INCIPIENT MOTION OF PARTICLES UNDER OSCILLATORY FLOW

by

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ABSTRACT

This paper is aimed at the establishment of a generally applicable criterion for the onset of grain motion under the influence of oscillatory flow. Data from previous studies are used in a dimensional analysis and an empirically derived relationship between the dimensionless parameters R* (shear Reynolds number) and D* (dimensionless grain parameter) is proposed as a criterion to be used in coastal engineering problems.

This study forms part of a larger programme by the Sediment Dynamics Division of the National Research Institute for Oceanology in Stellenbosch, RSA, which is aimed at the re-evaluation and updating of the input parameters and relationships for the predictive equations for coastal sediment transport.

1. INTRODUCTION

Coastal sediment transport formulae are based mainly on the transport formulae derived for uniform flow. In contrast with the initiation of sediment motion under wave action, the conditions for movement under unidirectional steady flow have been widely studied by numerous researchers during the past years and well established criteria for the beginning of movement exist. The applicability of the criteria for incipient motion under uniform flow to oscillatory motion is, however, questionable.

A number of excellent studies with respect to oscillatory flow have already been done. The problem, however, is a complex one, mainly due to the numerous variables involved. Therefore most authors preferred to base their criteria on experimental rather than purely theoretical analyses. Although each individual study was obviously well designed and the results were very convincing, the different criteria are not always compatible, mainly due to

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differences in experimental set-up and initial conditions employed. The subsequent different ranges of applicability also make direct comparisons difficult. Different authors also presented their results in different ways and for comparative purposes all the criteria should be written in the same form. This was done by Silvester (1974) who calculated the critical near bed velocity at the onset of movement under specific boundary conditions, using the empirical formulae derived in a number of previous studies. He concluded that little correlation exists between the different criteria, mainly because some criteria were used outside their ranges of applicability.

Despite this apparent lack of correlation, it was nevertheless felt that the vast amount of data which are available may still yield satisfactory results if they are analysed within a general theoretical framework and the results brought together on a single curve.

All the relevant available studies were subsequently investigated with special emphasis on the quality of the data. Aspects which were carefully looked at were, for example, the experimental procedure, sediment characteristics, parameters directly measured during the experiment and the authors' definition of incipient motion.

2. EVALUATION OF AVAILABLE DATA

The data which were used in this study can be broadly subdivided into four groups based on the experimental set-up, namely, those tested with:

(i) The simple harmonic motion of sediment particles through still water.

(ii) Water oscillating in a U-tube over bed material.

(iii) Progressive waves over bed material in a conventional wave flume.

(iv) Direct measurement of water movement on the sea floor due to surface waves, that is, prototype data.

According to these four types of data, the different data sources which were used in this study can be grouped as follows:

Oscillating bed material	U-tube	Wave flume	Prototype
Bagnold (1946) Manohar (1955)	Ishihara and Sawaragi (1962) Carstens *et al.* (1967) Rance and Warren (1968)	Vincent (1959) Horikawa and Watanabe (1967)	Davies and Wilkinson (1978)

3. DEFINITION OF INCIPIENT MOTION

The determination of the actual onset of movement during an experiment is normally done by visual observation. Unfortunately the definition of incipient motion is not clear cut. Normally it relates to the proportion of moving grains on the bed surface. Manohar (1955), for instance, found that at a certain critical velocity a few particles were dislodged from their position of equilibrium and moved a short distance from their initial position. This movement of a few particles in the top layer was defined as initial movement.

Vincent (1959) considered the initial motion condition to be characterised by the displacement of the very first grains.

Carstens *et al.* (1967) defined incipient motion on a flat bed when approximately 10 per cent of the surface particles were rolling back and forth.

Rance and Warrent (1968) considered the dislodgement and small downstream movement of the first one or two grains to be the onset of motion.

This variation in definitions is a major handicap in any comparative study. Even when the onset of motion is properly defined within a single experiment, the assesment of the exact onset of motion is difficult and subjective. In addition, the particles in the sandy bed are not of uniform size, and consequently the onset of motion will be largely dependent on the grain size distribution of the bed material. This is clearly illustrated by the prototype data of Davies and Wilkinson (1978). Although the wave period stayed essentially constant during their observations, the measured horizontal orbital velocity at incipient motion varied by a factor two.

