

## CHAPTER NINETY EIGHT

### A SYSTEM OF MATHEMATICAL MODELS FOR THE SIMULATION OF MORPHOLOGICAL PROCESSES IN THE COASTAL AREA

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#### ABSTRACT

A compound mathematical model (COMOR) for the simulation of morphological changes in the coastal area is being developed. The model is very flexible; it can be composed according to the specific demands of the problem which is considered. So far the model is being applied in the advisory practice to predict initial morphological changes. Recent experiences have shown that the model is quite useful, although at present the knowledge of the individual coastal processes and their interactions is still insufficient to utilize it optimally. However, it is foreseen that in the near future such mathematical models will become a powerful tool in coastal engineering.

#### 1 INTRODUCTION

Numerous physical processes, often at different time and length scales, play a part in coastal morphology. When attempting to understand these processes and their mutual interactions, computer simulations can be of great help, especially if the second horizontal dimension is involved.

The progress in mathematical modelling and the increased capabilities of modern computers have contributed substantially to the application of numerical models in coastal research. In the field of simulation of coastal morphological evolution this started with the strongly schematized single-line models (Grijm, 1964; Le Méhauté and Soldate, 1978), but via an extension to multiple-line models (Bakker, 1968; Boer, 1983) the second horizontal dimension came into the picture (Flemming and Hunt, 1976; O'Connor et al, 1981; Watanabe, 1982; Coeffé and Péchon, 1982; McAnally et al., 1984). Although most of these models were developed primarily for research purposes they have proved to be a useful tool in hydraulic advisory practice, as well.

Within the framework of the Coastal Research Group of the applied hydraulic research programme TOW, the Ministry of Transport and Public Works (Rijkswaterstaat), the Delft Hydraulics Laboratory and the Delft University of Technology are developing a system of computer programs (COMOR) which provides the possibility to compose compound mathematical models for the simulation of horizontally two-dimensional morphological changes in the coastal area. The basic idea of the COMOR-system

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is to assemble the presently available knowledge on the individual physical processes in morphological models.

COMOR-models, so far without time-dependency in the bottom topography, are being applied in hydraulic advisory practice at the Delft Hydraulics Laboratory. They have proved a good support of the usual investigation techniques (desk studies, scale models). Besides, experience with these applications helps to identify and weigh the needs for further research and hence to steer the future development of mathematical modelling in this field.

After a brief description of COMOR and some aspects of the modeling technique the present paper goes into some representative applications. Finally a discussion on the practical applicability is given.

2 DESCRIPTION OF COMOR

In order to meet the requirement of maximum flexibility COMOR was given a modular structure. This means that a model consists of a series of computer programs describing the waves, the net currents, the sediment transports and the bottom changes and that these programs are coupled in a neutral way, such that each of them can easily be replaced by another one. Thus it is possible to compose a model according to the specific demands of the problem to be considered. The neutral coupling of the constituent models is realized by using one central data base for all of them. Each model draws its input data from this file and adds its results to it. In practice this means that each computer program suited as a module in COMOR-models needs its own interface (input and output) with the central data base in order to be actually applicable.

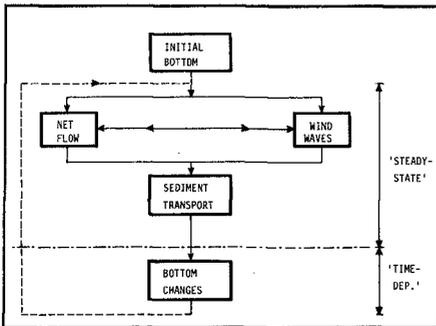


Fig.1 Computation procedure COMOR

Figure 1 outlines the computation procedure underlying COMOR. Starting from the initial bottom configuration in the area of interest and a given set of (boundary) conditions, the waves, the net current field, the sediment transport and the bottom level changes are computed, basically in a quasi-stationary procedure. This implies, that all kinds of composite wave, current and transport computations are allowable but that in any of them the bottom level is kept constant. In the computation of time-dependent morpho-changed after a certain time interval. Subsequently waves, currents and transports are computed for the new bottom. In some practical applications it is not necessary to predict the morphological evolution over a long period. The "initial" rate of change of the bottom level (considered at the morphological

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time scale) then gives sufficient information, provided that it is computed on the basis of a "mixture" of flow and wave conditions that is representative for the period considered. This implies that the effect of the bottom evolution on the waves, currents and transport is left out of consideration. In doing so a number of problems (stability, accuracy, computer-expenses) attending time-dependent computations can be avoided.

