CHAPTER THIRTY THREE

CALCULATION OF DIRECTIONAL WAVE SPECTRA BY THE MAXIMUM ENTROPY METHOD OF SPECTRAL ANALYSIS

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Abstract

Two analysis techniques for calculating directional wave spectra from measured pressure and biaxial current components were intercompared using data from the 25 October 1980 Atlantic Remote Sensing Land Ocean Experiment (ARSLOE) storm. The two methods are the conventional Fast Fourier Transform (FFT) method and a Maximum Entropy Method (MEM). The MEM is a nonlinear data adaptive method of spectral analysis which is capable of generating higher resolution spectral estimates from shorter data records than conventional FFT methods. The MEM has shown good agreement with the frequency and directional wave spectra calculated using conventional methods.

Introduction

The accurate calculation of directional wave spectra is important in the coastal zone for determination of coastal erosion, littoral sediment transport, diffraction and refraction of waves, and interaction with marine structures. One of the top research priorities of the National Research Council's Workshop on Wave Measurement Technology conducted in Washington in 1981 was the development of more efficient analysis techniques for the calculation of directional wave spectra (6). Considerable interest was generated at the ASCE Conference on Directional Wave Spectra Applications held in Berkeley on the Maximum Likelihood Method (MLM) for calculating directional spectra (8). Borgman (1) presented a proposal during the Second Workshop on Maximum Entropy and Bayesian Methods in Applied Statistics for comparisons of directional wave spectra instrument systems and analysis methods. Among the methods discussed were the conventional FFT, MEM, MLM, variational fitting, and linear programming. The four latter methods are all data-adaptive procedures which are capable of generating a higher resolution spectral estimate from shorter data records than conventional FFT methods. The primary objective of this paper is to demonstrate the promise of the MEM as a technique to increase directional resolution in directional wave spectra estimates.

Grosskopf (7) reported the results of an intercomparison of five different measurement systems and analysis techniques for calculating directional wave spectra from data obtained during the ARSLOE

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experiment. The five measurement systems included two pressure/biaxial current meter arrays belonging to the Coastal Engineering Research Center (CERC) and Woods Hole Oceanographic Institution (WHOI), respectively, a Scripps S_{xy} array, an NHL triaxial current meter array, and a CERC X-Band Surface imaging radar (XERB). In this paper, an intercomparison with the CERC pressure/biaxial current meter (PUV) array is presented. Data representative of storm conditions during 25 October 1980 of the ARSLOE experiment are used.

Techniques Used

In 1965 Cooley and Tukey sparked a revival of the Fast Fourier Transform (FFT) which had been known for years but was not practical until the advent of the high speed digital computer. The direct method of calculating spectral estimates involving magnitude squaring of the transform of windowed data records became popular. Unfortunately, this method unreasonably assumes the data to be zero outside the selected number of points and repeats itself periodically.

In 1967, Burg (3) introduced the concept of MEM of autospectral analysis. Entropy is a measure of the information content contained in a signal. Maximizing entropy, therefore, maximizes the information transmitted in a signal. The concept involves finding a spectral estimate corresponding to the most random or unpredictable time series whose extended correlation function satisfies the constraint that it agrees with known values. Since then researchers have successfully applied MEM to such diverse fields as geophysics, neurophysics, and ocean engineering. Campbell (4) accurately estimated natural frequency and damping ratio parameters and their 95 percent confidence limits for offshore platforms. The multichannel MEM was shown to be a useful tool in mode shape identification of offshore structures (2). Houmb (9) showed that MEM is a powerful tool for estimation of wave spectra and proposed a data acquisition system based on the technique.

In conventional FFT analysis of PUV arrays, the data are windowed and Fourier transformed; and the variance lost due to windowing is restored to the line spectra. The auto and cross-spectra terms are calculated from the line spectrum and used to obtain the first five directional Fourier series coefficients (truncated series representation for three independent measurements). These coefficients are usually band or ensemble averaged to decrease the variance of the estimate. Finally, the directional wave spectrum is calculated from the averaged coefficients using a smoothing or weighting function such as Longuet-Higgins, et al. (11) to eliminate negative side lobes.

Because the MEM is data adaptive, it does not suffer from the "bias vs. variance" tradeoff due to finite record length requirements of conventional methods. When calculating spectral estimates at one frequency, it is able to adjust itself to be least disturbed by power at neighboring frequencies. In this paper, the MEM method is used instead of conventional FFT techniques to calculate the auto- and cross-spectra for input to the directional Fourier series coefficient algorithm. Other portions of the algorithm are unchanged.

