NEW FRAMEWORK FOR PREDICTION OF LONGSHORE CURRENTS

by

C A Fleming[1] and D H Swart[2]

ABSTRACT

The accuracy of prediction of longshore sediment transport depends largely on the accuracy with which the wave-driven longshore currents within the breaker zone can be predicted. Longuet-Higgins (1970) developed a formulation for longshore transport which is widely used today. In the present paper the basic theory of Longuet-Higgins is re-examined. The effect of bed roughness on the magnitude of the longshore current is quantified with the aid of over 350 individual data sets and the theory is theoretically extended to include the effect of random waves, in a similar way to Battjes (1974), and higher-order waves. For this latter purpose the Vocoidal water wave theory of Swart (1978) is used. It is shown that the use of Vocoidal theory leads to a velocity distribution which is in closer correspondence to measured data than that predicted by using linear wave theory.

1. MOTIVATION AND BACKGROUND

The magnitude and distribution of the wave-driven longshore current in the breaker zone depends on the momentum balance, which in turn depends on the underwater profile, the incident wave characteristics and the wave breaking mechanism. In 1970 Longuet-Higgins solved this momentum balance equation in the longshore direction in the shore area by making specific assumptions regarding three individual terms, namely, the driving force or radiation stress term, the bed shear and the lateral mixing, the latter two being dissipative terms.

The momentum balance in the longshore direction x as given by Longuet-Higgins is:

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1) Sir William Halcrow and Partners, Swindon, UK

2) National Research Institute for Oceanology, CSIR, Stellenbosch, RSA

$$\frac{\partial R_{yx}}{\partial y} + B_x + \frac{\partial D_y}{\partial y} = 0 \qquad \ldots \quad (1)$$

where $\frac{\partial R_{yx}}{\partial y}$ = variation in flux of x-momentum with distance y offshore;

B_x = bed shear in the direction of the longshore current; and

$\frac{\partial D_y}{\partial y}$ = exchange of momentum due to lateral mixing.

The type of solution obtained, or more specifically the variation of the current with distance offshore, depends on the assumptions made regarding these three terms.

Predicted longshore sediment transport rates are very sensitive to the **distribution** and **magnitude** of the longshore current. An error in prediction of the longshore current of 10 per cent could cause an error in the prediction of the longshore sediment transport of as high as 70 per cent.

In a laboratory study into current patterns in the vicinity of a proposed coastal structure it was observed that the longshore current velocities generated by regular waves on a very flat beach (≈ 1 in 100) were between 2 and 5 times higher than predictions with the Longuet-Higgins model would tend to indicate (CSIR, 1978). This anomaly is seemingly coupled to the bed roughness.

In the shallow water region where sediment transport calculations are normally performed the waves are decidedly non-linear. Swart and Loubser (1979) have shown that linear theory does not allow for a proper representation of the time-dependent wave properties in this area. In a comparison of the applicability of 13 different wave theories they found that Vocoidal water wave theory, developed by Swart (1978), is the most applicable theory under all wave conditions tested in the shallow water region, and was significantly better than the linear theory. In addition, waves in prototype are not regular but random and do not always break as spilling breakers, as has to be assumed in order to obtain a solution for the longshore current profile.

Swart and Fleming (1980) already reported briefly on the effect of the bed roughness and the bed slope on the Longuet-Higgins solution without going into the background for the re-analysis. Subsequently more research was done into the effect of the non-linearity of the incident waves on the longshore current profile.

The objective of this paper is to report on progress made towards the understanding of the effect of the above-mentioned factors. It will deal mainly with

- the effect of bed roughness; and
- the effect of both higher-order and random waves on the type of solution obtained.

2. TREATMENT OF LINEAR, REGULAR WAVES

The theory of longshore current generation by linear, regular waves breaking, as spilling breakers, on a plane beach was covered authoritively by Longuet-Higgins (1970) and will not be repeated here. Only those points which are relevant to the present discussion will be highlighted.

Driving forces

The driving force for the longshore current is obtained by considering the variation in the flux of longshore momentum with distance y offshore.

$$\frac{\partial R_{yx}}{\partial y} = -\frac{5}{16}\gamma^2 \rho g d \tan\alpha \sin\Theta \cos\Theta \qquad \ldots \quad (2)$$

(Longuet-Higgins, 1970)

where γ = breaker index, that is, the ratio of breaking wave height H_b to corresponding water depth d_b; this index is assumed constant throughout the breaker zone;

ρ = mass density of water;

g = gravitational acceleration;

d = water depth;

α = beach slope;

Θ = angle of incidence of the waves, measured between the wave crest and the local bed contour.

