Abstract

A numerical model is presented for calculating beach profile development due to offshore sediment transport and tested with laboratory and field data. The model allows variable wave conditions, water level fluctuations due to tide and storm surge, arbitrary bathymetry, and arbitrary sediment size. The agreement between calculated and measured beach profile erosion is good.

Introduction

The interactions of waves and currents with beaches are not well understood. The complexities of the phenomena do not allow useful closed-form solutions. On the other hand, hydraulic models designed to simulate these phenomena are subject to fundamental limitations of laboratory test and scale effects. Various numerical models have been developed (e.g. 1,2) to describe beach profile development due to wave attack. Although some of these models provide reasonable qualitative results, none have been shown to produce good quantitative predictions. This paper describes a numerical model based upon concepts developed by Swart (3), to calculate beach profile development. The model was tested by comparison with laboratory experiments and field data. The response of beach profiles during a period of extreme tides and storm surge also was investigated and results are presented.

Governing Equations

The numerical model developed in this study uses the governing equations obtained by Swart (3). Swart (4) conducted model studies and used regression analysis to obtain equations governing development of beach profiles (4). The readers may refer to references 3 and 4 for details.

The beach profile development during a time step was calculated according to the following equation from Swart (3) (See Fig. 1):

$$h_{t+1} = h_{t} - \exp(-X_b t) + S^d - \exp(C - X^b)$$

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in which \( h_{t1} \) = depth of profile measured from the water level (MSL + Tide) at time \( t \) and at location \( i \) on the profile; \( h_{li} \) = depth of initial profile measured from the water level (MSL + Tide) at time \( t=0 \) and at location \( i \) on the profile; \( \delta_{Ai} \) = depth of the equilibrium profile measured from the same reference at location \( i \); and \( X_b \) is a time factor (rate coefficient) (5).

Method of Calculation

The method of calculation is as follows: first an initial profile \( (h_{li}) \) is selected. Then, from the known wave characteristics (height, period, and angle), sediment size, and time-varying tide, \( h_o \), \( X_b \), and \( \delta_{Ai} \) are calculated according to the method of Swart. Equation 1 is then solved to calculate \( h_{t1} \) during the first time step. At the beginning of the second time step, a new set of wave and tide conditions is input. Equation 2 is then used to calculate the new position of the developed profile with respect to the new datum (MSL + Tide),

\[
h_{li} = h_{t1} - RELV
\]  

in which

\[
RELV = (h_o + Tide)_t - (h_o + Tide)_{t+At}
\]

where \( h_o \) = elevation from the MSL to the upper boundary of developed profile, and \( At \) is the time step. A one-hour time step was used throughout the entire study. It should be noted that the elevation \( h_o \) corresponds to the most shoreward location that waves reach on the beach.
In the numerical model this location varied with time, because the tide datum and wave climate varied. Figure 1 presents a definition sketch showing the various parameters involved in Eqs. 1, 2, and 3. The governing equations (3) for $X$, $h$, $Q$, and $\delta$ were calculated and Equation 1 was then numerically solved. The above procedure was repeated during each time step, until the desired number of iterations for the final beach profile development was attained.

Model Testing

The accuracy of the numerical model was tested by comparison of calculations with laboratory experiments and with prototype measurements. Figure 2 shows a comparison of the beach profile measured in hydraulic model tests by Eagleson et al. (6), and calculations of the profile response numerical model. The parameters used in the numerical model calculations were identical to those of the laboratory tests (initial profile, wave height, wave period, diameter of testing material, and length of time of the test). Figure 2 shows that the major features of the measured profile were reproduced by the numerical model. Additional comparisons of hydraulic scale model tests with the results of the numerical model are given in Swain and Houston (5).