3 MODELLING TECHNIQUE

The scheme in Figure 1 outlines the computational procedure followed in COMOR-models, reduced to its basic form. It gives no information, however, on how a model is composed, how it is used and how its results are combined and interpreted. These activities, denominated "modelling technique" here, determine to a large extent the quality of a model application. As a discussion in detail on this modelling technique, however important, would go beyond the scope of this paper, only some of the most relevant aspects will be discussed briefly.

The COMOR-models for initial changes applied so far refer to situations in which, within the area of interest, the morphological changes are influenced by wave-induced as well as tidal currents. This leads to a wide variety of length scales which have to be considered. The area of interest of the morphological models is usually rather small (length scale a few kilometres) compared with the tidal wave length and the distance along which the wave propagation is influenced by the limited water depth. For economical reasons smaller and more refined models are often nested into larger models in order to arrive at a sufficiently accurate description of the tidal motion and the wave field in the area of interest. If this area is small compared with the tidal wave length, the tidal motion can be approximated by a series of unsteady rigid-lid or even steady flow models, with the flow rate imposed at the boundaries. In such cases the tidal model only serves to generate the boundary conditions for the tidal flow component in the detailed flow model covering the area of interest. This leads to a composed model as indicated in Figure 2, with separate modules for the computation of the overall tidal motion, the wave field, the combined wave- and tide- induced flow in the region of interest and the sediment transport there, in this sequence and without feedback. In practice it is not always feasible to follow this procedure throughout. Especially the largest

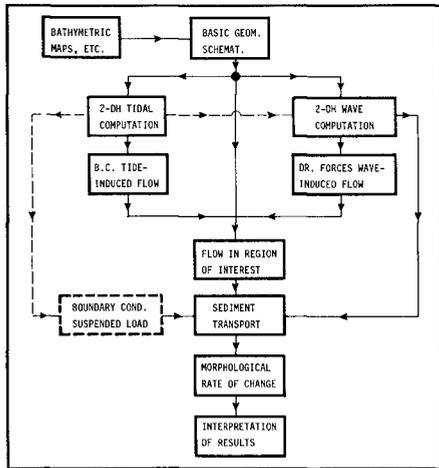


Fig. 2 Flow chart "initial" models

feasible to follow this procedure throughout. Especially the largest

tidal and wave models are rather expensive to be built and calibrated, whence in smaller projects existing models are used as far as possible. As these existing models have usually been developed for other purposes, they will not always fit in with the project objectives. Therefore these largest models are linked up with the detailed models via one or more intermediate sized models.

The selection of the computation methods for the currents, waves and transport to be used should be based on the ability to deal with the physical phenomena involved and the order of accuracy and reliability required. At this point the modellers have to rely heavily on experience, as systematic estimation methods for the relevance of elementary physical phenomena are hardly available. In general the models simulating the wave field, the coastal currents and the sediment transport in the region of interest should be in keeping with each other as well as with the geometrical schematization. At present a variety of computation methods and computer codes is available to describe waves, currents and transport rates. However, little is known yet about the morphological impact of the underlying elementary physical processes. Especially the state of knowledge on the sediment transport under the combined action of waves and currents is such that an accurate sediment transport model is still far from being available. The transport formulae presently applied in coastal morphology are based on the assumption of local equilibrium and relate the sediment transport to local time-mean parameters only. In view of the inaccuracy of the sediment transport model it may seem inappropriate to use highly sophisticated models for the simulation of the waves and coastal currents, as they will hardly increase the accuracy of the morphological computation.