Resistive forces: Bed shear

It was shown (CSIR, 1978) that the general expression for bed shear B_x in the longshore direction is:

$$B_x = \frac{2}{\pi}\rho\,(C_{LH1}\,u_o v \;+\; g v^2/C_n^2) \qquad \ldots \quad (3)$$

where C_{LH1} = a friction coefficient

$$= \left(\frac{f_w g}{2C_n^2}\right)^{0,5} \qquad \text{(CSIR, 1978)} \qquad \ldots \quad (4)$$

u_o = maximum value at the bed of the horizontal orbital velocity;

v = longshore current velocity at depth d;

C_n = Chezy roughness coefficient = 18 log (12d/r);

f_w = wave friction factor, as defined by Jonsson (1966).

Longuet-Higgins (1970) assumed that $v \ll u_o$ and obtained a simplified version of equation (3), namely,

$$B_x = \frac{2}{\pi} \rho C_{LH} u_o v \qquad \ldots \quad (5)$$

In the present study the simplified equation (5) as used by Longuet-Higgins will be employed. Any approximations introduced in this manner for cases where v/u_o is not negligible, which is normally the case, will exhibit itself as a scatter in the empirical values of C_{LH}.

Resistive forces: Lateral mixing

Longuet-Higgins (1970) assumed that Dy is given by:

$$D_y = -\mu_e d \frac{\partial v}{\partial y} \qquad \ldots \quad (6)$$

where μ_e = horizontal eddy viscosity

= $N \rho y (gd)^{0,5}$ (Longuet-Higgins, 1970)

N = dimensionless constant.

Although the eddy viscosity as defined by Longuet-Higgins increases indefinitely with distance from the shoreline, even outside the breaker zone where it would be expected to decrease markedly, the results obtained via this method are very realistic.

Solution

Using equations (2), (5) and (6) above, Longuet-Higgins obtained the longshore current profile:

$$v = v_{or} f(y,P) \qquad \ldots \quad (7)$$

where v_{or} = scaling velocity at the breaker line

$$v_{or} = \frac{5\pi}{16} \frac{\gamma(gd_b)^{0,5}}{C_{LH}} \tan\alpha \sin\Theta_b \cos\Theta_b \qquad \ldots \quad (8)$$

P = dimensionless lateral mixing parameter; and subscript b refers to the initial breaker line.

After investigating all available data sets on longshore current, as summarised by Galvin and Nelson (1967), Longuet-Higgins (1970) found that if he neglected those data sets termed questionable by Galvin and Nelson the parameter C_{LH} varied between 0,0036 and 0,012 with a mean value of 0,0082. He therefore concluded that "C_{LH} is of order 0,01". This has subsequently been interpreted by researchers to mean that C_{LH} is a constant. Since C_{LH} is by definition a roughness coefficient this is obviously an erroneous conclusion.

Re-analysis of data

To settle this matter, the data as tabulated by Galvin and Nelson were re-analysed, along with some newer data, in an approach similar to the earlier analysis performed by Longuet-Higgins. Only the mean longshore current velocity was available for most of the data. Longuet-Higgins showed that the mean current velocity is given by:

$$\bar{v} = \frac{5\pi}{16} \beta_m \frac{\gamma^{0,5}(gH_b)^{0,5}}{C_{LH}} \tan\alpha \sin\Theta_b \cos\Theta_b \qquad \ldots \quad (9)$$

where β_m depends on the intensity of lateral mixing, and has a mean value of 0,33 with a possible variation of plus/minus 30 per cent.

Equation (9) can be rewritten to obtain C_{LH} in terms of a dimensionless parameter Φ_o, namely,

$$C_{LH} = \frac{5\pi}{16} \beta_m \gamma^{0,5} \Phi_o \qquad \ldots \quad (10)$$

where $$\Phi_o = \frac{(gH_b)^{0,5}}{\bar{v}} \tan\alpha \sin\Theta_b \cos\Theta_b \qquad \ldots \quad (11)$$

For each data set the parameter Φ_o is a function of the bed roughness and, perhaps surprisingly, of the bed slope. If one, however, keeps in mind that the bed slope influences the breaker type which in turn is strongly related to the extent of lateral mixing, it becomes apparent that Φ_o implicitly contains the effect of lateral mixing.

