

## LABORATORY PHOTOGRAMMETRIC WAVE HEIGHT MEASUREMENT

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### ABSTRACT

The paper describes the experimental procedure used to produce a computer contour plot of the wave height distributions and wave directions in a model basin, using photogrammetric techniques. Only monochromatic waves are analysed. A technique is outlined to simulate and measure waves entering a basin of infinite extent, in other words to photograph the penetration of a wave train into a harbour basin before the pattern has been contaminated by reflections. Proof is offered that this infinite basin technique is a valid representation of the steady state situation of a continuous wave train entering an infinite basin.

### INTRODUCTION

Model harbours are used by coastal engineers as an aid in the optimization of harbour designs. The models predict wave heights in full scale harbours, the results being used to reduce these wave heights to a minimum in order to prevent damage to moored ships, to the wharf structure and to the mooring systems. Two main limitations in experimental procedures are encountered, however, when attempting to measure accurately the wave height distributions within model harbours; these are:-

- (1) Wave heights in model harbours are commonly measured using parallel wire resistance or capacitance wave probes. A number of these probes are usually mounted on a moveable instrument carriage which can traverse the wave basin to measure the wave heights. Such a configuration was used by Harms [3]. The disadvantage of this system is that the wave height at only a limited number of discrete locations can be measured at any one time. The system is also time consuming, since the instrument carriage has to be moved within the wave basin until the entire water surface has been measured. Furthermore, excessive spacing of these wave probes may result in points of maximum wave heights being overlooked.

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- (2) Wave measurements using the above system necessitate (in most cases) that the wave paddle must run continuously. This enables secondary effects, (such as wave reflections, basin oscillations, cross waves, etc.) to develop and distort the generated wave, thus causing marked anomalous wave height variations along the generated incident wave crests. These problems are discussed in depth by Harms [3].

The two abovementioned problems effectively prevent the achievement of accurate wave height and pattern measurement in model harbours. This paper describes research at U.C.T. directed towards solving these problems by using stereophotogrammetry. A photogrammetric technique is described which enables an instantaneous, synoptic and permanent three dimensional record to be obtained of the deformed water surface in a model harbour. A procedure is also described to achieve a computer generated contour plot of the experimental wave height distributions in a wave basin.

#### DEVELOPMENT OF THE TECHNIQUES USED

In some early attempts at using stereo-photography for the measurement of a deformed water surface, it was appreciated that in clear water it is difficult to distinguish surface features from bed details, thus some method was needed for making the water surface opaque. The substance used must not significantly affect the wave process under investigation, ideally it should also be cheap and non-contaminating with regard to laboratory installations. Surface applications such as aluminium powder and confetti were first tried and were found to be very inconvenient, then various solutions such as P.V.A. in water were tried but were found to settle out quite quickly. Eventually it was found that a low concentration of cutting oil in water, about 0.7 per cent, gave a suitably milky fluid which satisfied the above requirements.

It was found necessary to arrange for special illumination for the water surface and this was achieved by employing 4 conventional lecture room type overhead projectors placed 4m above the water surface in the plan positions shown in Fig. 1. However, when faced with the viewing and interpretation of stereopairs, it was found impossible to identify corresponding points in the photographs without introducing some optical contrast onto the water surface, and this was effected by placing transparencies in the projectors containing black geometrical shapes alternating with translucent areas. Initially, pattern geometry was used, for example concentric black rings, but this was abandoned because it was found that the pattern interfered with the viewer's three dimensional perception. Subsequently random assemblies of black stars and arrows were used, and this is now our standard practice. With regard to photography, two Zeiss Jena metric cameras were used at an elevation of about 5m above water surface and 1.9m apart. At first, the cameras, coupled to their synchronization device, were relied upon to give synchronized exposures of identical duration, but detailed examination of the camera performance showed that this was not reliable, and therefore the projectors were converted into flash units