Multichannel Maximum Entropy Method of Spectral Analysis

In order to assist understanding of the multichannel MEM algorithm, a brief review of the single-channel MEM model as a prediction error (PE) filter will be presented. An error series, $e(n)$, is defined as the difference between the desired signal, $d(n)$, and the actual signal, $y(n)$. The desired value is chosen as the input signal advanced one time unit ahead. The actual signal represents past values of the input signal. These past or previous values of the time series are used to predict the next value. According to least squares theory, a mean square error or error power, $P(L)$, is defined as the expected value of the square of the error signal. The energy contained in this error power must be minimized in such a way that the input signal is whitened as the filter order is increased. The Normal or Wiener-Levinson equations are obtained as a result of this minimization and are given by

$$[R]\{A\} = \{P\} \tag{1}$$

where

$[R]$ = matrix of autocorrelation coefficients, 0 to 1 lags

$\{A\}$ = column vector of prediction error filter coefficients

$\{P\}$ = column vector of prediction errors

The Normal equations are then solved by the Levinson-Durbin recursion to obtain the PE filter coefficients, A. The MEM spectral estimate, Sx, defined between the Nyquist frequency, f_{ny}, is then given by

$$Sx(f) = |A(f)|^2 \, Sw(f) \qquad -fny \leq f \leq fny \tag{2}$$

$$= \frac{2\sigma^2(L)\Delta}{\left|1 - \sum_{m=1}^{L} A(m) \exp(-j2\pi fm\Delta)\right|^2}$$

where $\sigma^2(L)$ or $Sw(f)/2\Delta$ is the prediction error or white noise variance and the denominator is the magnitude squared of the Fourier transform of the PE filter coefficients. The Δ is the time increment in seconds between sampled data points.

Thus, the single-channel MEM filter can be written in a form familiar to engineers. The MEM spectral estimate, $Sx(f)$, (i.e. output spectrum) is the product of the prediction error spectrum, $Sw(f)$ (i.e. input spectrum), and the magnitude squared of the transfer function of the PE filter, $A(f)$. The MEM spectral estimate is obtained by (1) calculating the PE filter coefficients out to the desired filter order of length L (as determined by Akaike's Final Prediction Error (FPE) or other suitable model order criterion), (2) calculating the PE due to a white noise signal at filter order L, (3) taking the magnitude squared of the Fourier transform of the PE coefficients, and (4) performing the operations indicated in Equation 2.

For the multichannel MEM algorithm, the development is analogous to the single-channel case. The expected mean square values of forward and backward errors of length M $(M \leq L)$ are minimized for the optimum filter. As a result, the Normal equations for the two-channel case forward filter coefficients, CF, are given by

$$[RF]\ \{CF(M,m)\} = \{V\} \tag{3}$$

where

$[RF]$ = forward R-matrix, Toeplitz, square block submatrices

$\{V\}^T$ = forward power matrix, $[P(M)\ 0\ 0 \ldots 0]$

m = coefficient number

The R4 element or 2x2 submatrix of the RF matrix for a lag of 4 for the two-channel case is

$$\{R4\} = \begin{bmatrix} R11(4) & R12(4) \\ R21(4) & R22(4) \end{bmatrix} \tag{4}$$

where the diagonals are the autocorrelations and the off-diagonals are the cross-correlations between channels 1 and 2.

The single-sided multichannel MEM spectral estimate matrix is a function of the Fourier transform of the forward filter coefficient matrix and is given by

$$G(f) = 2\Delta[CF^{-1}(1/z)]^{*}\ P(M)\ [CF^{-1}(1/z)] \tag{5}$$

where $z = \exp(-j2\pi f\Delta)$. Equation 5 reduces to Equation 2 for the single channel case if matrices are replaced by vectors and vectors by scalars. The inverse matrix operations become divisions, and the product of the filter coefficients with their complex conjugates gives the magnitude squared as before.

Directional Wave Spectral Theory

The directional wave spectrum is given by

$$S(\sigma,\theta) = S(\sigma)\ D(\sigma,\theta) \tag{6}$$

where $S(\sigma)$ is the one-dimensional frequency spectrum

$$S(\sigma) = \int_{0}^{2\pi} S(\sigma,\theta)\,d\theta \tag{7}$$

and $D(\sigma,\theta)$ is a directional spreading function which satisfies

$$\int_{o}^{2\pi} D(\sigma,\theta)d\theta = 1 \qquad (8)$$

For a pressure/biaxial current meter combination (PUV), the derivation of the wave directional spectra is analogous to that of Longuet-Higgins et al. (11) and Cartwright (5) for heave-pitch-roll buoys. The water surface elevation is given by

$$\eta(x,y,t) = \int_{-\infty}^{\infty} \int_{o}^{2\pi} A(\sigma,\theta) \exp\,[i\psi(x,y,t,\sigma,\theta)]\; d\theta\; d\sigma \qquad (9)$$

