CHAPTER FOUR

WAVE ATTENUATION AND SET-UP ON A BEACH

by

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ABSTRACT

A theoretical two-dimensional model for wave heights and set-up in a surf zone is described and compared to measurements. The integral wave properties energy flux E_f, and radiation stress S_{xx} are determined from crude approximations of the actual flow in surf zone waves. Some physical aspects of the outer region are discussed and found to agree with our knowledge of the waves seawards and shorewards of this region.

1.   INTRODUCTION

This paper examines waves in the surf zone on a beach with no long-shore bars. We also restrict the considerations to regular waves with constant period T. The theoretical results are compared with measurements on a plane beach but in general the results should be applicable to any bottom topography provided the waves continue to break shoreward of the breaking point.

We only consider integral properties of the waves and conservation equations time averaged over a wave period. Hence the only information that can be obtained from the model is the variation of wave height, the associated energy dissipation and the set-up. In this respect the model to be described follows a long tradition of earlier investigations, although some of those only consider the wave height variation, not the set-up. The model deviates, however, from earlier contributions in the way the basic properties of the broken waves are determined.

The time averaged properties we need for the broken waves are energy flux, E_f, radiation stress, S_{xx}, and energy dissipation D. In previous models various non breaking wave theories have been used to determine E_f and S_{xx} in combination with either elaborate turbulent mixing models (Horikawa & Kuo, 1966) or energy dissipation equal to or related to the dissipation in a bore of the same height. References are Le Mehauté (1962), Divoky et al.(1968), James (1974). A somewhat different approach has been used by Dally (1980) who assumes the energy dissipation is proportional to $E_f - E_{f,0}$ where $E_{f,0}$ represents the energy flux in the smallest possible breaking wave which is empirically determined to be about H = 0.4 h. Sine wave theory is used for the wave integral properties.

The model presented here follows the numerical part of the solution described by Svendsen (1984) who also showed, however, that an analytical solution is possible under certain conditions. The following presentation also includes a discussion of the conditions at the shoreline.

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2.   THE BASIC EQUATIONS

We consider the two-dimensional problem sketched in Fig. 1 which also shows the definition of variables.

The three basic equations to be satisfied represent the conservation of mass, momentum and energy, integrated over depth and averaged over a wave period T.

The conservation of mass will not be invoked explicitly but used in the way the particle velocities in the wave are determined.

We consider regular progressive waves only and hence the momentum equation simply reads:

$$\frac{\partial S_{xx}}{\partial x} = -\rho g(h_0 + b)\frac{\partial b}{\partial x} \tag{2.1}$$

where S_{xx} is the radiation stress defined (exactly) by:

$$S_{xx} = F_m + F_p$$

$$F_m = \overline{\int_{-h_0}^{\eta+b} \rho u^2 dz} \; ; \; F_p = \overline{\int_{-h_0}^{\eta+b} \rho_D dz} - \frac{1}{2}\rho g \overline{\eta^2} \tag{2.2}$$
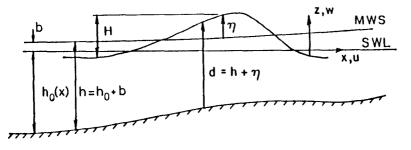

Fig. 1. *Definition sketch.*

with $^{-}$ denoting average over a wave period, and the dynamic pressure ρ_D given by:

$$p_D = \rho g(z-b) + p \tag{2.3}$$

i.e. p_D is defined on the basis of the local mean water depth. Notice that η is measured from the level $z = b$ so that $\overline{\eta} = 0$. In (2.1) we have also neglected the mean bed shear stress.