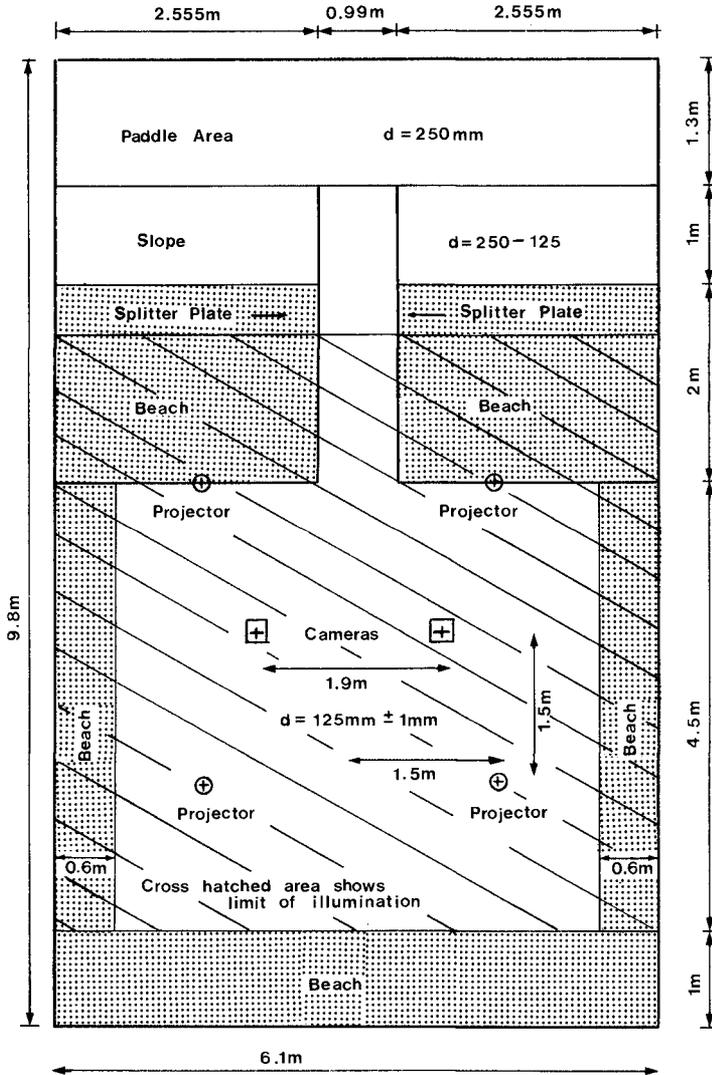


Fig.1 TYPICAL HARBOUR CONFIGURATION

by replacing the conventional bulbs with flash bulbs, placed at the focal points of the focusing lenses, and fired by a common capacitance. A shutter speed setting of 1/30 second was used, to allow both shutters to be open a sufficient time to encompass the flash duration (allowing for the variation in shutter speeds between the cameras and their lack of synchronisation) and to ensure that the illuminated control points were adequately exposed. All photography took place at night. Due to the very short flash duration, of less than a millisecond, the water surface detail was effectively "frozen" (i.e. very little image movement) in the stereopairs.

The next concern is to consider how the photographs are to be interpreted to yield information about wave heights at points in the field of view. Bearing in mind that a wave height is the vertical distance between a crest at a selected point and a subsequent (or preceding) trough at that point, it is clear that two stereopairs are needed, and these were arranged to be taken such that the waves imaged in the 2nd stereopair are  $180^\circ$  out of phase relative to the waves imaged in the 1st stereopair, making use of two electronic microswitches triggered by the wave paddle mechanism. Algebraic subtraction of the two resulting deformed water surfaces gave the wave heights at all points where a crest or trough appeared in either stereopair. In the earlier stages of the investigation a Zeiss (Jena) Topocart stereoplotter was used to analyse the photographic plates, and was found capable of measuring the wave heights with an average error of 2 mm. This method, however, requires the use of very expensive metric survey cameras in addition to the very expensive stereoplotter (plus a specially trained operator), and therefore a cheaper and slightly more accurate alternative approach was adopted. These earlier stages of the investigation are described in full by Adams and Pos [2].