In this context lateral mixing is quantified by means of the lateral mixing coefficient P, which was found by Longuet-Higgins to have a theoretical upper limit of 0,4. By inspection of the available data on longshore current profiles Longuet-Higgins concluded that the data are adequately bracketed by theoretical profiles with values of P = 0,1 and P = 0,4. Furthermore, the mean value of P appeared to be P = 0,2. In the present analysis it was therefore assumed that P = 0,2 and that any effect of a different mixing coefficient would be rectified for by the value obtained for C_{LH} (or Φ_o in the present stage of the computation). Keeping the comment about β_m in mind it is then reasonable to expect that C_{LH} will also exhibit a variation of plus/minus 30 per cent, which was indeed later shown to be the case.

For a value of P = 0,2 the function f(y,P) in equation (7) becomes:

$$f(y,P) = \begin{cases} -1{,}74(y/y_b)^{1{,}61} + 2(y/y_b) & \text{for } 0 \leqslant y/y_b \leqslant 1 \\ 0{,}26(y/y_b)^{3{,}11} & \text{for } 1 < y/y_b \leqslant \infty \end{cases} \qquad \ldots (12)$$

In analogy with the result in equation (4) it is assumed that the actual value of the roughness parameter C_{LH} is related to roughness by

$$C_{LH} = K\left(\frac{f_w g}{2C_n^2}\right)^{0{,}5} \qquad \ldots (13)$$

Parameter K is a proportionality constant which according to Figure 1 should be a function of the bed slope.

With the aid of the data in Figure 1 and equation (10) it was found that

$$K = 25(\tan\alpha)^{0{,}85} \qquad \ldots (14)$$

which means that

$$C_{LH} = 25\left(\frac{f_w g}{2C_n^2}\right)^{0{,}5} (\tan\alpha)^{0{,}85} \qquad \ldots (15)$$

A comparison between measured and predicted values of C_{LH} is given in Figure 2.

It is interesting to note that Komar and Inman (1970) in an analysis of the same data found that