Using E_f for mean energy flux and D for the gain in energy (i.e. $D < 0$ for dissipation), the energy equation (also averaged over a wave period) becomes:

$$\frac{\partial E_f}{\partial x} = D \tag{2.4}$$

The definition of E_f is

$$E_f = \int_{-h_0}^{\eta+b} [p_D + \frac{1}{2} \rho (u^2 + v^2 + w^2)] u \, dz \qquad (2.5)$$

(which with p_D from (2.3) presumes no net current)

In both eqs. (2.2) and (2.5) velocities and pressures are the instantaneous values, so that these definitions also cover the turbulent flow situations in a surf zone.

In equation (2.4) we may choose at will the division between which type of energy belongs to E_f, and which is already considered lost (and hence belongs to D). Since energy once turned into turbulence will be dissipated to heat mostly with one wave period we choose to consider turbulent energy as energy already dissipated.

This highly simplifies the computations since it implies that we do not have to keep trace of the amount of turbulent energy present at the different phases of the breaking process.

The drawback of this is of course that we cannot evaluate the contribution to the momentum balance from the turbulent velocity fluctuations (u',v',w'). These contributions, however, are proportional to $\widetilde{u'^2} - \widetilde{w'^2}$ where $\widetilde{}$ represents ensemble averaging. And the measurements of Stive & Wind (1982) shows that these contributions only increase the radiation stress by a few per cent, mainly because u' and w' are not very different.

The conclusion of this is that E_f in (2.4) is taken as the ordered wave energy defined as

$$E_f = E_{f,w} = \int_{-h_0}^{\eta+b} \left(\widetilde{p}_D + \frac{1}{2} \rho (\widetilde{u}^2 + \widetilde{w}^2) \right) \widetilde{u} \, dz \qquad (2.6)$$

and D represents minus the production of turbulent energy.

To facilitate the analysis we introduce non-dimensional measures of both $E_{f,w}$, S_{xx} and D using the definitions

$$B = E_{f,w}/(\rho g c H^2) \qquad (2.7)$$

$$P = S_{xx}/\rho g H^2 \qquad (2.8)$$

$$D = D \, (4hT/\rho g H^3) \qquad (2.9)$$

where T is the wave period and c the speed of propagation for the wave.

These definitions are inspired by our knowledge from, say, linear wave theory which for B would yield the result $c(1 + G)/8$ where $(G = 2kh/\sinh 2kh)$ and for P similarly $P = (1 + 2G)/16$. The form chosen for the definition of D is related to the energy dissipation in a bore or hydraulic jump.

Thus the idea behind the dimensionless quantities B, P, and D is that the major part of the variation of $E_{f,w}$, S_{xx} and D has been factored out so that B, P, and D may be expected to vary only slightly.

Substituting into (2.1) and (2.4) we therefore find the equations

$$\frac{d}{dx} (H^2 P) = - (h_0 + b) \frac{db}{dx} \quad \text{and} \qquad (2.10)$$

$$\frac{d}{dx} (c H^2 B) = \frac{H^3}{4hT} D \qquad (2.11)$$

Thus, provided we can describe B, P, D and c in terms of h_0, b, T and H then (2.10) and (2.11) represent two simultaneous equations from which E(x) and b(x) may be determined.

3.   THE INTEGRAL PROPERTIES OF SURF ZONE WAVES

The ideas used in the determination of the three quantities B, P, and D are associated with the observation (see e.g. Svendsen et al., 1978) that from a point somewhat after breaking the waves become bore-like irrespective of the initial type of breaking.

B & D are determined from the definitions of $E_{f,w}$ and S_{xx}, i.e. (2.6) and (2.2), respectively. In essence this means that we need relevant approximations for $\tilde{u}$, $\tilde{w}$ and p_D in these expressions.

The important feature dominating the bore-like wave motion is the surface roller, which in essence is a volume of water carried shorewards with the breaker. Figure 2a shows a typical situation, and also indicates a typical velocity distribution along a vertical at the front of the wave.

The roller is defined as the recirculating part of the flow above the dividing streamline (in a coordinate system following the wave). Since it is resting on the front of the wave, the absolute mean velocity in the roller equals the propagation speed c for the wave, and in the following we use this value for the velocity in the roller, neglecting the z-variation.

