a procedure that tries to emulate what would be achieved by cutting the marigram (tide record) into constituent periods (for example, every 25.82 h for O_1, a principal lunar diurnal constituent), piling the cut sections vertically and averaging a mean curve for the constituent period through the pile (Schureman, 1941). By means of stencil overlays, hourly heights (solar time) are identified as constituent hours (with errors not exceeding 0.5 h and algebraically averaging near zero), and then a Fourier analysis for the constituent frequency and possibly some harmonics is made of the constituent hourly means. An augmenting factor is applied to the amplitude to correct for a small reduction caused by using solar rather than constituent hours. Because it is impossible to have a finite length of observations that is commensurable for all tidal frequencies, the length of record chosen for Fourier analysis minimizes but does not completely remove the effect of interfering constituents. This effect is removed to a large extent by a further process called "elimination." In theory, repetitive elimination would refine the results even more, but this was not found necessary.

It is far more direct to solve for all constituents simultaneously, obtaining the complex amplitudes of cosine curves for each constituent that minimize the residuals in a least-square sense. For n constituents, we solve for $2n + 1$ coefficients (including the mean term) and then obtain the harmonic constants (amplitude and phase lag for each constituent) by simple trigonometry. For 37 constituents, a 75×75 matrix could not be solved without very large computers, and hence this approach was unthinkable until recent times. The procedure has been found to be more accurate for one-year analysis than the traditional methods, and it is now used in the National Ocean Survey (NOS) in this country and by the tidal authority in the United Kingdom, the Bidston Observatory of the Institute of Oceanographic Sciences. The process does not require equally spaced data (in the time sense) and will work with data in random time, although the software is more complicated as each data point must be identified in time (Zetler *et al.*, 1965). Comparative tests of the traditional and least-squares analysis procedures for 29-day series (approximate synodic period for phase, parallax, and declination of the moon) disclosed no advantage in the newer method; therefore the classical procedure has been retained in NOS, but the procedure has been modernized (in particular, stencil summing has been done away with). In 29-day analysis only K_1 and O_1 and M_2, S_2, and N_2 can ordinarily be resolved for species 1 and 2 (1 and 2 cycles per day, respectively).* Because the classical

*Principal tidal constituents: K_1 is lunisolar diurnal, O_1 is lunar diurnal, M_2 and N_2 are lunar semidiurnal, and S_2 is solar semidiurnal.

method infers the effect of other disturbing constituents in the elimination process, this probably gives it an advantage over the straightforward least-square analysis for the five constituents that offsets the improvements implicit in the latter process.

2. Extended harmonic analysis. Once tidal predictions were no longer constrained to a finite set of constituents, the door was opened to improving shallow-water predictions by adding additional compound tides (integral sums and differences of principal constituents). Two independent studies used 114 constituents with frequencies up to 12 cycles per day to improve predictions for Anchorage (Alaska) and for the Thames estuary (Zetler and Cummings, 1967; Rossiter and Lennon, 1968). Solving for 114 constituents simultaneously would involve a 229 × 229 matrix; fortunately this is not necessary as it has been demonstrated that sidebands are important only within each species (thus K_1 sidebands do not affect M_2), and therefore the solution may be simplified by requiring only that any particular matrix include all constituents within a species. A.S. Franco (Instituto de Pesquisas Tecnologicas, Sao Paulo, Brazil) has recently developed another approach using a matrix of the Fourier coefficients within a species to solve for constituent harmonic constants including numerous compound tides.