$$C_{LH} \approx 0{,}15 \tan\alpha \qquad \ldots (16)$$

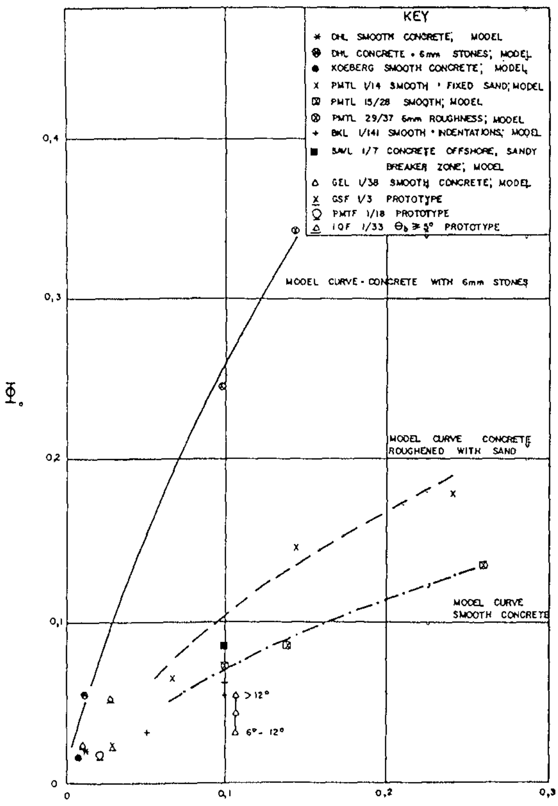

FIG. 1 RELATIONSHIP BETWEEN Φ_o, BEACH SLOPE AND BED ROUGHNESS

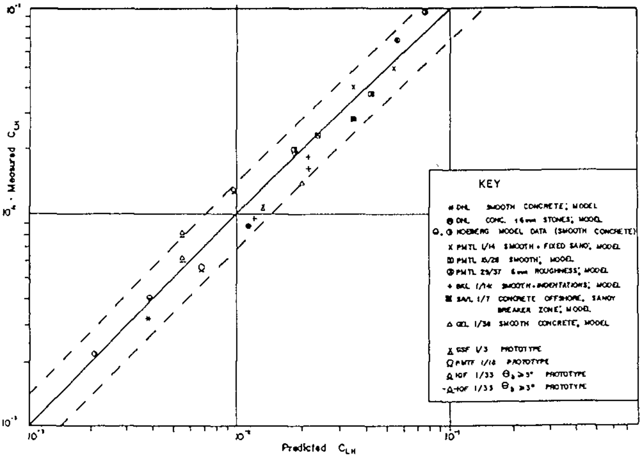

FIG. 2 RELATIONSHIP BETWEEN PREDICTED AND MEASURED C_{LH}-VALUES FOR MODEL AND PROTOTYPE CONDITIONS

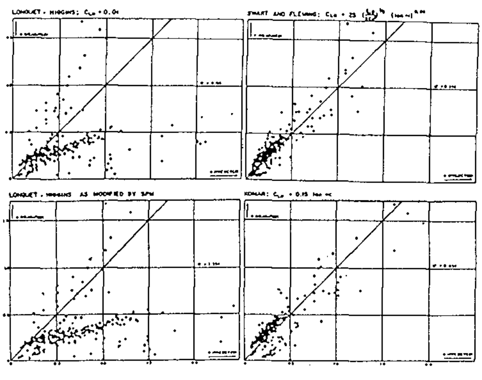

FIG. 3 PREDICTED VERSUS MEASURED LONGSHORE CURRENT VELOCITIES BY FOUR DIFFERENT METHODS

This would imply that the velocity is independent of bed slope.

Figure 3 gives a comparison of the "goodness of fit" to the data when using the present technique, Komar's method, the Longuet-Higgins method with the assumption of $C_{LH} \approx 0{,}01$ and a slight variation to this last approach, which also assumes a constant value for C_{LH} and which is the method recommended for use by the Shore Protection Manual (SPM, 1973). A root-mean-square error was computed for each of the four methods tested, based on the comparison between the theoretical/emprirical models tested and all 352 available data sets. The present method has a rms error of 0,248, which is only about 40 per cent of that found for the Komar method, and which is an order of magnitude better than the methods which assume a constant value for C_{LH}.

It is interesting to note that the two methods with the closest correspondence to the data, that is, the present method and Komar's method, suggest a very much lower dependence on bed slope than predicted by the classical theory. The explanation for this apparent anomaly lies in the fact, as was pointed out earlier, that the roughness coefficient contains, at least in part, the effect of the lateral mixing, which is a function of amongst other things the bed slope.

It should be borne in mind that although the method derived here is far superior to the method normally used, it is still not applicable to the case of waves breaking as plunging breakers or for waves breaking on a barred beach.

3. TREATMENT OF LINEAR, RANDOM WAVES

A theoretical framework for the prediction of longshore currents generated by random waves was developed by Battjes (1974) which yielded reliable results. It is comparable to the Longuet-Higgins approach for regular waves except that wave set-up was not neglected and for the obvious differences between regular and random waves. The method has to be applied numerically since no analytical solution was found. Battjes draws the very important conclusion that lateral mixing is not nearly as important in the determination of the velocity profile for random waves as it is in the case of regular waves.

In this section a derivation will be done for random waves at the same level of assumption as was done by Longuet-Higgins for regular waves.

Assumptions

(i) Linear wave theory is used;

(ii) Waves are random with a Rayleigh height distribution;

(iii) Waves break as spilling breakers with a constant breaker index $\gamma = H_b/d_b$ throughout the breaker zone;

(iv) The wave spectrum within the breaker zone is treated in the same way as done by Battjes (1974), that is, waves in excess of γ times the water depth are reduced to γ times the water depth;

(v) Wave set-up is initially neglected although its effect will be discussed later;

(vi) The bed slope α in the breaker zone is considered constant;

(vii) The bed roughness coefficient C_{LH} is constant over the breaker zone; and

(viii) Lateral mixing is neglected (Battjes, 1974).

Governing equations

The same momentum balance equation (equation (1)) applies as for regular waves. The assumptions outlined above lead to the following expression for the driving force term when shallow-water wave conditions are assumed:

$$\frac{\partial R_{yx}}{\partial y} = -\frac{5}{16}\,\rho\,\gamma^2\,(gd)\,\exp\left(\frac{-\gamma^2 d^2}{H_{frms}^2}\right)\sin\Theta\cos\Theta\tan\alpha \quad \ldots (17)$$

where H_{frms} = **fictitious rms wave height** at water depth d, which would have existed under the influence of shoaling, refraction and bed friction if **no wave breaking** had occurred.