The schematization of the wave climate (deep water wave heights, wave periods and directions) and the tidal motion (tidal cycle and modulation) is one of the major problems in modelling coastal processes, since Monte-Carlo or real-time simulations are far from feasible yet. Even when considering "initial" morphological changes the wave climate and the tidal motion have to be schematized to a limited number of conditions in order to obtain bottom change rates that are representative for the "initial" time span. In the applications so far a single tidal cycle is schematized to a small number of steady state conditions, which are combined with a series of wave conditions during that individual cycle. The horizontally two-dimensional morphological models contain many interactions some of which are non-linear. Therefore the criteria imposed at present on these schematizations, viz. a correct representation of the average annual longshore transport for the waves and the mean flood and ebb-phase transports for the tide, still contain a great deal of arbitrariness and should be applied with caution. The selected schematization of the tidal motion and the wave climate specifies a series of production runs from the results of which the expected net morphological changes in the area of interest are to be derived. To that end a weighting procedure has to be established, in accordance with the schematization procedure.

A practical point of concern is the conformity of the geometrical schematizations in the various constituent models. As a rule these models make use of different computational grids and hence their

geometrical schematizations will show differences. In order to limit these differences as much as possible the original bathymetric data are reduced to a basic geometrical schematization, representing the relevant properties of the actual bathymetry and disregarding irrelevant details. It may be remarked that this choice is by no means always simple. The geometrical schematizations in all constituent models are derived from this basic schematization rather than from the original bathymetric data.

A specific aspect of compound models is the processing of the intermediate model results. The sensitive interaction of waves, current and transport in the computation of the rate of bottom changes poses heavier demands on their accuracy and reliability than in the case of separate application. Besides, additional output quantities are often needed. The wave model for instance has to produce not only wave heights and directions, but also orbital velocities near the bottom and whatever other parameters are needed to model wave effects on the net current and the sediment transport. Similarly, the flow model has to yield not only the magnitude and direction of the net flow velocity, but also other parameters needed for the transport model such as the bottom shear stress.

The last aspect mentioned here concerns the computational procedures applied in the numerical models for the region of interest. Usually only a finite-element procedure or a curvilinear finite-difference procedure provides the possibility to deal with the capriciously shaped coastlines and structures encountered in practice. Moreover these procedures allow for local grid refinements such that the morphological processes in the breakerzone, which is usually rather narrow compared with the total area of interest can be reproduced with sufficient accuracy.

#### 4 PRACTICAL APPLICATIONS

COMOR has been applied in different studies with the following fields of interest:

- design of artificial coastal extensions and islands
- design of sea water intakes and outfalls
- analysis of autonome coastal changes
- design of coastal protection schemes.

Some representative applications in the advisory practice as well as in research projects will be treated hereafter. For each of the applications the problem statement, the predominant physical processes, the simulation of these processes and the principal conclusions are considered.

##### Morphological aspects of an artificial storage basin for the disposal of dredged spoil near Rotterdam, the Netherlands

In the morphological study of a storage basin for the disposal of dredged spoil, reclaimed from the sea by means of a sanddam (see Figure 3), COMOR has been applied to estimate the amount of regular beach nourishment which will be necessary to compensate for the erosion of the sea-exposed dam. The region of interest is a fairly