In the present two-dimensional study we assume a zero net mass flux which of course implies that there is a return flow compensating for the surface drift.

From observations we know that in the inner region the change in wave shape is slow so the instantaneous volume flux:

$$Q = \int_{-h_0}^{\eta + b} u(x,z,t)\,dz \tag{3.1}$$

may for $\overline{Q} = 0$ be determined as:

$$Q = c\eta = Ud \tag{3.2a,b}$$

where the surface profile is specified so that $\overline{\eta} = 0$. U is the wave particle velocity averaged over depth.

Thus assuming the particle velocity destribution shown in Fig. 2b we are able from (3.2) at any phase of the wave to express the velocity u_0 below the roller in terms of η and e (e = e(x,t) being the vertical thickness of the roller. The resulting full expression is

$$u_0 = c\,\frac{\eta - e}{d - e} \tag{3.3}$$

where e = 0 away from the wave front. We also assume that $w^2 (<< u^2)$ can be neglected.

The pressure is assumed to be hydrostatic. This is of course not quite correct, but in combination with the rather crude assumptions for u and in view of the very small deviations from hydrostatic pressure actually measured by Stive (1980) this is the most relevant approximation. Thus we have

$$p_D = \rho g \eta \tag{3.4}$$

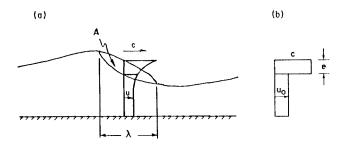(a)   (b)

Fig. 2a. *Velocity distribu-*   Fig. 2b. *Approximations for the*
tion under the front of a   *horizontal velocities in the*
breaking wave.   *surf zone waves .*

When these approximations are substituted into (2.6) we find after
some manipulations and further omission of small terms, that the leading
approximation for B may be written

$$B = B_0 + \frac{1}{2} \frac{A}{H^2} \frac{h}{L} \tag{3.5}$$

where

$$B_0 \equiv \overline{(\eta/H)^2} \tag{3.6}$$

and A is the vertical cross sectional area of the roller (see Fig. 2a).
The mentioned omission of small terms are based on the following approx-
imations

$$\overline{(\eta/H)^3}, \ \overline{(\eta/H)^4} << \overline{(\eta/H)^2}$$

and $\overline{\eta^3 e/h^4} << \overline{(\eta/H)^2}$

The "wave length" L in (3.5) is defined as L(x) = c(x) T. Hence L
is not the distance between two consecutive wave crests.
We have also assumed that $c \sim \sqrt{gh}$ based on the measurements by
Svendsen et al (1978) who found $c \sim 1.05 - 1.10 \ \sqrt{gh}$.
A similar procedure yields P by substitution of the assumptions for
u and p_D into (2.2). The result may be written

$$P = \frac{3}{2} B_0 + \frac{A}{H^2} \tag{3.7}$$

Here the small terms omitted are of the type $\overline{(e/H)^2}$, $\overline{\eta e/\eta^2}$.
Experimental information on the roller area A is only available for
waves breaking behind a hydrofoil (Duncan, 1981). Fig 3 shows a plot of
Duncan's data for A which suggests that we can use the approximation

$$A = 0.9 \ H^2 \tag{3.8}$$

by which B and P reduce to

$$B = B_0 + 0.45 \ h/L \tag{3.9}$$

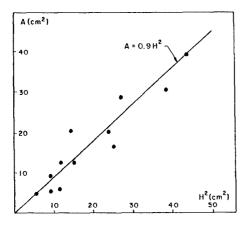$$P = \frac{3}{2} B_0 + 0.9 \ h/L \tag{3.10}$$