3. Determination of the continuum. On occasion, Walter Munk has used the expression, "Noise exists everywhere but in textbooks on tides." He was referring to the tendency to look only at the tidal lines in the spectrum, disregarding the level of the continuum between the lines. In a plot of energy (or amplitude) versus frequency, the extent to which the value at a tidal line protrudes above the continuum is a measure of reliability of the tidal constants for the constituent. Over a period of a few years, Munk and various associates made a determined effort to evaluate the level of the continuum for a wide band of frequencies. Very sharp filters discriminating against tidal lines were used to establish the continuum between tidal species (Munk and Bullard 1963). Then it was demonstrated that if the noise level is sufficiently low, two tidal constituents can be resolved in a record of length shorter than their synodic period (Munk and Hasselmann, 1964). An extremely fine-resolution analysis in the low frequencies (0 to 0.75 cycle per day) showed the continuum decreasing monotonically in energy for higher frequencies and no significant peaks other than previously determined tidal constituents (Groves and Zetler, 1964). Finally cusps of energy were found between tidal groups (separations of 1 cycle per month) and even between tidal lines within a group (Munk *et al.*, 1965). The knowledge derived in these studies was particularly important in the development of response analysis and prediction (see below) and serves to establish a limit on the accuracy to which one may aspire for barotropic tide predictions in the open ocean.

4. Response analysis and prediction. For any linear system, a input function $\chi_m(t)$ and an output function $\chi_n(t)$ are related according to

$$\chi_n(t) = \int_{-\infty}^{\infty} \chi_m(t-\tau)\omega_{mn}(\tau)d\tau + \text{noise }(t) \ ,$$

where $\omega_{mn}(\tau)$ is the "impulse response" of the system, and its Fourier transform

$$Z_{mn}(f) = \int_{-\infty}^{\infty} \omega_{mn}(\tau)e^{-2\pi i f\tau}d\tau = R_{mn}(f)e^{i\phi_{mn}(f)}$$

is the system admittance (coherent output/input) at frequency f.

Response tidal analysis and prediction (Munk and Cartwright, 1966) uses as input functions the time-variable spherical harmonics of the gravitational potential and of radiant flux† on the earth's surface. In practice, the integrals are replaced by summations; χ_m, ω, and Z are complex. The discrete set of ω values are termed response weights.

This method is the first successful major departure from the traditional solutions in which the tide oscillations are described by the amplitudes and phase lags for a finite set of predeter-

†A function designed to vary with the radiant energy falling on a unit surface in a unit time; it is related to daily atmospheric pressure and wind variations and to seasonal changes in ocean temperature.

mined frequencies. Subsequently it was recommended that response analysis of short records (about one month) of pelagic tidal measurements use a response prediction at a nearby coastal station as the reference series (Cartwright *et al.*, 1969). The calculation of traditional harmonic constants from response admittances made results of the two methods compatible (Zetler *et al.*, 1969). The optimum number of weights in response analysis depends directly on the length of record and inversely on noise level in a tidal band; more weights degrade the prediction and generate an artificial wiggliness in the admittance (Zetler and Munk, 1975). This study showed for the first time that better results are obtained by centering the lags to a potential retarded by the age of the tide‡ rather than to a potential centered on the prediction time.

SCOR* Working Group 27, "Tides of the Open Sea," conducted an analysis workshop in conjunction with an intercomparison of open sea tidal pressure sensors. The report (Unesco, 1975) shows a clear superiority for response procedures as compared with classical methods used by various national tidal groups. In addition to this statistical advantage, response analysis is more intellectually pleasing in that one uses the entire tide-producing potential rather than having arbitrarily to choose a finite set of tidal frequencies.

The ability to separate gravitational and radiational contributions to the tide resolves an unsatisfactory aspect of results from classical analysis. If one plots the phase angles or the amplitude admittances (ratio of analyzed to equilibrium amplitudes) from traditional analysis against frequency, one usually finds a sharp bend or even a discontinuity at S_2 (30°/solar hour). It is not reasonable that the ocean oceans should exhibit such abrupt changes, particularly always at the same frequency. The smoothness of the plots obtained for the gravitational admittances in Figure 3 (Zetler and Munk, 1975) using response procedures is undoubtedly due to the separation of radiational from gravitational inputs, whereas classical analysis produces combinations of the two.