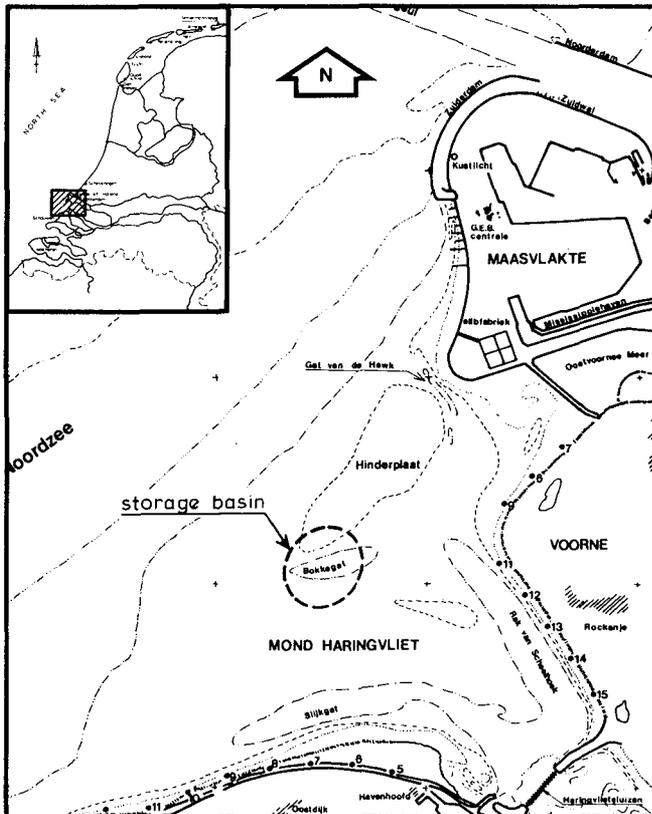


Fig. 3 Future situation

shallow area with a shoal, the Hinderplaat, falling partly dry at low tide. The foreshore shows almost straight and parallel depth contours. The storage basin is projected just at the southernmost edge of the Hinderplaat. The coastal area is exposed to deep water waves coming from directions between N and SW. The wave height distribution near the Hinderplaat is dominated by wave breaking. As such the transport by wave-induced currents is an important phenomenon in the erosion of the sanddam. However, the transport by tidal currents is expected to be important as well as the basin is situated in the immediate vicinity of a tidal gully, the Bokkegat. For this reason, the detailed flow model describes the combined wave-induced and tidal current.

Figure 4 shows the sequence of the constituent models of COMOR for this application. Existing calibrated models have been used for the simulation of the "far field" tidal motion and the wave propagation there. The tidal computations are based on the numerical solution of the vertically integrated two-dimensional long wave equations, discretized on a grid of square meshes (for instance, see Langerak,

1978). Figure 5 shows the tidal flow pattern at the moment of maximum flood flow, computed with the small tidal model.

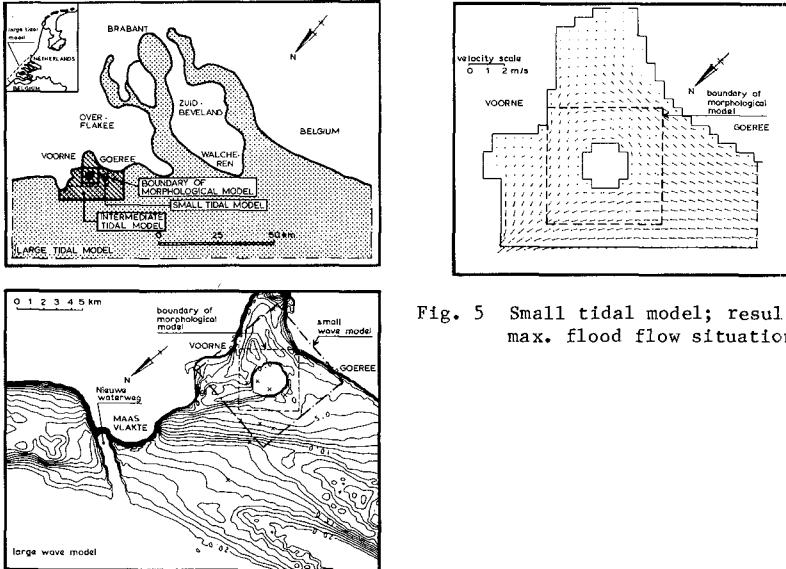


Fig. 5 Small tidal model; results max. flood flow situation

Fig. 4 Constituent tidal and wave models

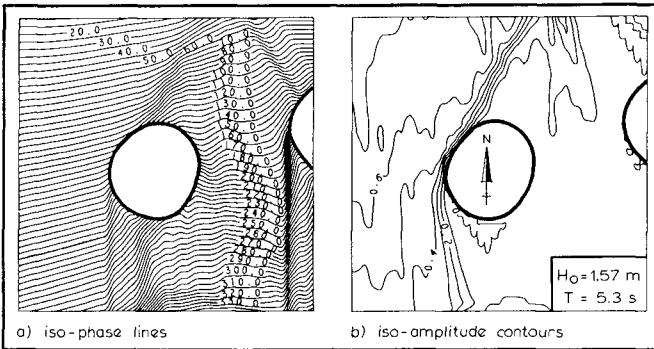


Fig. 6 Small wave model; results for waves from the North

The wave field has been computed with a model based on the parabolic approximation for the propagation of time-harmonic water waves (see Radder, 1979). In the model shoaling, refraction, diffraction and energy dissipation due to bottom friction, currents and wave breaking are included (see Dingemans et al., 1984). In Figure 6 the wave field for waves from the North computed with the small wave model are shown. To simulate the combined wave-induced and tidal flow in the area of interest, a finite element model for friction dominated steady flow (see Wind and Perrels, 1982) is applied. Figure 7 shows the