It was decided that the water surface elevations would be measured using projective transformations. A brief description of the theory of projective transformations is given by Adams [1]. The advantages of this method are that the plates can be viewed by a relatively inexperienced operator using a less expensive Zeiss (Jena) stereocomparator, (i.e. less expensive than a stereoplotter). The water surface elevations are determined by microcomputer using the theory of projective transformations. This method is inherently more accurate than the previous one since the human error involved in the relative and absolute orientation of the stereoplotter is eliminated. Also the projective transformation calculations enable the inner orientation elements (principal distance and principal point) of the cameras to be determined for each stereopair. (Thus far cheaper non metric cameras can be used).

#### PROJECTIVE TRANSFORMATION THEORY

The theory and mathematical techniques employed are described by Adams [1] and Welham [4]. For ease of reference the appropriate formulae and their solutions are summarised below.

Calculation of Transformation Parameters

$$X_i b_{11} + Y_i b_{12} + Z_i b_{13} + b_{14} - x_i X_i b_{31} - x_i Y_i b_{32} - x_i Z_i b_{33} = x_i, \quad (1)$$

$$X_i b_{21} + Y_i b_{22} + Z_i b_{23} + b_{24} - y_i Y_i b_{31} - y_i Y_i b_{32} - y_i Z_i b_{33} = y_i, \quad (2)$$

where

- $X_i, Y_i, Z_i$  are free net space co-ordinates of point  $P_i$ ,
- $x_i, y_i$  are comparator image co-ordinates of point  $P_i$  referred to an arbitrary origin,
- $b_{ij}$  are transformation parameters.

In the solution of the transformation parameters, both the image and object co-ordinates are assumed to be error free and the best mean  $b_{ij}$  terms are determined using normal equations formed from quasi-observation equations as follows:

|          |          |          |          |          |          |          |          |            |            |            |          |
|----------|----------|----------|----------|----------|----------|----------|----------|------------|------------|------------|----------|
| $b_{11}$ | $b_{12}$ | $b_{13}$ | $b_{14}$ | $b_{21}$ | $b_{22}$ | $b_{23}$ | $b_{24}$ | $b_{31}$   | $b_{32}$   | $b_{33}$   | $= 1$    |
| $X_1$    | $Y_1$    | $Z_1$    | 1        | 0        | 0        | 0        | 0        | $-x_1 X_1$ | $-x_1 Y_1$ | $-x_1 Z_1$ | $x_1$    |
| $X_2$    | $Y_2$    | $Z_2$    | 1        | 0        | 0        | 0        | 0        | $-x_2 X_2$ | $-x_2 Y_2$ | $-x_2 Z_2$ | $x_2$    |
| $\cdot$    | $\cdot$    | $\cdot$    | $\cdot$  |
| $\cdot$    | $\cdot$    | $\cdot$    | $\cdot$  |
| $X_n$    | $Y_n$    | $Z_n$    | 1        | 0        | 0        | 0        | 0        | $-x_n X_n$ | $-x_n Y_n$ | $-x_n Z_n$ | $x_n$    |
| 0        | 0        | 0        | 0        | $X_1$    | $Y_1$    | $Z_1$    | 1        | $-y_1 X_1$ | $-y_1 Y_1$ | $-y_1 Z_1$ | $y_1$    |
| 0        | 0        | 0        | 0        | $X_2$    | $Y_2$    | $Z_2$    | 1        | $-y_2 X_2$ | $-y_2 Y_2$ | $-y_2 Z_2$ | $y_2$    |
| $\cdot$    | $\cdot$    | $\cdot$    | $\cdot$  |
| $\cdot$    | $\cdot$    | $\cdot$    | $\cdot$  |
| 0        | 0        | 0        | 0        | $X_n$    | $Y_n$    | $Z_n$    | 1        | $-y_n X_n$ | $-y_n Y_n$ | $-y_n Z_n$ | $y_n$    |
|          |          |          |          |          |          |          |          |            |            |            |          |
| A matrix |          |          |          |          |          |          |          |            |            |            | L matrix |

From these quasi-observation equations, the 'B' matrix can be solved as usual from  $B = (A^T A)^{-1} A^T L$ .

Most modern desk top or microcomputers can deal with a matrix inversion of this size (11 x 11) and the solution equations adopted take into account any redundant control points imaged and used for calculating the most likely  $b_{ij}$  values.