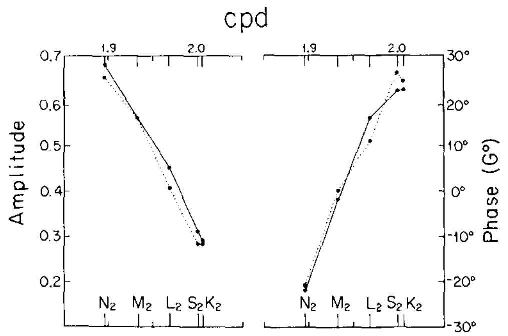

FIGURE 3. Amplitude and phase admittances for Bermuda referred to the gravitational potential. The solid lines are grav admittances from (grav and rad) response analysis; the dotted lines are admittances from traditional analysis. The response analysis was for three 355-day series over a 9-year period. The traditional analysis (1 year, 1934, IHB Spec. Pub. 26, #600) furnishes a lumped (vector) sum of grav and rad admittances.

‡The time interval between a maximum range in the equilibrium tide and a comparable range at a particular place. For example, equilibrium spring tides occur at new and full moon; in the ocean they occur 0.984 $(S_2°-M_2°)$ hours later. There are comparable ages for maximum ranges related to perigee and to maximum declination of the moon.

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B. *Numerical Modeling*

A cotidal chart is one showing an areal distribution of times of high tide for particular constituent relative to the equilibrium (theoretical) time of high tide for the same constituent. The cotidal lines, labeled in either solar or constituent hours, identify the locus of points at which high tide for the constituent occurs simultaneously. A feature of such maps is a system of nodal points, known as amphidromes, at which there is no vertical rise or fall. Some charts also include corange lines, contours of equal constituent amplitude. Near an amphidrome, corange lines tend to be concentric about the amphidrome with amplitudes increasing with distance from the zero amplitude point.

Historically, cotidal charts have been based on seaward extrapolation from coastal and island observations supplemented by general knowledge of how tides behave in mathematically described basins. Inasmuch as ocean tides are modified significantly in the continental coastal zones, use of coastal and, even worse, harbor and estuary observations make these ocean projections quite speculative. Nevertheless, until huge electronic computers became available, these empirical efforts were as good as could be done. In retrospect, some of the early cotidal charts have been found to be remarkably good and have served many useful purposes; those of Rollin Harris are outstanding examples of an advanced state of the art at the beginning of this century (Figure 4).

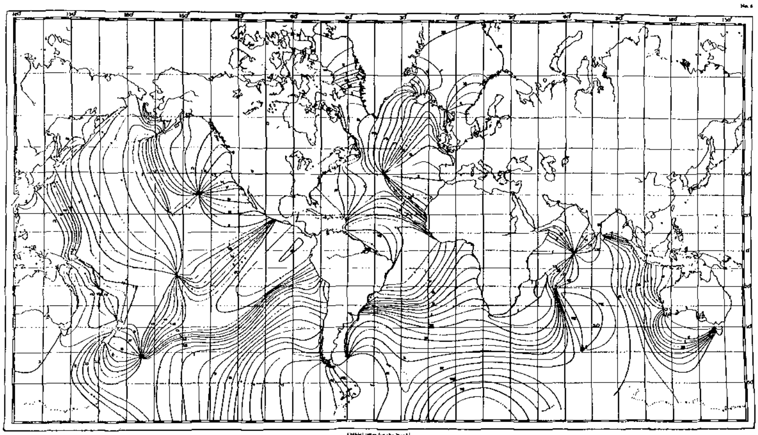

FIGURE 4. Harris cotidal chart for M_2.

In a parallel but quite different effort, hydrodynamicists have traditionally been concerned with solving the tidal equations developed by Laplace in 1775. Because of their complexities, various simplifying assumptions for hypothetical basins (such as flat bottom, boundaries, and stability) have been necessary (Hendershott and Munk, 1970). Although much has been learned in these earlier scientific efforts, the solutions have had little or nothing to do with tides in the real oceans. Realistic calculations require detailed bathymetry and boundaries, inconceivable before the advent of large electronic computers and marginal even today. When Pekeris presented a global solution for M_2 at an international meeting about 15 years ago, Joseph Proudman, a prominent tidal authority, commented that he had not anticipated seeing this degree of success in his lifetime.