The dynamic wave pressure and horizontal u- and v-water particle velocities in the x and y directions are, respectively,

$$p(x,y,t) = \int_{-\infty}^{\infty} \int_{o}^{2\pi} A(\sigma,\theta)K_p(\sigma) \exp\,[i\psi(x,y,t,\sigma,\theta)]\; d\theta\; d\sigma \qquad (10)$$

$$u(x,y,t) = \int_{-\infty}^{\infty} \int_{o}^{2\pi} A(\sigma,\theta)K_u(\sigma) \cos\theta \exp\,[i\psi(x,y,t,\sigma,\theta)]\; d\theta\; d\sigma \qquad (11)$$

$$v(x,y,t) = \int_{-\infty}^{\infty} \int_{o}^{2\pi} A(\sigma,\theta)K_u(\sigma) \sin\theta \exp\,[i\psi(x,y,t,\sigma,\theta)]\; d\theta\; d\sigma \qquad (12)$$

where

$$A(\sigma,\theta) = \text{amplitude spectrum} = \sqrt{2S(\sigma)d\sigma}\;\;\sqrt{D(\sigma,\theta)d\theta} \qquad (13)$$

$$\psi(x,y,t,\sigma,\theta) = \text{random phase angle} = \sigma t - k_x x - k_y y \qquad (14)$$

$$K_p(\sigma) = \gamma\,\frac{\cosh k\,(h+z)}{\cosh\,(kh)} \qquad (15)$$

$$K_u(\sigma) = \sigma\,\frac{\cosh k\,(h+z)}{\sinh\,(kh)} \qquad (16)$$

The single-sided autospectra for pressure and u- and v-velocities are, respectively,

$$G_{pp}(\sigma) = \int_{o}^{2\pi} S(\sigma,\theta)\ K_p^2(\sigma)\ d\theta \qquad (17)$$

$$G_{uu}(\sigma) = \int_{\sigma}^{2\pi} S(\sigma,\theta)\ K_u^2(\sigma)\ \cos^2\theta\ d\theta \qquad (18)$$

$$G_{vv}(\sigma) = \int_{\sigma}^{2\pi} S(\sigma,\theta)\ K_u^2(\sigma)\ \sin^2\theta\ d\theta \qquad (19)$$

The single-sided cross-spectra are given by

$$S_{pu}(\sigma) = \int_{o}^{2\pi} S(\sigma,\theta)\ K_p(\sigma)\ K_u(\sigma)\ \cos\theta\ d\theta \qquad (20)$$

$$S_{pv}(\sigma) = \int_{o}^{2\pi} S(\sigma,\theta)\ K_p(\sigma)\ K_u(\sigma)\ \sin\theta\ d\theta \qquad (21)$$

$$S_{uv}(\sigma) = \int_{\sigma}^{2\pi} S(\sigma,\theta)\ K_u^2(\sigma)\ \sin\theta\ \cos\theta\ d\theta \qquad (22)$$

Since a PUV gage has only three independent measurements, only the first five directional Fourier coefficients can be derived. They are expressed as

$$A_o(\sigma) = \frac{S_{pp}(\sigma)}{2\pi\ K_p^2(\sigma)} = \frac{S_{uu}(\sigma) + S_{vv}(\sigma)}{2\pi\ K_u^2(\sigma)} \qquad (23)$$

$$A_1(\sigma) = \frac{S_{pu}(\sigma)}{\pi\ K_p(\sigma)\ K_u(\sigma)} \qquad (24)$$

$$B_1(\sigma) = \frac{S_{pv}(\sigma)}{\pi\ K_p(\sigma)\ K_u(\sigma)} \qquad (25)$$

$$A_2(\sigma) = \frac{G_{uu}(\sigma) - G_{vv}(\sigma)}{\pi K_u^2(\sigma)} \tag{26}$$

$$B_2(\sigma) = \frac{2S_{uv}(\sigma)}{\pi K_u^2(\sigma)} \tag{27}$$

Ideally, the number of directional Fourier coefficients would be infinite, and perfect frequency and directional resolution would be obtained. However, due to the limited number of coefficients, a loss of resolution and negative side lobes results in a broadened spectra with lost total energy (variance). A binominal weighting function given by Longuet-Higgins et al. (11), although ensuring a non-negative directional spectra, still results in some loss of directional resolution. It is given by

$$S(\sigma,\theta) = A_0 + 2/3(A_1 \cos\theta + B_1 \sin\theta) + 1/6(A_2 \cos 2\theta + B_2 \sin 2\theta) \tag{28}$$

Some of the statistical parameters which are compared in this paper are significant wave height, peak frequency, peak wave direction, and peak directional spread. The significant wave height is related to the zeroth moment of the directional wave spectrum by

$$H_{mo} = 4.0\sqrt{M_o} \tag{29}$$

where M_o is the sum of the energy or variance over all frequencies and directions. The peak frequency is the center frequency of the band containing the maximum energy. The peak wave direction is the mean direction the waves travel toward in the band of maximum energy. It is defined as

$$\bar{\theta}(\sigma) = \arctan\left[\frac{B_1(\sigma)}{A_1(\sigma)}\right] \tag{30}$$