Calculation of Space Co-ordinates using Stereopair

$$\left. \begin{aligned} (x_1 b_{31} - b_{11})X_1 + (x_1 b_{32} - b_{12})Y_1 + (x_1 b_{33} - b_{13})Z_1 &= b_{14} - x_1 \\ (y_1 b_{31} - b_{21})X_1 + (y_1 b_{32} - b_{22})Y_1 + (y_1 b_{33} - b_{23})Z_1 &= b_{24} - y_1 \\ (\bar{x}_1 \bar{b}_{31} - \bar{b}_{11})\bar{X}_1 + (\bar{x}_1 \bar{b}_{32} - \bar{b}_{12})\bar{Y}_1 + (\bar{x}_1 \bar{b}_{33} - \bar{b}_{13})\bar{Z}_1 &= \bar{b}_{14} - \bar{x}_1 \\ (\bar{y}_1 \bar{b}_{31} - \bar{b}_{21})\bar{X}_1 + (\bar{y}_1 \bar{b}_{32} - \bar{b}_{22})\bar{Y}_1 + (\bar{y}_1 \bar{b}_{33} - \bar{b}_{23})\bar{Z}_1 &= \bar{b}_{24} - \bar{y}_1 \end{aligned} \right\} \quad (3)$$

where the unbarred elements refer to the left hand picture and the barred elements to the right hand picture. Let 'C' be the matrix of the left hand coefficients and 'L' the matrix of the right hand terms. The space co-ordinates of an image point (X, Y, Z) can be found from

$$\begin{pmatrix} x \\ y \\ x \end{pmatrix} = (C^T C)^{-1} C^T L.$$

It will be noted that the traditional elements of inner and relative orientation are not apparent, but these can be obtained from a knowledge of the  $b_{ij}$  elements if so desired.

Summarised, the appropriate inner orientation equations are:

Non-zero scalar  $\lambda$ :

$$\lambda^{-2} = b_{31}^2 + b_{32}^2 + b_{33}^2 \quad (4)$$

Principal point co-ordinates (comparator system):

$$x_p = (b_{11} b_{31} + b_{12} b_{32} + b_{13} b_{33}) \lambda^2, \quad (5)$$

$$y_p = (b_{21} b_{31} + b_{22} b_{32} + b_{23} b_{33}) \lambda^2, \quad (6)$$

Equivalent principal distance:

$$c_x^2 = (b_{11}^2 + b_{12}^2 + b_{13}^2) \lambda^2 - x_p^2, \quad (7)$$

$$c_y^2 = (b_{21}^2 + b_{22}^2 + b_{23}^2) \lambda^2 - y_p^2, \quad (8)$$

Use average of  $c_x$  and  $c_y$ .

Unless it is intended to plot from the stereopair using a stereoplottor, the inner orientation elements are of academic interest only and therefore only equations (1), (2) and (3) are relevant.