A recent summary of the activities of SCOR Working Group 27, "Tides of the Open Sea" (Cartwright, 1975) noted various problems in numerical modeling of tides: "There are numerous esoteric sub-problems concerning stability, mesh size, boundary and depth topography, friction, but the most important and difficult appear to be representation of energy losses at the boundaries of shallow seas and at places of steep topography where internal tides are generated, and the solution of the equations modified for a yielding Earth."

Hendershott's more recent numerical studies include energy dissipation, ocean self-attraction, and the solid earth yielding to the weight of the oceanic tidal column. Figure 5 is a recent M_2 cotidal chart by Hendershott and Parke. A description of other contemporary numerical tidal solutions (Hendershott, 1973) furnishes information on the tidal frequencies considered, boundary conditions, dissipation mechanisms (if any), and whether earth tides are considered

Thus, theoretical studies are now using real data in realistic ocean models. Solutions are improving; in particular, it appears that an amphidrome first identified in a numerical study is real. However, results indicate that many ocean areas are nearly resonant at semidiurnal

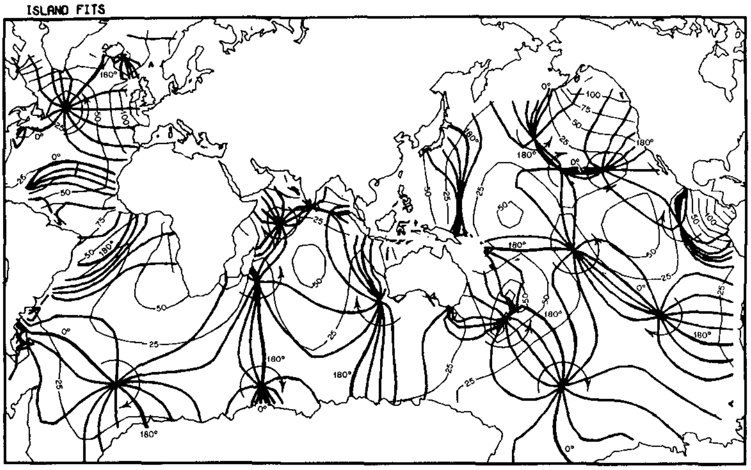

FIGURE 5. Hendershott and Parke cotidal and corange chart for M_2.

periods. As a result, a variation in mean depth of only several hundred meters may change the M_2 amplitude by a factor of 2 or 3. Furthermore, it has been found that changing the coastline slightly or refining the grid from a 2° to a 1° mesh changed the M_2 amplitude by as much as 3 m (Pekeris and Accad, 1969). This extreme sensitivity make it difficult to evaluate empirically the contribution of various parameters.

In the past, physical models of various estuaries have been built in order to determine the effect of proposed man-made structures on the tidal regime in the estuary. For example, the model of San Francisco Bay (built in Sausalito by the U. S. Army Corps of Engineers) has been used in studying the effect of proposed salinity barriers on the tide in the Bay. Garrett (1977) recently used a numerical model to study the effect of a proposed power-generating system in the Bay of Fundy, with the large ranges of tide as the energy source. Ordinarily one would expect such a utilization to diminish the tidal range and would need to consider the economic consequences of the anticipated change. However, Garrett found the resonant peak of the basin to be only slightly higher than the M_2 frequency (thus accounting for large tides) and that the proposed structure would reduce the resonant frequency, bringing it even closer to the M_2 frequency, and so would lead to very little, if any, reduction in amplitude.

C. Recording

Pelagic pressure sensors, capable of measuring the tide on the sea floor in the deep parts of the oceans are an important technological development in recent years (Unesco, 1975). Pioneered by Frank Snodgrass at Scripps Institution of Oceanography and Marc Eyries in France, the state of the art has rapidly advanced to the point that only financial considerations deter us from obtaining the global grid of tide observations needed for an optimum set of cotidal and corange charts. A recently published global compendium of tidal harmonic constant for pelagic stations contains information for more than a hundred places, (Cartwright *et al.*, 1979).