4. RE-ANALYSIS OF DATA

As it is the purpose of this paper to bring data from different sources together on one universal curve, a considerable scatter must therefore be expected. The question at this stage would be which parameters to use in the dimensional analysis to obtain the most acceptable results for application in coastal engineering studies. Various relationships have been used in the past to present the results of studies on incipient motion. A typical example is the critical sediment number (N_{SC}) as defined by Carstens *et al.* (1967), namely,

$$N_{SC} = \frac{U_{OC}}{(\Delta_S\, d\, D_{50})^{1/2}} = f(T, D_{50}, \rho_W, \rho_S) \qquad \ldots \quad (1)$$

where U_{OC} = horizontal orbital velocity at bed at onset of movement

D_{50} = median grain diameter of bed material

ρ_W = density of fluid

ρ_S = density of material

Δ_S = relative buoyant sediment density $= (\rho_S - \rho_W)/\rho_W$.

The main disadvantage of this parameter in defining incipient motion lies in the fact that it does not contain the bed roughness in any way and can therefore not be used for a unique determination of the critical shear stress. Since the sea bottom is normally rippled in the areas of coastal engineering interest, the ultimate goal would be to extend any incipient motion criterion to ripple bed conditions. The sediment number can therefore hardly be unique for flat bed as well as rippled bed conditions.

The parameters which were chosen for the graphical presentation of the data in this study are the shear Reynolds number R* and a dimensionless grain diameter D* (by analogy with Bonnefille and Pernecker (1966) where

$$R_* = \frac{U_* D_{50}}{\nu} \qquad \ldots \quad (2)$$

and

$$D_* = \left(\frac{\Delta_S g}{\nu}\right)^{1/3} D_{50} \qquad \ldots \quad (3)$$

with U_* = shear velocity

ν = kinematic viscosity of fluid.

The choice of R_* enables the inclusion of the bed roughness since

$$U_* = \left(\frac{\bar{\tau}}{\rho_w}\right)^{1/2} \qquad \ldots (4)$$

with $\bar{\tau}$ = mean shear stress = $^1/_4\ \rho\ f_w\ U_o^2$ ... (5)

and $f_w = f(^{a_o}/r)$ where

a_o = maximum orbital excursion from mean position of the motion of water particles at the bed

r = bed roughness

f_w = wave friction factor

The procedure by which the data were re-analysed depended for each set on the parameters which were directly measured during the original experiment. In the case of oscillating bed material as well as the oscillating water tunnel, the angular velocity (ω) and the horizontal displacement (a_o) were usually directly measured at the onset of motion. The subsequent determination of R_* and D_* in such cases is straighforward, using the standard relationships for simple harmonic motion and equations (2) an (3).

In experiments which were done in conventional wave flumes the wave height (H), wave period (T) and the water depth (h) at the incipient motion condition were normally observed. It was felt that all the data which were obtained by means of progressive waves should be treated in the same manner and for this purpose the Vocoidal water wave theory of Swart (1978) was employed. A computer program which used the Vocoidal theory was written to calculate R_* and D_* for any combination of T, d, H, D_{50} and ρ_s.

The generalised procedure of analysis may therefore be summarised as follows:

(i) Computation of the wave properties U_o and a_o.

(ii) Computation of the bed roughness r using the following formulae:

$$r = 2\ D_{90} \quad \text{(flat bed; Kamphuis, 1975)}$$

$$r = 25\ \Delta_r \left(\frac{\Delta_r}{\lambda_r}\right) \quad \text{(rippled bed; Swart, 1976)}$$

where Δ_r = ripple height

λ_r = ripple length.

(iii) Computation of $^{a_o}/r$.

(iv) Computation of orbital Reynolds number

$$R = \frac{U_o a_o}{\nu}$$

(v) Determination of the wave friction factor f_w using empirical relationships based on more than 600 data sets (Swart, in preparation, 1982).

(vi) Calculation of R* and D* using equations (2), (3), (4) and (5).

5. RESULTS

A total number of 643 data points consisting of combinations of R* and D* at the onset of movement were calculated in the above manner. Figure 1 shows a logarithmic plot of the results together with a best fit curve through the data points. This empirical curve was found to be very closely simulated by the following parabolic equation:

$$\log_{10} R_* = 0{,}092(\log_{10} D_*)^2 + 1{,}158 \log D_* - 0{,}367 \quad .. (6)$$

It is also possible to test the validity of equation (6) by making use of laboratory and field measurements of sediment load under waves in conjunction with appropriate predictive sediment load equations which include an incipient motion criterion. For this purpose the Ackers and White sediment load formula for steady flows, as modified for application in the coastal area (see Swart and Fleming, 1980) will be used.