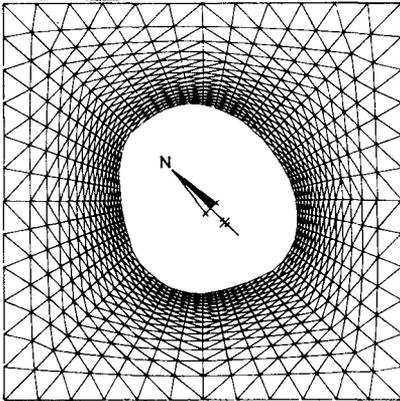


Fig. 7 Finite-element mesh

finite-element mesh used here. In the detailed flow model the tidal constituents of the flow are imposed via boundary conditions prescribing the inflow and/or the outflow velocities. The wave-induced current is driven by forces inside the area derived from the radiation stresses in the wave field (Longuet-Higgins, 1970; Battjes, 1974). The sediment transport model is based on the local equilibrium transport formulation due to Bijker (Bijker, 1967; Bijker, 1971). The bottom change rates follow from the conservation of sediment mass.

In Figure 8 the results of the computation of the detailed models for the waves from a Northern direction and maximum flood flow are outlined. The current pattern shows a longshore current along the seaward side of the dam, directed to the South-west. Around the basin contraction of the tidal currents takes place. Additional calculations have shown that, as a result of this phenomenon, the longshore current is strongly counteracted by the flood flow. The sediment transport pattern shows no significant transport rates in the sheltering area of the basin. Here the pronounced influence of the wave heights on the transport rates is expressed. Around the basin the bottom changes in a small strip are mainly determined by the longshore current, whereas at higher waterdepths the influence of the tidal motion is predominant: erosion occurs where the tidal flow accelerates and sedimentation where it decelerates. The contour plot of the bottom change rate in Figure 8 exhibits a rather irregular pattern with alternating sedimentation and erosion zones, which is hard to interpret qualitatively. For this reason the initial bottom contours have also been compared with the ones after a given time interval.