When looking at the resistive forces, the bed shear is written in the simplified form of Longuet-Higgins (see equation (5)), which after substitution of shallow-water wave conditions reduces to

$$B_x = \frac{1}{\pi}\,C_{LH}\,\rho\left(\frac{\bar{H}}{d}\right)\,v(gd)^{0,5} \quad \ldots (18)$$

where $\bar{H}$ = mean wave height at depth d, after including the effect of wave breaking.

Solution

The combination of equations (17) and (18) yields an expression for the longshore current velocity at water depth d:

$$v = \frac{5\pi}{16}\ \frac{\gamma^2(gd)^{0,5}}{C_{LH}}\ \left(\frac{d}{H}\right) \exp\left(-\frac{\gamma^2 d^2}{H^2_{frms}}\right) \tan\alpha \sin\Theta \cos\Theta \qquad (19)$$

If, in analogy with regular waves, one writes, with lateral mixing now being neglected

$$v = v_{oir} f(y) \qquad \ldots (20)$$

it follows that

$$v_{oir} = \frac{5\pi}{16}\ \frac{\gamma(gd_{bs})^{0,5}}{C_{LH}} \tan\alpha \sin\Theta_{bs} \cos\Theta_{bs} \qquad \ldots (21)$$

and
$$f(y) = \left(\frac{d}{d_{bs}}\right) \left(\frac{d}{H}\right) \gamma \exp\left(-\frac{\gamma^2 d^2}{H^2_{frms}}\right) \qquad \ldots (22)$$

where subscript bs refers to the significant breaker line.

An example of the comparison between the velocity profiles as predicted for regular waves with lateral mixing and for random waves without lateral mixing is given in Figure 4.

In the above derivation the wave set-up was neglected. If, however, the depth d is taken to be the still-water depth plus the wave set-up, the slope $\tan\alpha$ should actually read $(\partial d/\partial y)$, then a very good first approximation of the actual effect of wave set-up is obtained. For the present equations (21) and (22) can be used together with the value of C_{LH}, as given by equation (15). However, it is deemed advisable to obtain more data on longshore currents generated by random waves, especially under controlled conditions.

Furthermore, in the case of longshore currents generated by random waves breaking as plunging breakers on a barred beach, the longshore current profile will not be as smooth as indicated by, for example, Figure 4. In this case one should for the present return to the Battjes approach. However, the effect of lateral mixing has to be included since the lateral velocity gradients, which according to equation (6) are not small any more, will lead to higher lateral mixing, especially near the breaker line.

4. TREATMENT OF REGULAR, VOCOIDAL WAVES

The only known case in the literature of a derivation for longshore currents based on a higher-order theory, is that of James (1974). James based his model on hyperbolic waves in the nearshore region and on Stokes waves further out to sea. Due to the disadvantage of coupling different wave theories at a given water depth, the limited range of applicability of the hyperbolic theory and the fact that