The original formula for the prediction of sediment transport rates under steady-state conditions was (Ackers and White, 1973):

$$S = 1{,}45\ U\ D_{35}\ C\left(\frac{P_{fg}/\rho}{U^3_{fg}}\right)^n \left(\frac{P_{cg}/\rho}{U^2_{cg} U_r}\right)^{1-n} \left(\frac{F_{gr}}{(F_{gr})_{crit}} - 1\right)^m \quad ... (7)$$

where C, n, m and $(F_{gr})_{crit}$ are grain size dependent parameters, for which empirical relationships are given by Ackers and White; F_{gr} is the sediment mobility = $(U^n_{*fg}\ U^{1-n}_{*cg})/(\Delta_s\ g\ D_{35})^{1/2}$;
P is the stream power; U_r the resultant velocity (= U for steady state) and U* is the shear velocity = $(\tau/\rho)^{1/2}$; subscripts "fg" and "cg" respectively denote the "fine grain" and "coarse grain" versions of the properties. The

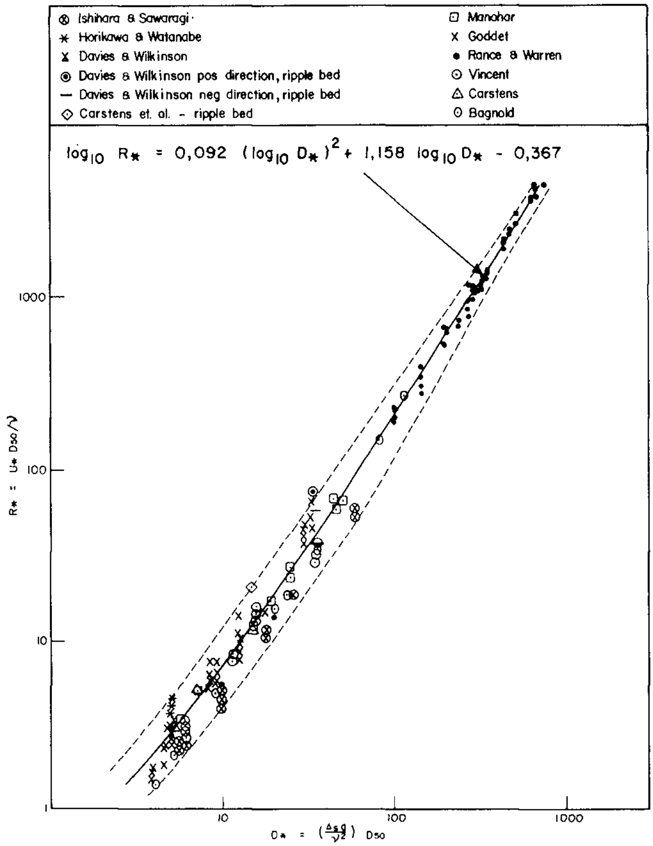

FIG. 1 INCIPIENT MOTION OF SEDIMENT PARTICLES UNDER OSCILLATORY FLOW - EXPERIMENTAL RESULTS

value for fine-grained sediment is obtained by using the actual bed roughness r, as described before, whereas the value for coarse-grained sediment is obtained by using the grain size D_{35} instead of r in the appropriate equations. For steady state conditions the efficiency term reduces to $(U/U_{*fg})^n$.

Various adapted versions of the original steady-state Ackers and White formula were in use in 1980, varying only in the way in which the effect of waves was included in the stream power and mobility terms (Swart, 1976; Willis, 1978; Van de Graaf and Van Overeem, 1979). Swart and Fleming (1980) discussed these versions and came to the conclusion that all three methods quoted above had shortcomings. They then suggested a fourth alternative which used the better aspects of each method and discarded the rest. The main points of improvement of this latter method as opposed to the earlier modifications were the following:

(1) The average effect of the inclusion of waves on the mean mobility number is calculated by integrating the instantaneous mobility number with respect to time, that is,

$$F_{wc} = \overline{F_{gr}(t)} = {}^1/T \int_0^T F_{gr}(t)dt \qquad \ldots \quad (8)$$

where F_{wc} is the mean sediment mobility for combined current and wave action.