Fig. 3. *The cross sectional area for A for the roller.*
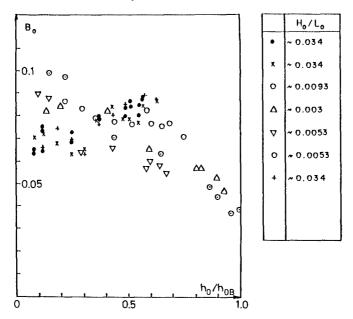   *Measurements by Duncan 1981.*

Fig. 4. *Measured values of B_0 defined by (3.6.). Hansen (1982).*

The energy dissipation D was analysed theoretically by Svendsen et al (1978) and more explicitly by Svendsen & Madsen (1981). The conclusion which can be derived from the expression they find for D is that the energy dissipation will normally be almost equal to that in a hydraulic jump of the same height. Deviations (in particular in the upward direction) depend on the detailed velocity and pressure distributions in the wave - particularly in the wave trough, but normally they do not seem to exceed 20%.

For the present rather crude model it is natural simply to use D equal to the value in a hydraulic jump of the same height, and this yields

$$D = \frac{-h^2}{d_c d_t} \tag{3.11}$$

where d_c and d_t are the water depths under wave crest and wave trough, respectively.

It is convenient to express d_c and d_t in terms of the crest elevation η_c and the wave height H. With

$$d_c = h + \eta_c, \quad d_t = h + \eta_c - H$$

(3.11) may be written:

$$D = - \left[\left(1 + \frac{\eta_c}{H} \frac{H}{h} \right) \left(1 + \frac{H}{h} \left(\frac{\eta_c}{H} - 1 \right) \right) \right]^{-1} \tag{3.12}$$

which shows that for fixed (η_c/H), D depends slightly on H/h. Figure 5 shows the variation and Fig 6 gives values of η_c from the experiments by Hansen quoted above. As was the case for B_0 the results for η_c/H show significant scattering but in the inner region of the surf zone the value is mostly 0.6-0.7 which from Fig. 5 is seen to represent a D nearly independent of H/h.

Figure 5 also shows that D only varies slightly with η_c/H. In other words the primary variation of the energy dissipation is represented by the H^3/h dependence already accounted for in the definition (2.9).
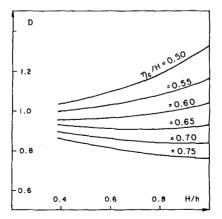

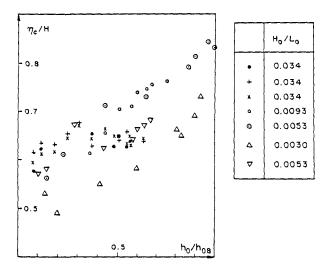Fig. 5. *The variation of D with H/h and η_c/H according to (3.12).*

	H_0/L_0
•	0.034
+	0.034
x	0.034
o	0.0093
⊙	0.0053
△	0.0030
▽	0.0053

Fig. 6. *Measurements of* η_c/H *in the surf zone. Hansen (1982).*

4.   COMPARISON WITH MEASUREMENTS

The outer and the inner region

The original concept of an outer (transition) region and an inner
(bore) region was primarily based on the visual observations of wave
behaviour after breaking (see Svendsen et al., 1978). The impression is
one of a gradual change towards the bore shape found in the inner region.
Consequently, no attempt was made to define a proper limit between the
two regions, and wave height measurements truly do not suggest a natural
definition.