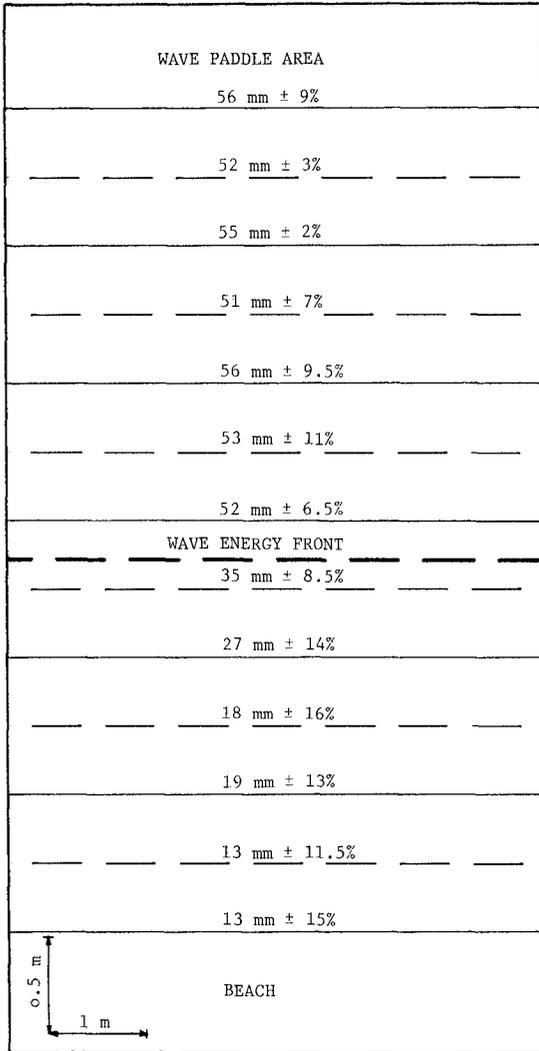


Fig.2 WAVE ENERGY FRONT

#### CONTROL POINT CONFIGURATION

Initially 12 control points were used (a minimum of 6 control points imaged in the overlap area of the stereopairs are required), 8 at levels above the water surface and 4 at levels below the wave surface (in open canisters). This was done since it was postulated that more accurate results would be obtained if the surface to be measured was straddled by control points, i.e. interpolation, rather than extrapolation as was the case for the first configuration (as described in [2]) which had 4 control points above the water surface. Stereopairs, using the infinite basin technique, were taken of the energy front of a plane long crested wave train entering an area of still water. On analysing the stereopairs it became obvious that the 280 mm maximum vertical distance between the two planes of control points was inadequate to determine the projective transformation parameters with sufficient accuracy. This problem was subsequently overcome by constructing an additional plane of 4 control points approximately 540 mm above the original upper control points.

To determine the XYZ positions of the 16 control points they were first levelled and then 48 inter control points measurements were taken using a steel tape. The corrected data was analysed using a least square adjustment program written for the Tektronix 4051 by Dr. H. R ther of the Survey Department, U.C.T., and an accuracy in X, Y of better than 1 mm was achieved. Research work done by Welham [4] indicates that the control configuration used was more than adequate for the accurate determination of the projective transformation parameters. Since photography took place at night each control point was individually illuminated.

#### THE INFINITE BASIN TECHNIQUE

An analysis of the steady state results coupled with visual observations of the deterioration of the wave field in the basin after a very short period of wave paddle action lead to the development and adoption of the infinite basin technique. It was observed that when the wave paddle was subsequently stopped and the main wave train had traversed the basin, a marked reflection and resonance mode was evident in the basin, which took a few minutes to dissipate. This reflection and resonance mode was obviously superimposed upon the wave field in the basin under steady state conditions.

It was hypothesized that it would be possible to simulate the steady state situation of a continuous wave train entering an infinite basin by sending a wave train into a model basin of still water and taking a stereopair of the water surface just as the wave energy front reached the peripheral beaches. To test this hypothesis, two stereopairs were taken of the wave energy front region of a typical wave train entering an open basin of initially still water. On analysis it was found that the wave heights of the crests immediately behind the wave energy front were closely equal to the mean wave height between the front and the generator. The experimental configuration analysed and the results obtained is shown in fig. 2. A wave train of period  $T=0.74$  seconds was propagated in a water depth  $d=138$  mm. The crestlines observed in the 1st stereopair are shown as solid lines, while the trough lines