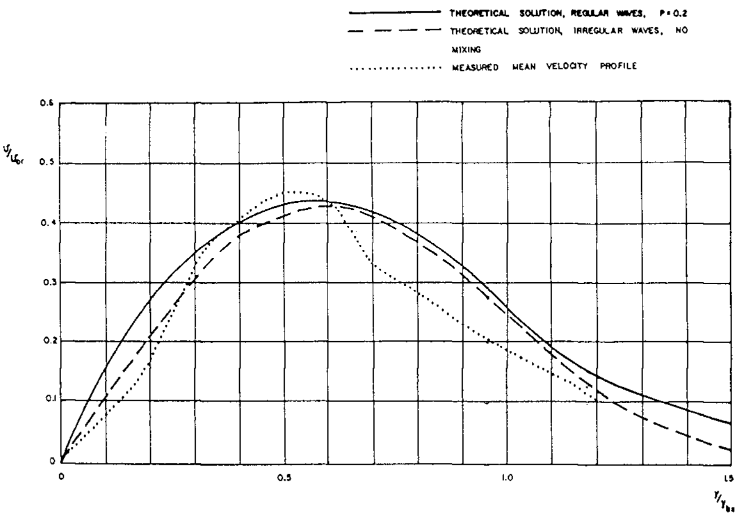

FIG. 4 COMPARISON BETWEEN THEORETICAL SOLUTION FOR REGULAR AND IRREGULAR WAVES RESPECTIVELY AND MEAN MEASURED PROFILE

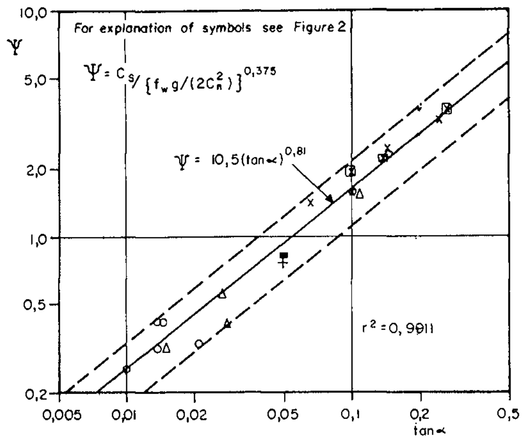

FIG. 5 RELATIONSHIP BETWEEN Ψ AND TANα

the longshore current model of James is numerical, this method is not considered further.

Swart and Loubser (1979) made a comprehensive comparison of measured wave properties with predictions from 13 different theories. The results indicate that although linear theory provides an adequate description of wave profile, wave celerity and orbital motions in deep water, the comparison deteriorates as the relative water depth becomes less. Swart and Loubser concluded that linear theory gives satisfactory results only for values of a non-linearity parameter $F_c \leqslant 200$ and should not be applied for $F_c >$ 200. The parameter F_c is defined by

$$F = (H/d)^{0,5} \left(T(g/d)^{0,5}\right)^{2,5} \quad \ldots (23)$$

If one assumes breaking wave conditions with $H/d = 0{,}7$, the above restriction would imply that linear theory can only be applied for $T\sqrt{g/d}$ less than 8,9. The following table shows values of d, below which the theory should then not be applied, for different values of wave period T.

T (s)	d (m)	T (s)	d (m)
6	4,4	14	24,1
8	7,9	16	31,4
10	12,3	18	39,8
12	17,7	20	49,1

It is therefore clear that the use of the linear theory to predict breaker-line wave data is not advisable.

On the other hand, the same study by Swart and Loubser indicated that Vocoidal theory adheres well to data for all values of F_c covered by the data, which extended to F_c = 60 000. In fact, they finally concluded that Vocoidal theory is the only readily applicable analytical wave theory having a good correspondence to measured data and a good adherence to the original wave boundary value problem for the whole range of non-breaking waves. This theory therefore not only provides a good correspondence to data but also a sound framework for the derivation of expressions for the prediction of secondary wave-induced phenomena, including longshore currents.

In this section those aspects of the derivation of the longshore current as driven by Vocoidal theory will be highlighted which differ from the theory outlined in Section 2.

Driving forces

The radiation stress term R_{yx} for Vocoidal waves is given by

$$R_{yx} = 2E_{ku} \sin\Theta \cos\Theta \qquad \text{... (24)}$$

as compared to

$$R_{yx} = En \sin\Theta \cos\Theta \qquad \text{... (25)}$$

for linear waves.

In the above E_{ku} is the kinetic energy per unit surface area due to the horizontal orbital velocity, E is the total wave energy per unit surface area and n is the group velocity/celerity ratio.

In shallow water the ratio R_1 between R_{yx} as determined for the Vocoidal and linear theories respectively becomes, if one assumes for the moment that H and Θ are equal for the two theories:

$$R_1 = (R_{yx})_{voc}/(R_{yx})_{lin} = 16e_{ku} \qquad \text{... (26)}$$

where e_{ku} is defined by $E_{ku} = e_{ku}\ \rho g H^2$.

As an example of the variation in R_1, consider the case of H/d at breaking = 0,8.

$T(g/d)^{0,5}$	R_1	$T(g/d)^{0,5}$	R_1
5	broken waves	25	0,36
10	0,74	30	0,31
15	0,56	35	0,27
20	0,44	40	0,23

It is thus apparent that the coefficient of the radiation stress term is appreciably lower for Vocoidal theory than for linear theory.