In the Unesco report on an international intercomparison of open-sea tidal-pressure sensors, five types of sensor were found to intercalibrate closely (Unesco, 1975). Among the several mooring techniques used in the experiment, the self-contained acoustic "pop-up" unit showed a clear superiority. In the MODE (Mid-Ocean Dynamics Experiment) exercise, tidal data were obtained for the first time from an array of pressure sensors on the sea floor. Figure 6 shows that the phase angles obtained for MODE stations match Dietrich's M_2 and K_1 cotidal charts well (Zetler *et al.*, 1975). The M_2 tidal currents inferred from the gradients of observed

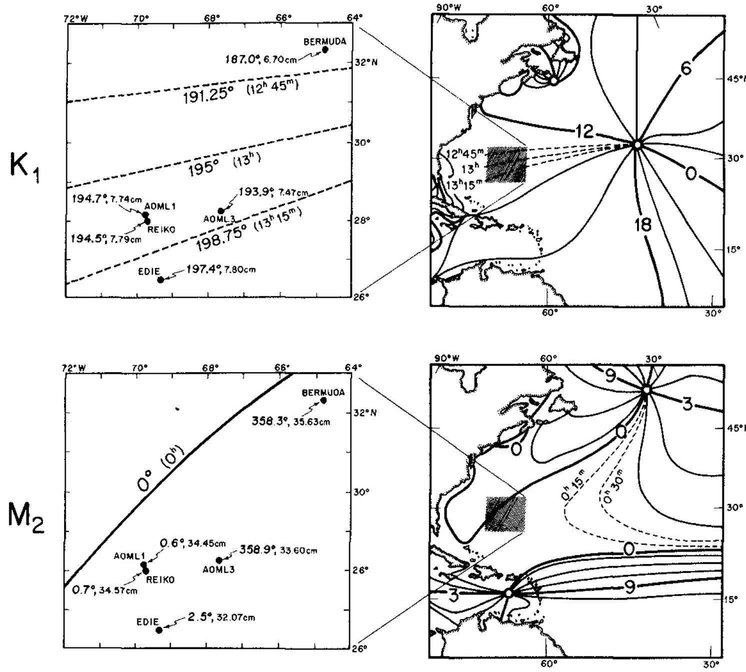

FIGURE 6. Right panels show Dietrich's (1944) cotidal lines in the North Atlantic for K_1 and M_2 tides, respectively. Values are in solar hours, with dashed curves designating interpolation by Zetler *et al.*, (1975). The MODE area falls within the shaded square, which is shown on an enlarged scale in the left panels, for comparison with the results at MODE stations (•).

pressures fit the barotropic component of the observed tidal currents reasonably well; the latter also contain large baroclinic components, whereas the small baroclinic contributions to the pressure gradients have been further reduced because the station spacings are roughly comparable to the baroclinic wavelength.

Numerical modelers have recommended observations at "antiamphidromes" (locations of maximum tidal amplitude) in order to calibrate their results. Furthermore, to work with intermediate scale models (portions of the world's oceans), because of the need for more dense topographic grids, they will need tidal observations at the boundaries of sections in the open ocean. The technology for this is now available, and some programs along these lines are already under way at the Institute of Oceanographic Sciences in England.

Time has passed by the 2000 or so conventional tide gauges used throughout the world; there have been few changes since Lord Kelvin argued the case for using a pencil before the Institution of Civil Engineers in 1882. The gauges continue to measure the height of a float in a stilling well (cylinder with a small orifice near the bottom to filter out wind waves). The only significant change has been the utilization of digital recording on magnetic and punched paper tape; these have significantly reduced tabulation and analysis costs, making it possible to obtain and process more observations with available resources.

CONCLUSION

Subsequent to the preparation of the abstract, a new publication (Zetler, 1982) describes modifications to the computing processes for tide analysis and prediction at the U.S. National Ocean Survey. This publication serves as a supplement to various National Ocean Survey manuals, in particular to Schureman (1941).

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