The situation is quite different, when the variations in mean water
level are considered. Figure 7 shows some examples from Hansen and
Svendsen (1979) covering a wide range of deep water steepnesses. Most
of them exhibit a marked change in the slope of the mean water level
at some distance shoreward from the breaking point. A similar variation
can also be seen in other investigations such as Bowen et al. (1968) and
Stive and Wind (1982). The mean water level is horizontal or weakly
sloping after the start of breaking over a distance of 5-8 times the
breaker depth and then a rather sharp increase in slope occurs. The
distance of nearly horizontal mean water level is comparable to the
distance of the most obvious transformations of the wave shape following
after the initiation of breaking, and so it will be coherent with the
original concept to *define* the limit between the outer and the inner
region as the point where the slope of the mean water level changes.
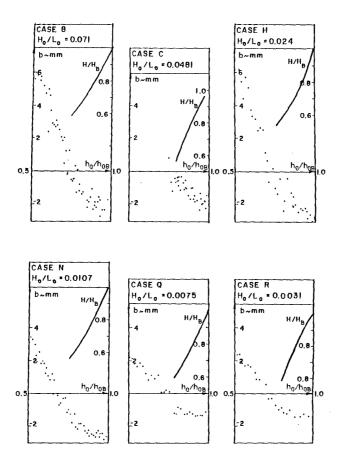In the following this is termed the *transition point*.

Fig. 7. *Measurements of the mean water level shoreward of the breaking point. The figure also shows the simultaneous change in wave height relative to the value HB at the breaking point (Measurements from Hansen & Svendsen, 1979).*

Wave conditions in the inner region

Physical explanations for these changes are sought in section 5. First, however, we notice that since the results derived above for the parameters B, P and D are based on the wave properties in the inner zone, comparisons with experimental data should start at the transition point.

Numerical solution of equations (2.10) and (2.11) using (3.9),
(3.10) and (3.12) then yields results for H and b in the inner surf zone.
In the computations we have neglected the variation of B_0 and n_c/H
using the constant values $B_0 = 0.075$ and $n_c/H = 0.6$. This, however, may
not always be sufficient for obtaining reliable results but a more sys-
tematic investigation of the variation of B_0 and n_c/H with wave param-
eters and bottom topography is required. For h/L is used $T\sqrt{g/h}$ corre-
sponding to $c = \sqrt{gh}$, with $h = h_0 + b$.

Discussion of results

Figures 8, 9, and 10 show a comparison with results for three
rather different wave steepnesses, all on a plane slope 1/34.3. In
general the agreement is quite good, particularly for the set-up. The
latter is of particular interest because the calculations show that b
is much more sentitive to the assumptions made than is the wave height
variation. As can be expected from what was said above about D, the H
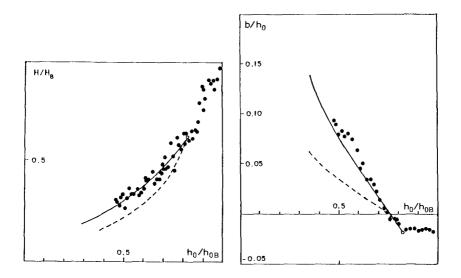variation turns out to be virtually independent of the choice of n_c/H.

Fig. 8. *Wave heights and set-up for a wave with deep water*
        steepness $H_0/L_0 = 0.071$ - theory using eqs. (3.9) and
        (3.10)— — —theory without a surface roller; • measurements
        by Hansen and Svendsen (1979), Case B.

It is noticed that in some of the cases the H-variation is slightly less curved than corresponding to the best fit of measurements, and the values of H become a little too large. This can be adjusted by using a value of D perhaps 20-30% larger than given by eq. (3.12), which is quite consistent with the results reported earlier(see Svendsen et al., 1978; Svendsen and Madsen, 1981) that the actual energy dissipation in a surf zone wave is generally larger than in a hydraulic jump of the same height.
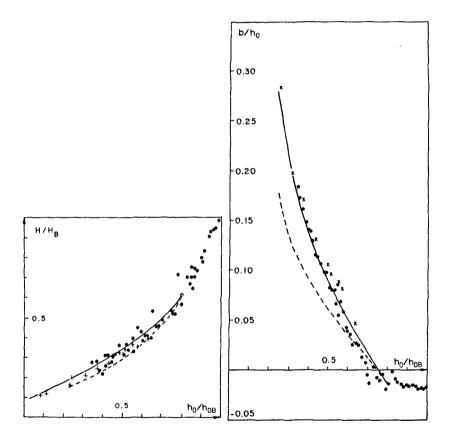