(2) The instantaneous value $E_f(t)$ of the efficiency term, as given in equation (9) is averaged to obtain the mean value E_{fwc} of the efficiency term for combined current and wave action, namely,

$$E_{fwc} = \overline{E_f(t)} = {}^1/T \int_0^T E_f(t)dt \qquad \ldots \quad (9)$$

$$\text{where } E_f(t) = \left(\frac{P_{fg}(t)/\rho}{U_{*fg}(t)^3}\right)^n \left(\frac{P_{cg}(t)/\rho}{U_{*cg}(t)^2\, U_r(t)}\right)^{1-n} \qquad \ldots \quad (10)$$

Values for the instantaneous resultant velocity at the bed $U_r(t)$ and the instantaneous shear stress at the bed $\tau(t)$ are found by vector addition of the contributions by the waves and the currents.

By equating measured sediment load data under known wave conditions to the right-hand side of this adapted version of equation (7), values for the critical sediment mobility $(F_{gr})_{crit}$ were calculated. Since $(F_{gr})_{crit}$ is simply related to R_* by

$$(F_{gr})_{crit} = R_* \, D_*^{-3/2}, \qquad \ldots (11)$$

corresponding values for R_* could be computed.

The results of this calibration technique for the threshold of motion can be seen in Figure 2. The threshold values calculated in the above manner agree very well with the threshold criterion proposed in terms of equation (6). The result is also extremely significant in the sense that it illustrates the efficiency of the adapted Ackers and White formula for the prediction of sediment load under waves, provided that equation (6) is used to define the critical sediment mobility.

Equation (6) can be rewritten in terms of a non-dimensional shear velocity V_*,

$$\text{where } V_* = U_*/(g\Delta_s \nu)^{1/3} \qquad \ldots (12)$$

This implies that

$$R_* = U_* \, D_*/(\Delta_s g \nu)^{1/3} = V_* \, D_* \qquad \ldots (13)$$

Equation (5) therefore reduces to

$$\log V_* = 0{,}092(\log D_*)^2 + 0{,}158 \log D_* - 0{,}367 \qquad \ldots (14)$$

The relationship between V_* and D_* as obtained from equation (14), as well as the scatter observed in the data, is indicated in the following table where V_* represents the value obtained from equation (14), while V_{*max} and V_{*min} corresponds to the upper and lower envelopes of the experimental data:

D_*	V_*	V_{*max} 1)	V_{*min} 1)
4	0,51	1,00	0,31
10	0,63	1,35	0,40
20	0,83	1,75	0,55
50	1,22	2,70	0,86
100	1,75	3,20	1,50
200	2,75	3,75	2,50
500	4,60	6,00	4,00
700	5,71	6,43	4,86

Comment: 1) These numbers reflect the maximum variability in the data.

The scatter indicated by this table may look appreciable for low values of D_* but is in fact of the same order of magnitude as that observed in the Shields diagram for steady flow and may be attributed to experimental errors and the use of different definitions for incipient motion.