The driving force term in the momentum balance equation then becomes:

$$\frac{\partial R_{yx}}{\partial y} = -\frac{5r_{yx}}{2}\gamma^2 f_{yx}\ \rho(gd)\ \tan\alpha \sin\Theta \cos\Theta \qquad \text{... (27a)}$$

where $R_{yx} = r_{yx}\ \rho g H^2$ defines r_{yx}; and

$$f_{yx} = 1 + \frac{d}{r_{yx}} \frac{\partial r_{yx}}{\partial d} \quad \ldots (27b)$$

Resistive forces

Similarly, the bed shear as given by Vocoidal theory can be written as

$$B_x = \frac{2}{\pi} a_{voc}\ \rho\ C_s\ u_o\ v \quad \ldots (28)$$

as opposed to equation (5), where a_{voc} is the ratio between the bed shear in the longshore direction as predicted by Vocoidal and linear theory respectively and C_s is the roughness coefficient applicable to Vocoidal waves, as opposed to C_{LH} for linear waves.

Again using H/d = 0,8, one finds:

$T(g/d)^{0,5}$	10	15	20	25	30	40
a_{voc}	0,91	0,65	0,50	0,40	0,34	0,26

Solution

In setting up a momentum balance equation, the fact that the coefficient of R_{yx} is also a function of distance offshore should be kept in mind. When neglecting lateral mixing, one finds for the Vocoidal theory at the breaker line, that

$$v_{or} = \frac{5\pi}{16}(P_{reg}) \frac{\gamma (g d_b)^{0,5}}{C_s} \tan\alpha \sin\Theta_b \cos\Theta_b \quad \ldots (29)$$

where P_{reg} is a parameter containing the effect of the non-linearity (vocoidalness) of the waves.

The following table gives values for P_{reg} in terms of γ and T_c, where $T_c = T\sqrt{g/d}$.

T_c	Values of P_{reg}									
	γ=0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
5	1,84	1,83	1,82	1,81	1,80	1,79	-	-	-	-
10	1,77	1,77	1,74	1,68	1,63	1,61	1,61	1,65	1,70	-
15	1,79	1,77	1,71	1,64	1,58	1,54	1,56	1,63	1,69	1,77
20	1,93	1,87	1,80	1,72	1,66	1,65	1,70	1,79	1,84	1,90
25	2,08	2,03	1,95	1,86	1,78	1,78	1,88	1,99	2,06	2,10
30	2,31	2,24	2,15	2,04	1,95	1,98	2,11	2,24	2,31	2,35
35	2,60	2,53	2,42	2,29	2,17	2,22	2,40	2,54	2,60	2,63
40	2,98	2,90	2,77	2,62	2,46	2,56	2,75	2,89	2,95	2,95

The above table shows that P_{reg} is a function of the deep-water wave steepness and the breaker index.

Equation (29) is identical to its linear theory counterpart (equation (8)) except for the inclusion in equation (29) of the parameter P_{reg}. As a first approximation to the effect of non-linear waves on the longshore current profile one can write

$$v = v_{or} \{(P_{reg})_d/(P_{reg})_{d_b}\} f(y,P) \quad \ldots (30)$$

where subscript d (or d_b) means that P_{reg} is computed at depth d (or d_b), v_{or} is given by equation (29) and f(y,P) is the same function as before (equation (12)) with P = 0,2. The original longshore current data as compiled by Galvin and Nelson (1967) were re-analysed to obtain C_s in an analogous fashion to that described in Section 2 for the computation of C_{LH}. The equation of best fit is given by:

$$C_s = 10,5 \left(\frac{f_w g}{2C_h^2}\right)^{0,375} (\tan\alpha)^{0,81} \quad \ldots (31)$$

(see Figure 5)

A comparison of values of C_s obtained from equation (31) with those derived directly from the data resulted in a correlation coefficient r^2 = 0,9911. On the ohter hand, a comparison of values of C_{LH} as obtained from equation (15) with values of C_{LH} derived directly from the data yielded a correlation coefficient r^2 = 0,9471, that is, the correlation is not as good as that for C_s. Not only does the Vocoidal approach therefore yield a description of the current profile which is more in line with shallow-water waves than the linear theory approach, but it also yields a better estimate of the mean longshore current velocity.