Fig. 9. *Wave heights and set-up for a wave with deep water steepness $H_0/L_0 = 0.024$. — theory using eqs. (2.10) and (2.11) --- theory without a surface roller; · Measurements by Hansen & Svendsen (1979) Case H.+Measurements by Hansen (1982).*

In Fig. 8-10 are also included results obtained by omitting the
surface roller (dotted curve corresponding to B = B_0 and P = 3/2 B_0).
The effect is quite appreciable. On the other hand, considering that
the presence of the surface roller significantly increases the energy
flux and radiation stress, the difference between the full and the
dotted lines in these figures indicates that the effect of also including
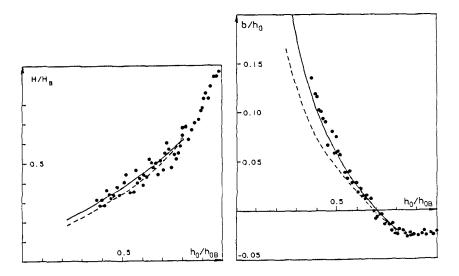turbulence, deviation from static pressure, etc. would hardly be dis-
cernible.

Fig. 10. *Wave heights and set-up for a wave with deep water*
steepness $H_0/L_0 = 0.010$. —— theory using eqs. (2.10)
and (2.11); —— theory without a surface roller;
• measurements by Hansen and Svendsen (1979), Case N.

The situation at the shoreline

Svendsen (1984) found that for D and B constant an analytical
solution can be obtained to the energy equation. This solution is given
by

$$\frac{H}{h} = \frac{H_r}{h_r} \frac{1}{h'^{5/4}[1 + KD(h'^{-3/4}-1)]} \qquad \text{with } h' = \frac{h_0 + b}{h_r} \qquad (4.1)$$

where H_r/h_r corresponds to values at a reference point (boundary condi-
tion) and K is a constant also depending on the wave properties at that
boundary point.
    This solution predicts a (very flat) minimum for H/h at some point
and shoreward from that point H/h increases with H/h → ∞ as h → 0.
    Clearly this is not in accordance with reality and turns out to be
associated with the assumption that D is constant. As h → 0, D will

increase similarly and prevent the singularity. This can be seen by
considering the energy equation in the form

$$\left(\frac{H}{h}\right)_x = -\left(\frac{h_x}{h} + \frac{c_x}{2c} + \frac{B_x}{2B}\right)\frac{H}{h} + \frac{D}{8cTB}\left(\frac{H}{h}\right)^2 \qquad (4.2)$$

(For derivation see Svendsen et al., 1978). As $h \to 0$ we assume the wave
has deformed to a perfect sawtooth so that $\eta_c = -\eta_t = \frac{1}{2}$ H. Thus we get
from (3.12)

$$D = \frac{-1}{1 - \frac{1}{4}(H/h)^2} \qquad ; \qquad B_x = 0 \qquad (4.3)$$

With $c \sim \sqrt{gh}$ we also have $c_x/c \sim h_x/4h$. Substitution of (4.3) into
(4.2) yields the relation

$$\left(\frac{H}{h}\right)_x = -\frac{5\,h_x}{4h}\frac{H}{h} - \frac{1}{1 - \frac{1}{4}(H/h)^2}\,\frac{1}{8T\sqrt{gh}\,B}\left(\frac{H}{h}\right)^2 \qquad (4.4)$$

$h \to 0$ yields H/h increasing. But as $H/h \to 2-$ the dissipation grows.
Hence

$$\frac{H}{h} \to 2 \quad \text{as } h \to 0 \qquad \text{i.e.} \quad d_t \to 0 \qquad (4.5)$$

     Thus the model described above has the limiting value of H/h = 2
at the shoreline, not ∞.
     Even this limit is considerably higher than the observed values.
As $h \to 0$, however, we also get $L/h \to \infty$ which implies that the parame-
ter h_x L/h is no longer small. That is the bottom slope is not negli-
gible and the assumption of locally horizontal bottom does not hold.