The peak directional spread is an estimate of the spread of energy about the peak wave direction. It is given by

$$\theta_s(\sigma) = \left\{ 2 - \frac{2\left[A_1^2(\sigma) + B_1^2(\sigma)\right]^{1/2}}{A_o(\sigma)} \right\}^{1/2} \tag{31}$$

Description of Arsloe Experiment

The ARSLOE was conducted during October and November 1980 by CERC's Field Research Facility (FRF) at Duck, NC (see Figure 1). The purpose was to evaluate various types of in situ and remote sensing

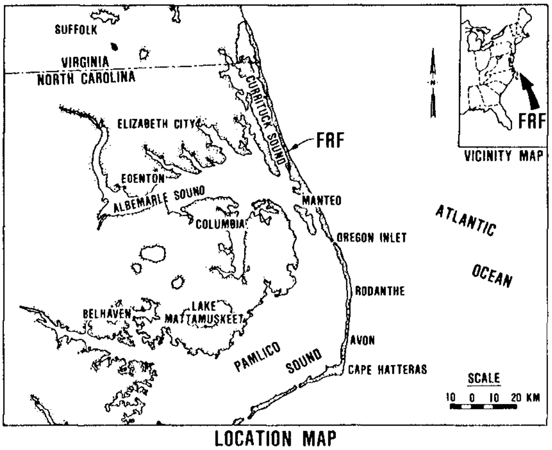


Figure 1. CERC Field Research Facility (FRF)

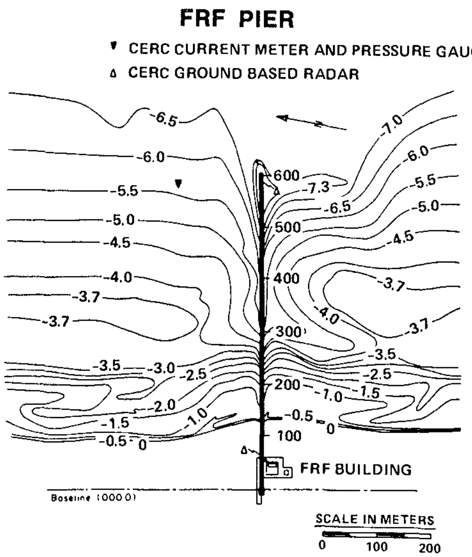


Figure 2. CERC FRF pier bathymetry

devices for measuring directional wave spectra in shallow water. All in situ instruments were installed on or near the end of the FRF pier (see Figure 2).

A major storm moved through the area during the period 23-25 October 1980. A total of 15 data records during the hours of 0915 to 1415 on 25 October was selected for intercomparisons with the CERC FFT and MEM directional wave spectra programs. The weather conditions during this time period were characterized by winds of 10-15 m/s (22-34 mph). Wind directions were initially from the northeast, then from the east followed by a rapid shift southward then around to the west and northwest (12). Figure 3 shows the mean current speed and direction during this time as determined from the PUV data records.

The gage used for software comparisons in this paper is the CERC PUV gage: a Model 551 Marsh McBirney biaxial current meter with a Bell and Howell pressure gage. The table below summarizes the sampling parameters used. The water depth at this location was 5.5-6 meters (18.0-19.7 ft) (see Figure 2).

Sampling and Analysis Parameters

Sampling Frequency, Hz	4
Burst Interval, min	20
Total Number of Points per Record	4096
Record Length, sec	1024
Bandwidth, Hz	0.00781

Results and Discussion

Intercomparisons are made between the directional wave spectra and moment generated parameters calculated by the CERC and MEM analysis methods for the 15 data records collected during the period 25 October 1980. For the MEM auto- and cross-spectra estimates, a constant model order (filter length L) was selected for each record based on the results for each from Akaike's FPE criterion. Figures 4, 5, and 6 are sample plots of frequency and peak directional wave spectral estimates for records 12, 13, and 14 (i.e., 1315, 1335, and 1355), respectively. The solid curve in all figures corresponds to the FFT-generated spectral estimates; the dashed line represents the MEM spectral estimates. The table on the following page compares the results for the significant wave height, peak frequency, peak wave direction, and peak directional spread for these three data records. Figures 7 and 8 are time series plots of significant wave height, peak frequency, peak direction, and peak directional spread for all of the 15 data records intercompared.