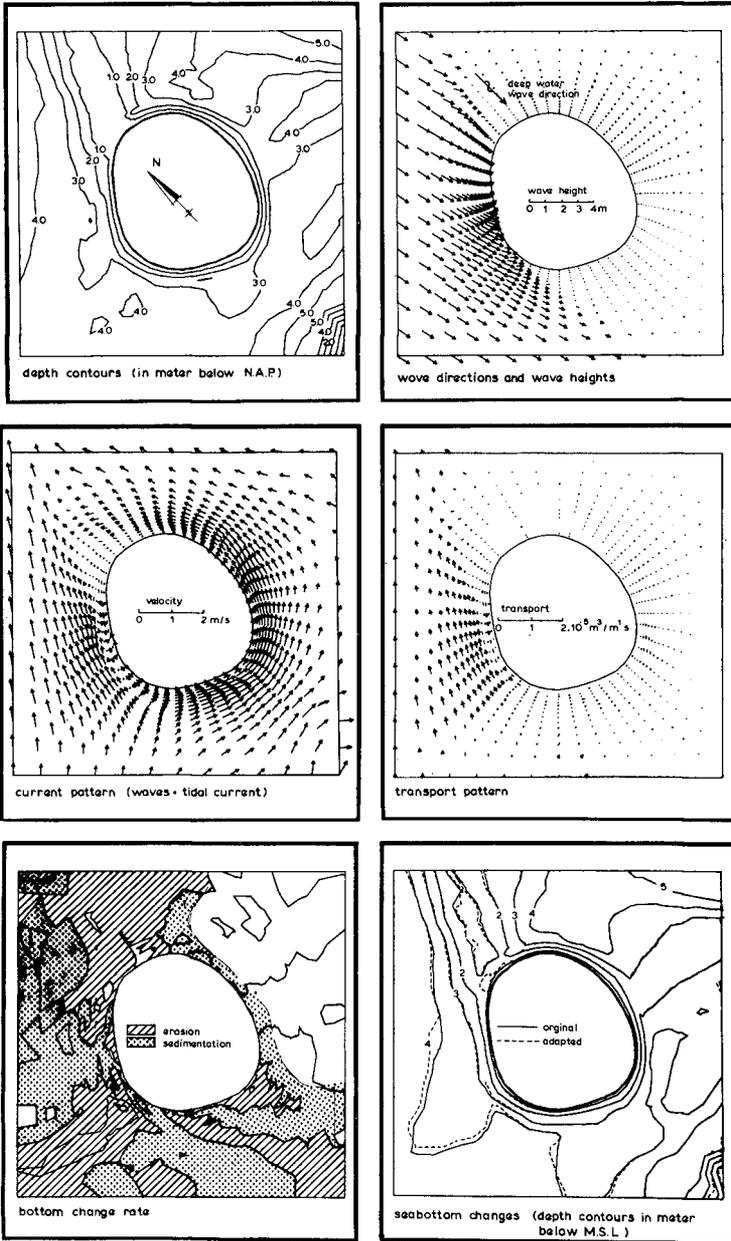


Fig. 8 Detailed models; results at moment of maximum flood and waves from the North

The final computational results of COMOR indicate that the resulting sediment transport pattern and the amount of beach nourishment is strongly influenced by onshore and/or offshore directed sediment transport resulting from the interaction between longshore current and tidal flow. The traditional computation methods viz. the computation of the resulting longshore sediment transport rates for successive cross sections, generally disregard this aspect. As such the COMOR model has contributed considerably to the understanding of the relevant phenomena for this situation.

Sedimentation study of sea water intake for the West Coast Steam Power and Desalination Complex, Umm al Qaiwain, United Arab Emirates