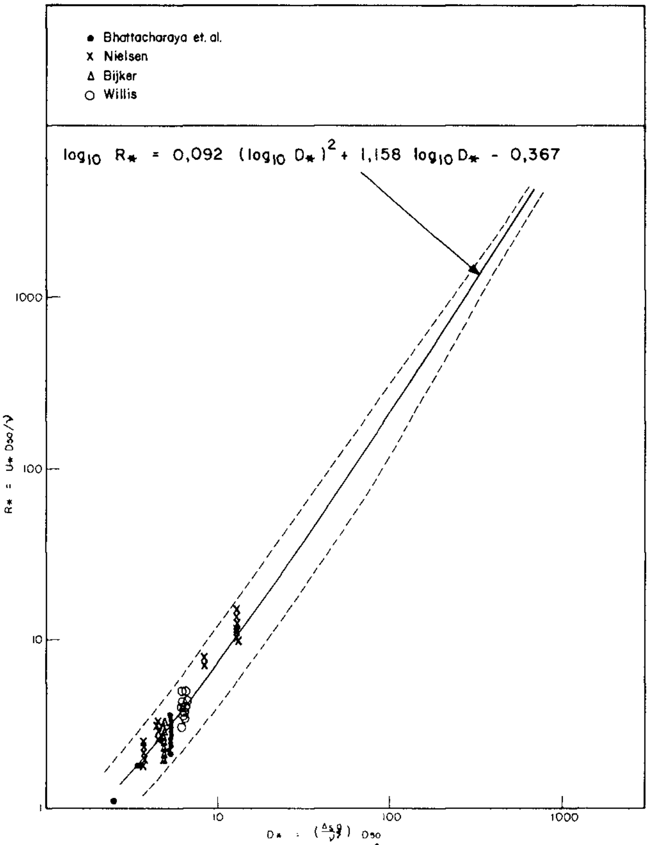

FIG. 2 COMPARISON WITH THRESHOLD VALUES COMPUTED FROM SEDIMENT LOAD DATA

6. COMPARISON WITH UNIDIRECTIONAL FLOW CRITERIA

The conditions for movement under unidirectional steady flow have been widely studied by numerous researchers and well established criteria for the beginning of movement exist. The work by Shields (1936) and Ackers and White (1973) are probably the most widely used and acknowledged in this respect. A direct comparison between these two criteria and the results of this study can now be made. Figure 3 shows the original Shields and Ackers and White data points together with the curve defined by equation (6). The envelope encompasses data scatter for oscillatory flow. The unidirectional data can be seen to fall well within the scatter of the oscillatory flow data points. It can therefore be concluded that there is no significant difference between the criteria for uniform flow and the criterion of oscillatory flow as derived in this study. A single criterion can be applied with the same degree of accuracy for both flow conditions. Consequently it is reasonable to assume that the criterion developed in this study can be applied for wave-generated flows, current-dominated flows and all combined flow modes in between, provided that the correct definition of the shear velocity is used in equations (6) and (14).

7. CONCLUSIONS

The main conclusions drawn from the present study are as follows:

(1) An empirical criterion, equation (6) or (14), was derived for the onset of grain motion on either a flat or a rippled bed under the influence of wave action.

(2) It was shown that the new criterion is not only valid for incipient motion due to waves but also does not differ from widely used incipient motion criteria for steady-state conditions (Shields, 1936; Ackers and White, 1973). In addition, provided that the correct definition of the bed shear is used, it also applies to the case of combined current and wave action. It can therefore be seen as a fairly universal criterion for the onset of grain movement.

(3) Comparison to sediment load data via the adapted Ackers and White sediment load formula given by Swart and Fleming (1980) yielded values of the incipient motion mobility number which would have been required to allow the sediment load formula to predict the measured sediment load. These indirectly calculated incipient motion data fall within the scatter of the original data used to derive equation (6). This serves as a very strong independent proof of the adapted Ackers and White sediment load formula given by Swart and Fleming (1980).

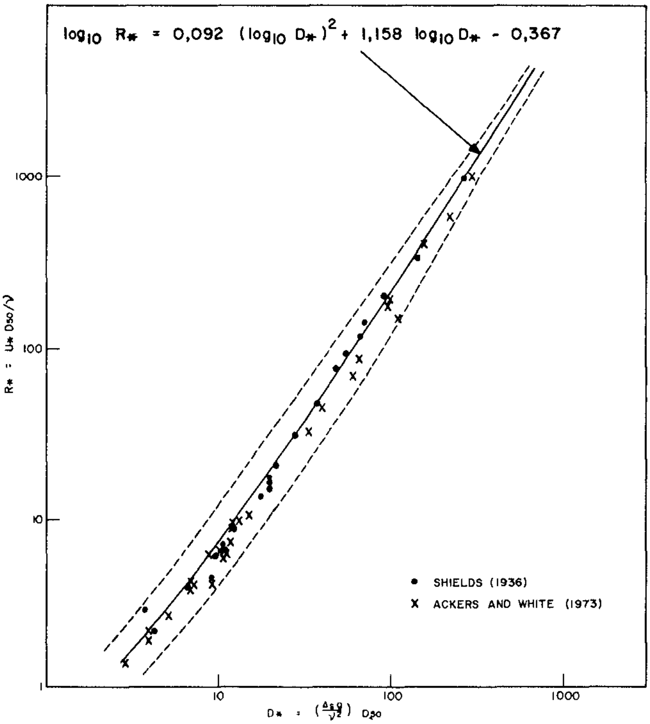

FIG. 3 COMPARISON WITH UNIDIRECTIONAL FLOW CRITERIA

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