In general, the MEM frequency spectral estimates are smoother than those of the FFT, with the peaks shifted slightly in frequency. Records 13 and 14 peak frequencies differ by one bandwidth interval (which is the minimum resolveable with the band averaging procedure used). As a

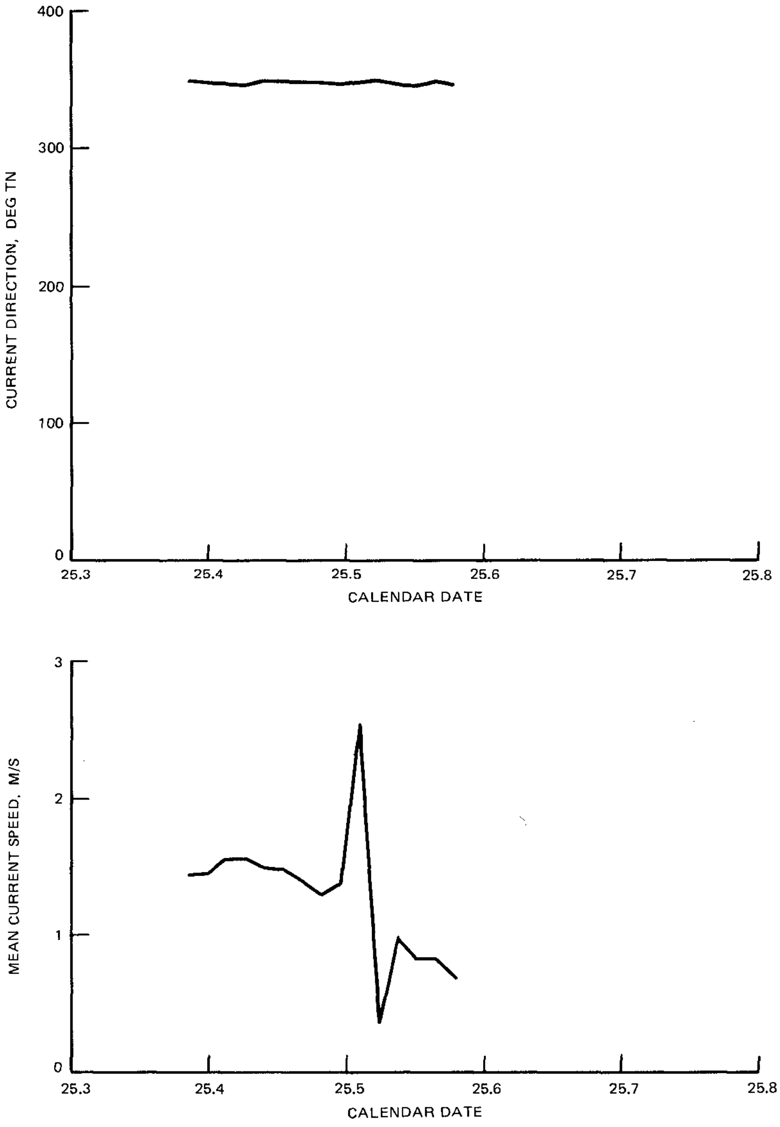
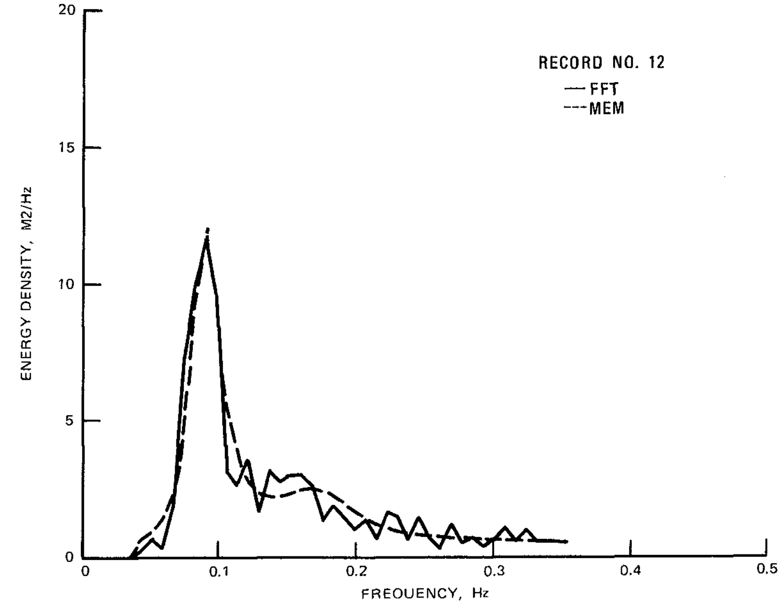
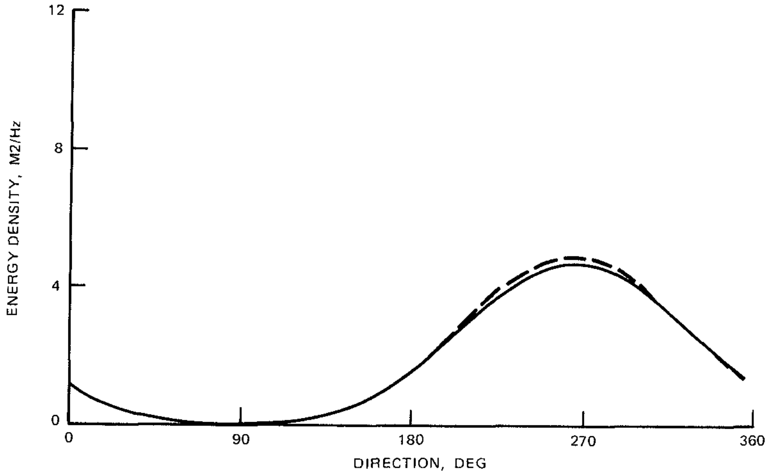