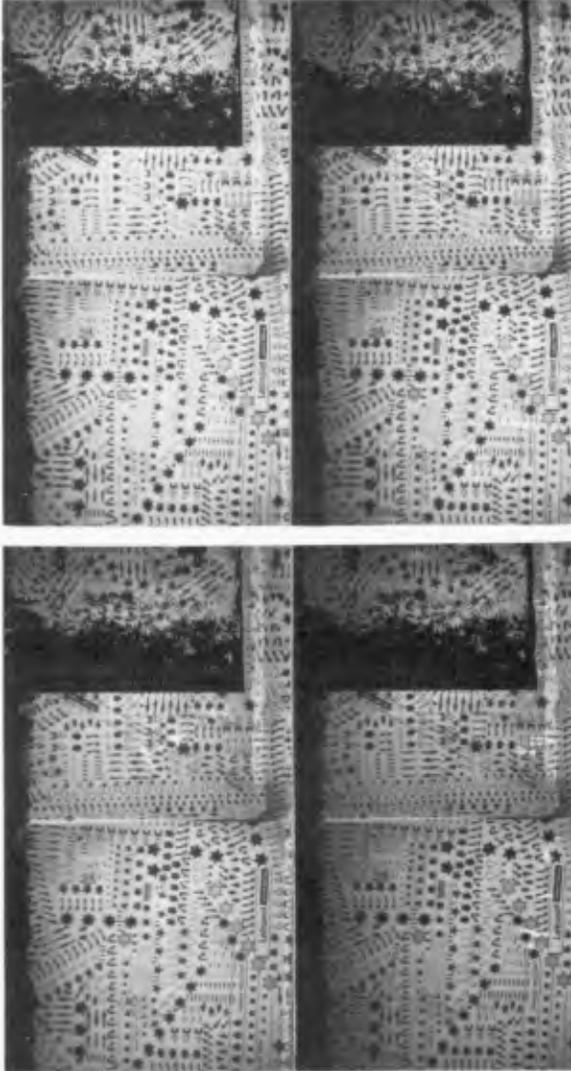


FIG.3 STEREO PAIRS TAKEN OF MODEL HARBOUR CONFIGURATION

observed in the 1st stereopair are shown as dashed lines. The calculated position of the wave energy front is shown as a thick dashed line. The mean wave heights plus the observed percentage variation from the mean is shown for each crest and trough line. Note the trough lines observed in the 1st stereopair are coincident with the crest lines observed in the 2nd stereopair and vice versa.

#### MODEL HARBOUR CONFIGURATION ANALYSED USING INFINITE BASIN TECHNIQUE

The stereopairs in fig. 3 show waves entering the model harbour basin shown in fig. 1. The basin is 4,5 m long and 4,8 m wide. (These dimensions do not include the crushed stone beaches along the side and back walls of the basin.) The incident wave train has the following characteristics: wave period 0,67 seconds, wave height approx. 55 mm, and wave length (calculated using Airy wave theory) 604 mm. The gap to wavelength ratio (i.e. B/L ratio) is 1,64. The water depth is 125 mm  $\pm$  1 mm. The stereopairs shown were taken 14,5 seconds after starting the wave paddy at which stage the wave energy front was at the toe of the back wall beach. The second stereopair was taken with the waves in the basin 180° out of phase relative to the waves imaged in the first stereopair. This means that the troughs imaged in the second stereopair occupy the positions of the crests imaged in the first stereopair. If one subtracts the crest and trough elevations of the first stereopair from their corresponding elevations in the second stereopair, one will achieve a plot of wave height distribution within the basin. Since the harbour configuration is symmetrical about the centre line, only the left hand side of the basin is shown in fig. 3.

#### WAVE HEIGHT MEASUREMENT PROCEDURE

The process of achieving a plot of wave height distributions within the model basin is briefly summarised in fig. 4. The stereopairs are analysed using a Zeiss (Jena) stereocomparator which is interfaced via an analog to digital converter and a data communications interface with a Tektronix 4051 microcomputer. The microcomputer, under program control, stores the X, Y and PX, PY data obtained from the stereocomparator for each point observed. The stereopairs can also be analysed on a digitiser tablet using a modified stereoscope and a special double cursor (as described by Welham [4]). Using this alternative technique the wave heights can be measured with an accuracy of 5 to 6 mm. Once a set of points has been observed, the raw data is edited, prepared for transfer to flexible disc, and then stored on tape. The tape is then transferred to a second Tektronix 4051 which has a flexible disc file manager, and the data transferred from tape to disc, under program control. All subsequent analysis is performed on the second Tektronix 4051.

The analysis procedure which is outlined by the procedural flowchart in fig. 5 is as follows: the control points and crests imaged in the first stereopair and the control points imaged in the second stereopair are observed using the stereocomparator. The observed data is then analysed using the program "Waveheight", written by the first author, to yield:- the inner orientation elements and projective transformation parameters for the 1st and 2nd stereopairs, the crest elevation data