More research is needed where the deep-water wave steepness is varied in a more systematic manner to improve this relationship. Nevertheless equation (31) constitutes the best estimate of C_s available at present and when used in conjunction with equation (30) it should yield better results than those predicted with the aid of the linear theory.

In the above it was implicitly assumed that the mean value of P remains 0,2 as was the case for linear waves. This assumption may not be correct, as pointed out by James (1974) and still needs to be carefully investigated with the aid of well-designed experiments.

5. TREATMENT OF RANDOM, VOCOIDAL WAVES

The same assumptions as those for the case of longshore currents generated by random, linear waves apply in the present study of random, vocoidal incident waves. The resulting shape of the longshore current profile after analogous deductions as in Sections 2 to 4 is:

$$v = \frac{5\pi}{16} P_{ran} \frac{\gamma^2 (gd)^{0,5}}{C_s} \left(\frac{d}{H}\right) \exp\left(\frac{-\gamma^2 d^2}{H_{rms}^2}\right) \tan\alpha \sin\Theta \cos\Theta \quad \ldots (32)$$

where P_{ran} is a Vocoidal parameter known simply in terms of Vocoidal wave properties.

Equation (32) is identical to its linear, random theory couterpart, equation (19), except for the inclusion of P_{ran} in equation (32).

In the following table values of P_{ran} are given in terms of T_c and γ.

T_c	Values of P_{ran}									
	H/d=0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
5	1,53	1,53	1,52	1,51	1,50	1,49	-	-	-	-
10	1,46	1,46	1,38	1,39	1,34	1,31	1,33	1,36	1,40	-
15	1,63	1,61	1,56	1,49	1,44	1,41	1,42	1,49	1,54	1,61
20	1,74	1,68	1,65	1,55	1,49	1,48	1,53	1,61	1,66	1,71
25	1,77	1,73	1,65	1,58	1,51	1,52	1,60	1,69	1,75	1,79
30	1,79	1,75	1,67	1,59	1,51	1,53	1,64	1,74	1,80	1,83
35	1,80	1,75	1,68	1,59	1,51	1,55	1,66	1,76	1,81	1,82
40	1,70	0,11	1,67	1,58	1,49	1,55	1,66	1,75	1,78	1,78

In analogy with linear waves a dimensionless scaling velocity v_{oir} is defined (see equation (21)):

$$v_{oir} = \frac{5\pi}{16} (P_{ran})_{bs} \frac{\gamma (g d_{bs})^{0,5}}{C_s} \tan\alpha \sin\Theta_{bs} \cos\Theta_{bs} \quad \ldots (33)$$

It follows that

$$v = v_{oir}\, f(y) \quad \ldots (34)$$

where $f(y) = \left(\frac{d}{d_{bs}}\right) \left(\frac{d}{H}\right) \gamma\left[(P_{ran})_d / (P_{ran})_{d_{bs}}\right] \exp\left(\frac{-\gamma^2 d^2}{H_{rms}^2}\right)$

$\ldots (35)$

The value of the roughness coefficient C_S for Vocoidal waves given by equation (31) applies. The same comments about wave set-up which were valid before are valid here, except that it has to be borne in mind that a new theory for the prediction of wave set-up due to random, vocoidal waves will have to be developed and tested.

6. SUMMARY AND CONCLUSIONS

The following is a summary of the main findings from the present study:

(1) The friction coefficient C_{LH} in the Longuet-Higgins model for longshore current prediction is not a constant, as was suggested previously, but varies with roughness and breaker zone beach slope. This supports earlier work by Komar and Inman.

(2) Using random incident waves a model was developed with the same approximations as for regular waves, except that lateral mixing is neglected, which gives very realistic profiles in close similarity with previous numerical work by Battjes.

(3) Using Vocoidal theory, explicit equations were derived for the longshore current profile under both regular and random wave attack. The friction coefficient is redefined and although it has a similar dependence on roughness and slope as in the linear case the actual values are higher.

(4) Although not discussed here, these four models, that is, for longshore current prediction under regular or random wave attack using linear or Vocoidal wave theory, have been extended to include the effect of mildly varying bed slope and mildly varying breaker index.

(5) Although the development of these models constitutes an appreciable advance, the models are only valid for the rare case of spilling breakers on a beach for which the water depth shows a monotonic decrease from breaker line to shore. Extensive research is still required on, for example, the non-steady nature of the longshore current, longshore currents on barred beaches and the effect of rip currents on longshore currents.

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