Figure 3. Mean current speed and direction estimates

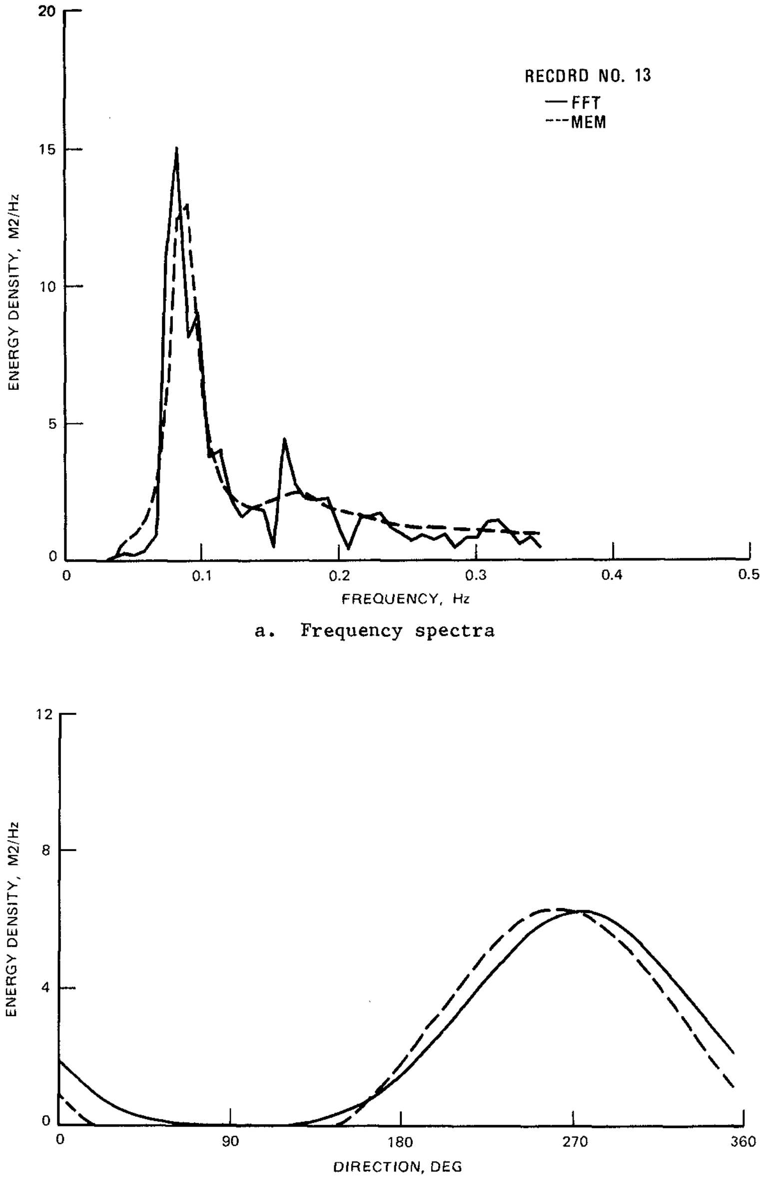


a. Frequency spectra



b. Directional wave spectra

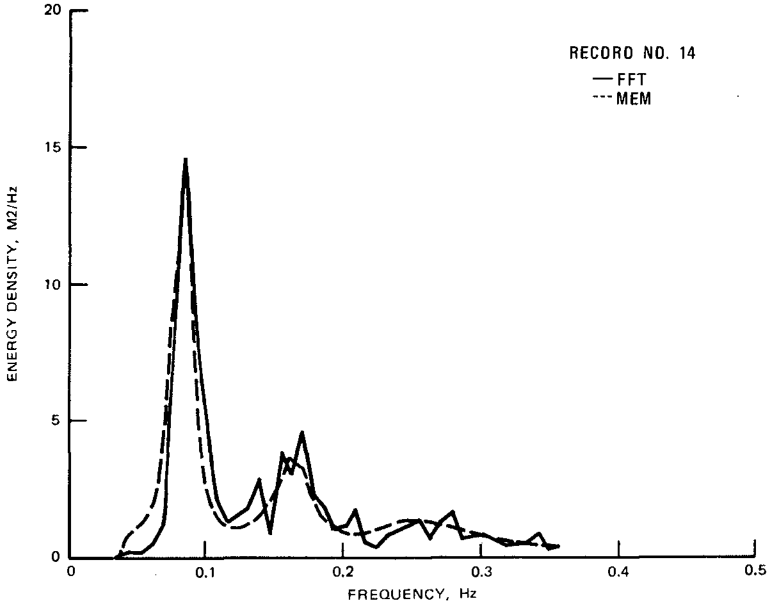
Figure 4. Record 12 spectra intercomparison