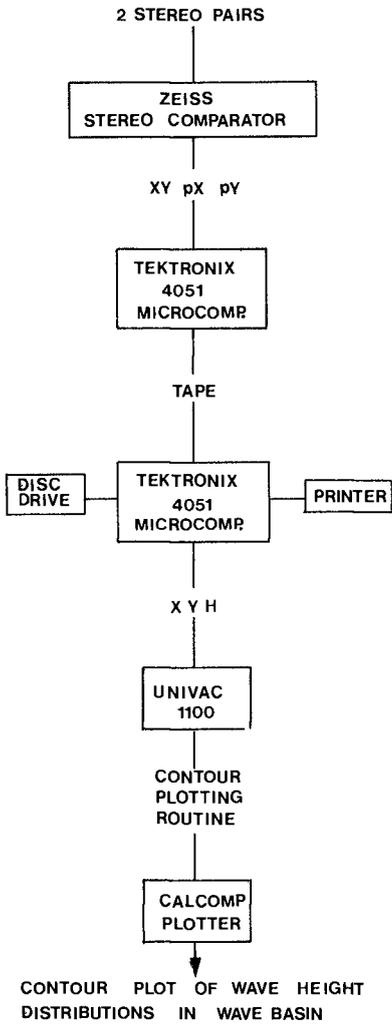


Fig.4 SEQUENCE OF ANALYSIS

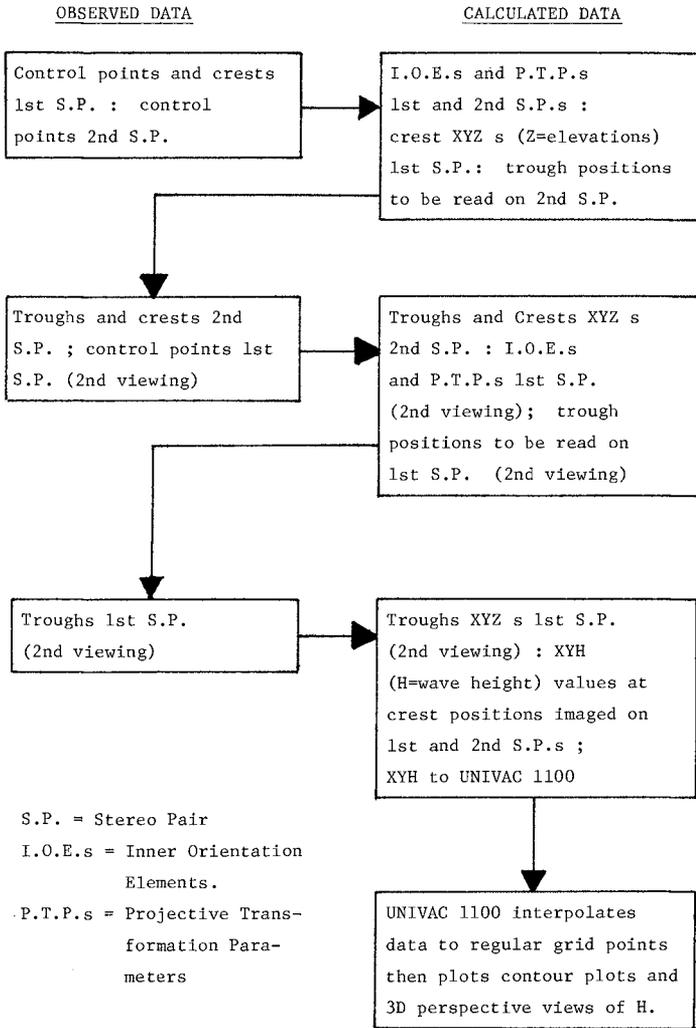


Fig.5 PROCEDURAL FLOWCHART

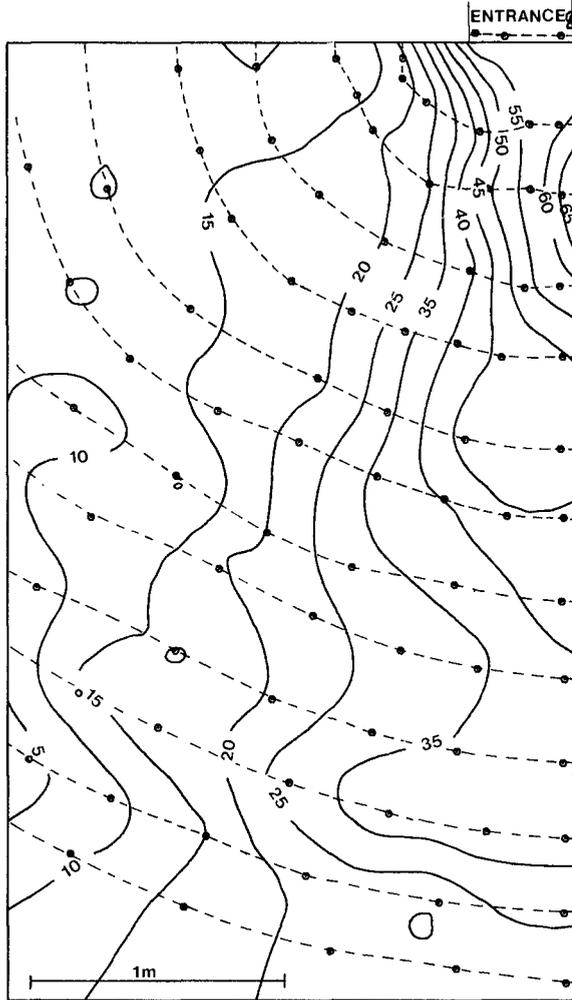


Fig.6 COMPUTER CONTOUR PLOT OF WAVE HEIGHTS  
IN A MODEL HARBOUR (HEIGHTS IN MM)

(crest XYZ data) for the 1st stereopair, and the positions at which the troughs must be observed on the 2nd stereopair. The troughs imaged in the 2nd stereopair are then observed at the predetermined points, while the crests are observed at arbitrary points selected by the observer. The 1st stereopair is again placed in the stereocomparator and the imaged control points are observed. This observed data is analysed using the programme "Waveheight" to yield: trough and crest XYZ data for the 2nd stereopair, inner orientation elements and projective transformation parameters for the 1st stereopair (2nd viewing), and the trough positions to be observed in the 1st stereopair (2nd viewing). The troughs imaged in the 1st stereopair are then observed. The observed data is then analysed using the program "Wave height" which yields: The trough XYZ data for the 1st stereopair and the XYH values along the crest lines imaged in the 1st and 2nd stereopairs, H is the wave height at a point within the basin. From the results obtained it is estimated that the wave heights can be measured with an accuracy of better than 2 mm.

The XYH data is then transferred to a file on the UNIVAC 1100 at U.C.T. The Saclant Graphics Package installed on the UNIVAC is then used to interpolate the raw data onto a rectangular equidistant grid. The interpolated data can then be used to plot a contour plot or a 3 dimensional perspective picture of the wave height distributions within the model harbour basin. The stereopairs, shown in fig. 3, were analysed using the procedure described above to yield the contour plot of wave heights within the basin, shown in fig. 6. Since the harbour configuration is symmetrical about the centre line, only the left hand side of the basin is shown. The dashed lines indicate the crest lines plotted from both stereopairs. The black dots indicate the crest line sampling points plotted from both stereopairs.

#### CONCLUSIONS

The two major problems which prevent the accurate measurement of wave heights in model harbours using conventional techniques (as described in the introduction) have been successfully solved using the photogrammetric technique. Problem (1) is solved since the two stereopairs of photographs can be taken in a much shorter period than is required for a scan using wave probes, and the information contained on the plates is permanent, synoptic and detailed. Furthermore, there is no instrumental interference in wave processes being observed. Problem (2) can be overcome by using the infinite basin technique, that is by photographing before the wave energy is reflected from internal walls, thus eliminating the distorting effects of wave reflections within a model basin. The infinite basin technique effectively enables the researcher to accurately model the situation of a continuous wave train entering a basin of infinite extent.

Three areas of utilisation for the photogrammetric wave height and pattern recording technique appear possible, the first is the routine use in applied investigations where refraction or diffraction effects are important, for example, the analysis of model harbour configurations. The second is to make use of the procedures as a check on the validity of various existing wave theories, where the information obtained (instantaneous and high accuracy) should be decisive. The third area of utilisation is to apply the technique in close conjunction with

mathematical modelling (such as finite element modelling) in order possibly to calibrate the models and thus improve their predictions.

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