Figure 3 presents a comparison between numerical model calculations and measured profile change in the prototype. The field data include a 16-21 February 1980 storm off the west coast of the United States, at Leadbetter Beach, Santa Barbara, California. The Nearshore Sediment Transport Study (NSTS) documented daily profile measurements in addition to complete directional spectral wave data during the storm (7). This was a large storm that produced approximately 40 m (122 ft) of shoreline erosion. The inputs to the numerical model were sand grain size, initial profile, and hourly values of significant wave height, period, direction, and tide level. Figure 3 shows good agreement between measured profiles and the numerical calculations over the 5-day period of the storm. An important result found in the numerical calculations was that tidal fluctuations were a first-order effect in the mechanism of offshore sediment transport.
transport (8). The sensitivity of the numerical model to other important parameters is discussed by Swain (9).

An additional comparison between measured profile modification during the Currituck Sand-Bypass Study (10), and the profile response numerical model calculations was made. This study involved placement of 26,750 cu m (34988 cu yd) of sediment on the coast near New River Inlet, North Carolina, using the split-hull dredge CURRITUCK. A profile was chosen through the center of the dump to avoid 'end effects.' Wave characteristics were obtained during the study using the Littoral Environment Observation (LEO) technique. The mean diameter of the disposal
material was 0.23 mm. Figure 4 shows the initial profile measured after the dump was completed, and measured and calculated profiles after 36 days. The numerical calculations predicted that there would be little modification in the profile over the time period except for some erosion of the break point bar and filling of the adjacent trough. The measured profile confirms the numerical prediction. Figure 4 shows the calculated and measured profiles differ at most by a few tenths of a meter in elevation. This difference is within the level of accuracy of the profile measurements.

As a final test for the model, beach erosion caused by the 1962 Ash Wednesday storm along the Outer Bank Barrier Islands of North Carolina on the east coast of the United States was simulated. This study area includes the vicinity of the Oregon Inlet which is presently the only inlet along the Outer Banks between Cape Hatteras and Chesapeake Bay. Bodie and Pea Islands border the north and south sides of the Oregon Inlet respectively. Figure 5 presents a portion of the barrier islands system. Wave characteristics were obtained from the U. S. Army Engineer Waterways Experiment Station (WES) Wave Information Study (11). Tide and surge elevations were obtained from the WES Implicit Flooding Model, a tidal circulation and storm surge model (12). Wave conditions were available at three-hour intervals for a period of 4 days. However, tidal and surge elevations were generated at intervals of one hour for the same number of days. Thus the tide and surge level was updated each time step and the wave conditions updated every third time step. Figure 6 shows a comparison of the calculated and the measured shore-normal erosion along Bodie Island. Measurements show that beach erosion varied from a few hundred feet along much of the island to approximately 1500 ft near the north spit. The north spit was a low-lying area that was
inundated during the storm. The numerical calculations predict a similar trend. Figure 7 presents a similar plot along the Pea Island. It is seen that the calculated erosion agrees well with the measurements.

Figure 6. COMPARISON OF CALCULATED AND MEASURED SHORE-NORMAL EROSION OREGON INLET, NORTH CAROLINA (BODIE ISLAND, 1962 ASH WEDNESDAY STORM)

Figure 7. COMPARISON OF CALCULATED AND MEASURED SHORE-NORMAL EROSION OREGON INLET, NORTH CAROLINA (PEA ISLAND, 1962 ASH WEDNESDAY STORM)
However, the comparison is not a rigorous test of the model since there was a constraint that stopped further erosion. The storm produced large waves over such long duration that erosion continued until stopped by the large sand supply of the high dunes. The north spit did not have high dunes and the erosion distance shown in Fig. 6 is the width of the barrier island since the spit was completely eroded.

Conclusions

A time-dependent numerical model to predict beach profile erosion was developed. The accuracy of the model was tested with laboratory experiments and with prototype measurements. It was found that the beach response model can accurately predict profile response to waves and fluctuating water levels.

Acknowledgments

The authors wish to acknowledge the U. S. Army Engineer District, Wilmington, for funding the study on which this paper is based and the Office, Chief of Engineers, U. S. Army Corps of Engineers for authorizing publication of this paper.

Appendix.-References

7. Gable, C. G., "Report on Data from the Nearshore Sediment Transport Study Experiment at Leadbetter Beach, Santa Barbara, California, January-February, 1981, IMR Ref. No. 80.5, Scripps Institution of Oceanography, La Jolla, California 92093.