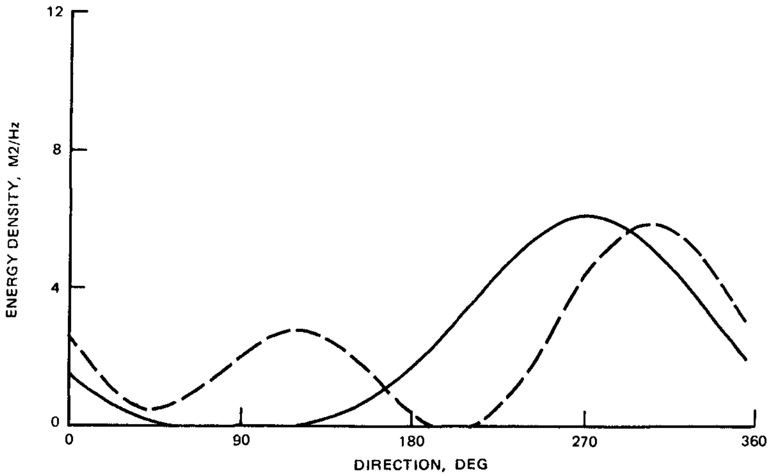
a. Frequency spectra

b. Directional wave spectra

Figure 5. Record 13 spectra intercomparison



a. Frequency spectra



b. Directional wave spectra

Figure 6. Record 14 spectra intercomparison

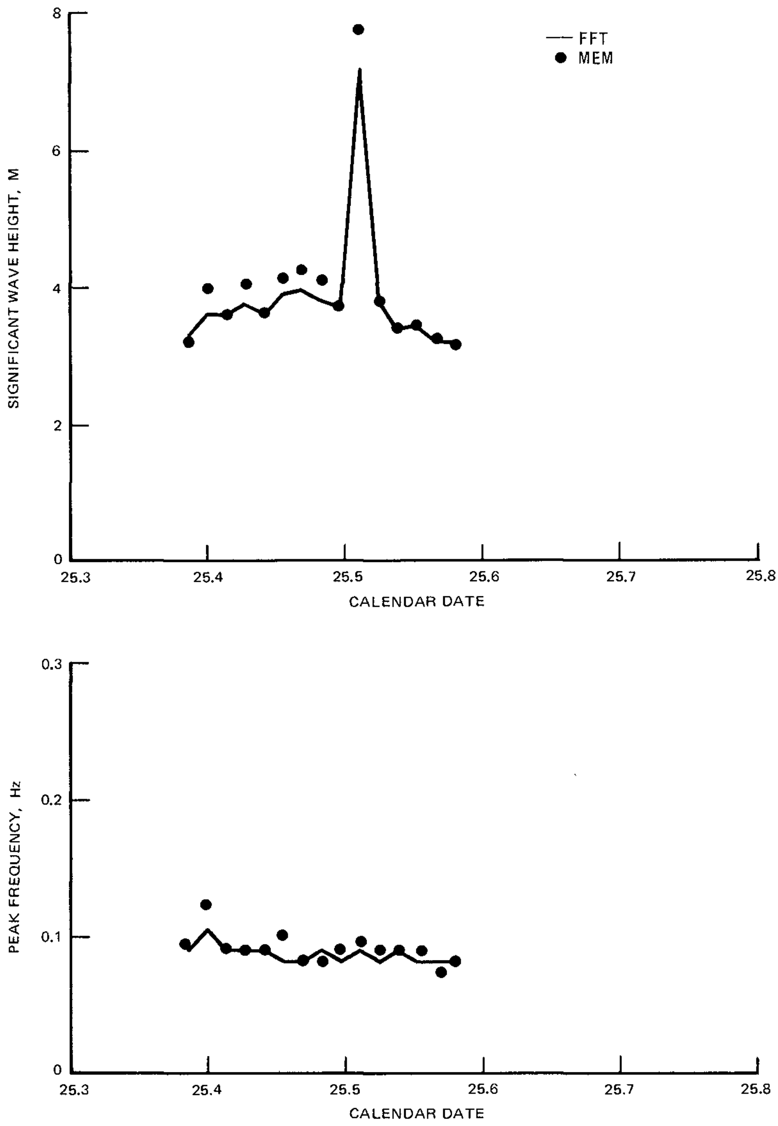


Figure 7. Significant wave height and peak frequency estimates

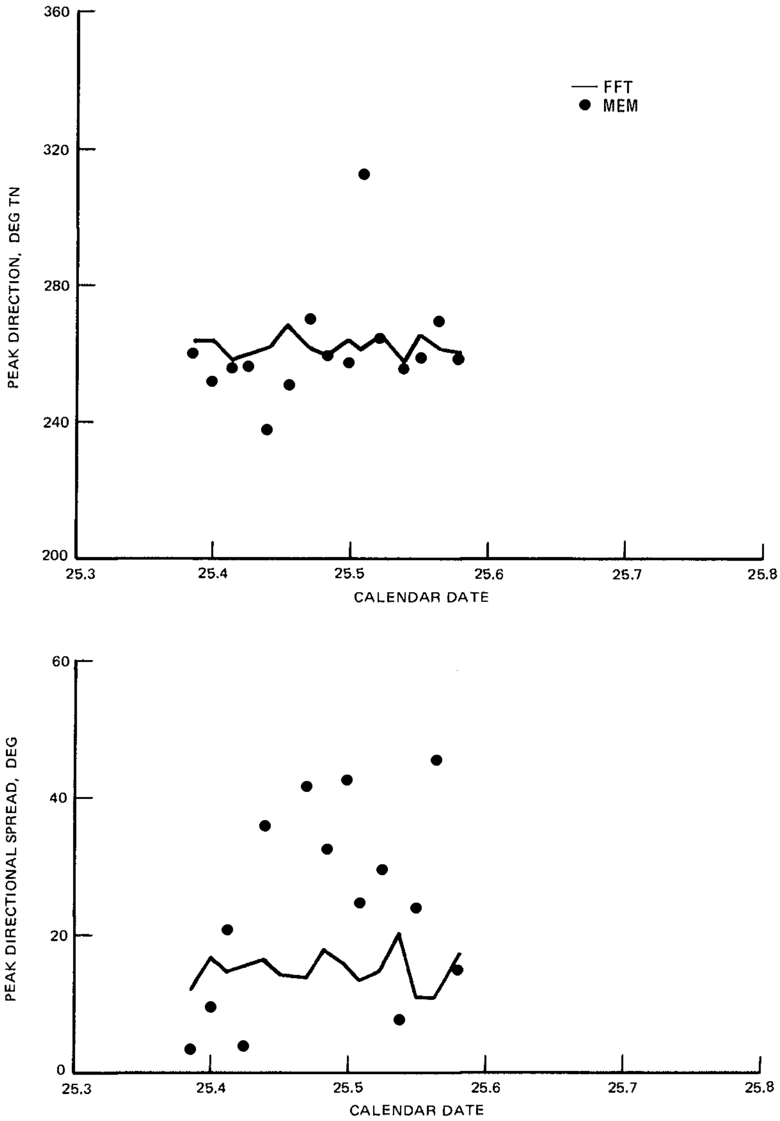


Figure 8. Peak direction and peak directional spread estimates

result, the significant wave height and peak frequency time series curves (Figure 7) agree quite well.

Comparison of MEM and FFT Method

Record No.	Hs, M		fp, Hz		$\bar{\theta}$, deg TN		θ_s, deg	
	FFT	MEM	FFT	MEM	FFT	MEM	FFT	MEM
12	3.39	3.39	0.0898	0.0898	256.9	255.1	20.2	8.3
13	3.45	3.53	0.0820	0.0898	265.0	257.3	10.7	24.1
14	3.21	3.20	0.0820	0.0742	261.7	269.0	10.7	45.7

The MEM peak directional spectral estimates intercompare less favorably, however. The ARSLOE data used here were characterized by a combination of factors which would tend to increase the variability of the peak directional spread estimates (see Figure 8). Among these factors were unsteady wind conditions, reflections from the beach, and pier effects. Irregular bathymetry, short-term disruptions in wave patterns, and shallow-water induced spatial variability as reported by Grosskopf et al. (7) have a pronounced effect on the data analysis.

Figures 4 and 5 agree reasonably well with some slight shifts in direction being apparent. Figure 6, however, indicates two peaks of differing amplitude occurring at the same peak frequency. This bidirectional wave spectrum of two wave trains agrees with results obtained by XERB radar during the ARSLOE experiment. More than one wave train occurring at the same wave frequency was resolved by the XERB but not by the other systems using conventional FFT analysis methods (12).

Jefferys (10) demonstrated the substantial effect reflections and phase locking have on the spread predictions even for MLM methods. Phase locking occurs when two waves of the same frequency but differing amplitudes and directions are produced. Even when the amplitudes were equivalent, only a unimodal spectrum was produced. As the amplitude of one of the waves is decreased, the peak remains unimodal but shifts slightly. When the waves were no longer identical in frequency but still equal in amplitude, the bidirectional nature of the waves could be resolved. This would explain the differences in the MEM-generated peak directional spectra of Figures 4, 5, and 6.

Conclusions

In general, the MEM and CERC FFT method intercompared reasonably well. Comparisons were made with frequency and directional wave spectral estimates, significant wave height, peak frequency, peak wave direction, and peak directional spread. The MEM spectral estimates were generally slightly shifted in frequency and direction. The MEM tended to resolve multiple wave trains of similar frequency coming from different directions better than did the FFT method. The lack of band averaging in the MEM also tended to make the spread calculations more erratic. Further research with MEM and other data adaptive methods should be encouraged.

Acknowledgements

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