5.   THE WAVE MOTION IMMEDIATELY AFTER BREAKING

     It is tempting and illustrative to try if the solution presented
in the previous chapters also applies to the region of rapid transition
right after the initiation of breaking.
     Figure 11a shows a computation of the wave height variation, start-
ing at the breaking point. The agreement is surprisingly good. This,
however, does not apply to Fig. 11b which gives a similar comparison for
the set-up b/h_0. The two figures together show the paradoxical fact
already hinted at earlier that the radiation stress in the transition
region stays nearly constant even with a 30-40% decrease in wave height.
Recalling eq. (2.8) this can only be true if P is increasing, roughly
as H^{-2}.
     By considering what happens when the breaking starts, it becomes
clear that the overturning of the wave cannot immediately be matched by
dissipation of a similar amount of energy. In the first transformation
a large amount of the lost potential energy is converted into forward
momentum flux which eventually is concentrated mainly in the roller, and
this must be the reason for the simultaneous increase in P.
     This is also consistent with the fact that P for very high waves
is rather small. There are no results available for the skew waves at
the breaking point, but the high order results for Stokes waves presented

by Cokelet (1977) can be used to determine P for very high, symmetrical waves. Values found are typically around P = 0.07, i.e. less than half the value of 3/16 for linear long waves and considerably less than for cnoidal waves of the same height.
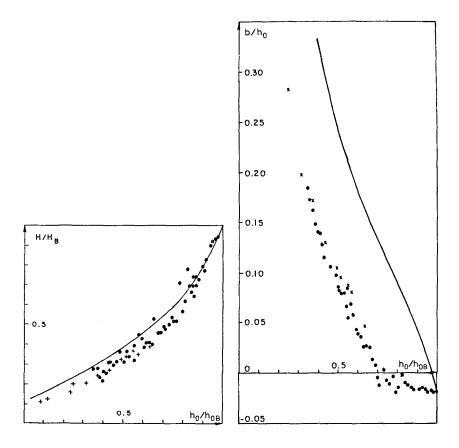

Fig. 11a and b. *Wave height and set-up using the theoretical results for B, P and D from the breaking point.*
· Measurements by Hansen & Svendsen (1979), Case H,
+ measurements by Hansen (1982).

The increase in P, however, is inevitably associated with a similar increase in B, the energy flux for a wave of unit height and propagation speed. The mechanism is the same as for P: very steep waves with peaky crests represent a very small energy flux relative to their height and the collapse of the crest in the initial stage of breaking leads to a significant increase in B.

It may be shown that these shifts in P and B are also consistent with the result found in section 3, that waves in the inner region represent rather high values of radiation stress and energy flux relative to their height and speed.

But even with no energy dissipation an increase in B will in itself require a decreasing wave height. Hence the question arises: how much of the wave height decrease in the outer transition region is actually due to redistribution of momentum and energy (represented by the changes in P and B) and how much is real energy dissipation?

This problem and the change in B and P can be analysed by considering the conservation of momentum and energy over the transition region as a whole in analogy to the jump conditions which apply to bores and hydraulic jumps in open channel flow and to shocks in compressible flows.

Svendsen (1984) found for a specific example that the wave height in the outer region with S_{xx} = constant decreased to 0.65 H_B at the transition point that is $H^2 = 0.423 \, H_B^2$ corresponding to an apparent energy reduction (had B been constant) of 57.7% of the energy at the breaking. Due to the simultaneous increase in B, however, the actual energy dissipation is only about 20%, or 1/3 of the 57.7%.

6.   ACKNOWLEDGEMENT

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