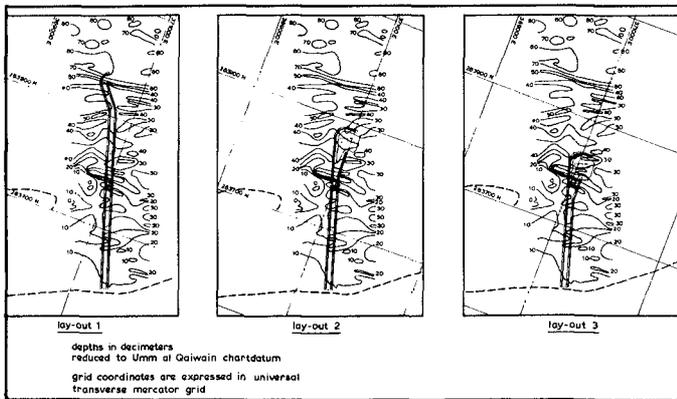


Fig. 9 Evaluated intake layouts

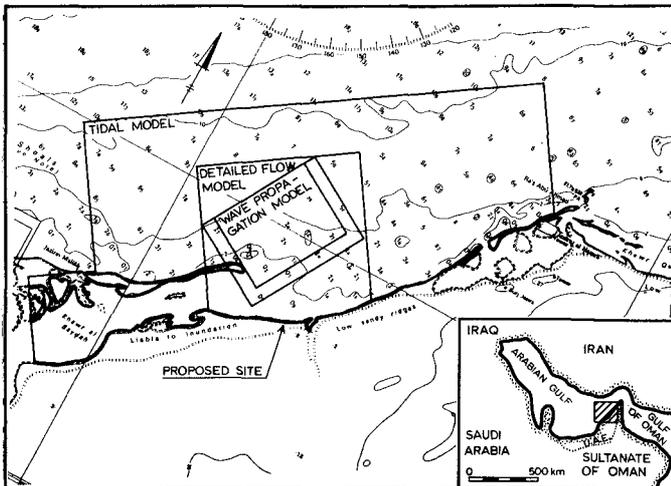


Fig. 10 Site location and constituent models

The sedimentation study was made to select the technically and economically optimal intake layout enabling the continuous withdrawal of up to  $53 \text{ m}^3/\text{s}$  of seawater. Three different intake layouts all consisting of two more or less parallel breakwaters forming a protected intake channel, have been evaluated (see Figure 9). COMOR is applied to determine the general morphologic changes after the construction of the seawater intake. The proposed site of the intake channel is a coastal stretch with a relatively wide and flat beach (Figure 10) exposed to predominant wave action from directions between W and NW. Along this coast a net Eastward longshore transport occurs, forming a spit at the extremity of the island Jazirat-Mallah. In addition, tidal and wave-induced currents carry a significant part of the longshore transport from Jazirat-Mallah towards the Chor Umm al Qaiwain. Part of this sediment is settling down in the area in front of the proposed site. Another part is transported North-east by the ebb current and picked up by the wave-induced longshore current East of the site.

The COMOR model applied in this project includes wave-induced as well as tidal currents. Figure 10 also shows the constituent models, varying in size between  $5 \times 17 \text{ km}^2$  for the tidal model to  $3 \times 5 \text{ km}^2$  for the wave propagation model. The numerical solution procedures in the various models are identical to those in the former application. Tidal computations were carried out for an average tidal situation of which the boundary conditions were derived from field measurements. For the deep water boundary conditions of the wave field, the predominant wave direction was used in combination with the annual extreme wave height. Special attention was given to the reproduction of the diffraction of waves around the head of the intake structure. To reduce computer costs the wave diffraction model covers only part of the detailed model area. In the remaining parts, simple refraction and shoaling computations were performed. The computations with the detailed flow model and the morphological model were carried out for two steady state conditions representing the mean ebb and flood currents. As an illustration, Figure 11 shows the computed wave field and the combined wave-induced and ebb current pattern, for the medium size inlet configuration.

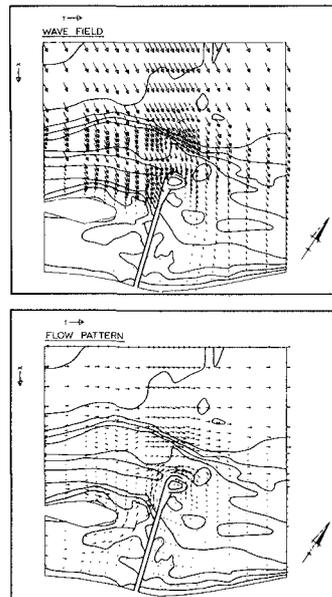
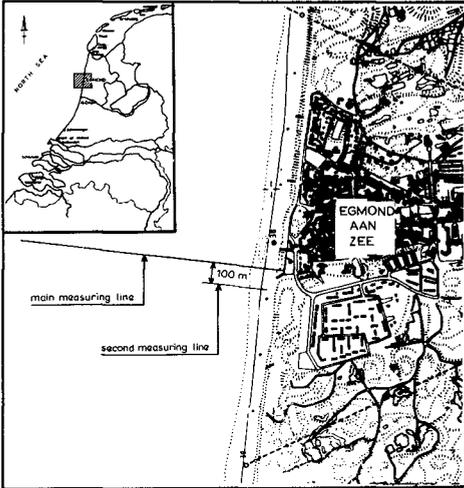


Fig. 11 Computed wave field and current pattern

The principal result of the application of the COMOR-model in this case was the indication that, depending on the layout of the construction, the tidal current and the wave-induced circulation in the

shadow zone could interact in a very unfavourable way, meeting just in front of the inlet (layout 1 in Figure 9) or the inlet could be situated entirely in the longshore transport zone (layout 3 in Figure 9). This indication, rather than the quantitative predictions of the morphological changes resulting from the model, has been used in the advice.

Current and sediment transport analysis at Egmond, the Netherlands



Recently within the framework of TOW two field campaigns on the Dutch Coast near Egmond (see Figure 12) have been conducted. The aims of the field measurements were the following

- to test instruments for the measurements of wave heights, currents and sediment transport in the surf zone;
- to collect data of hydraulic and morphologic phenomena in the near-shore zone.

For details of the 1982 campaign reference is made to Stive and Derks (1984). The location near Egmond was selected because of its longshore uniformity of

Fig. 12 Site location at Egmond

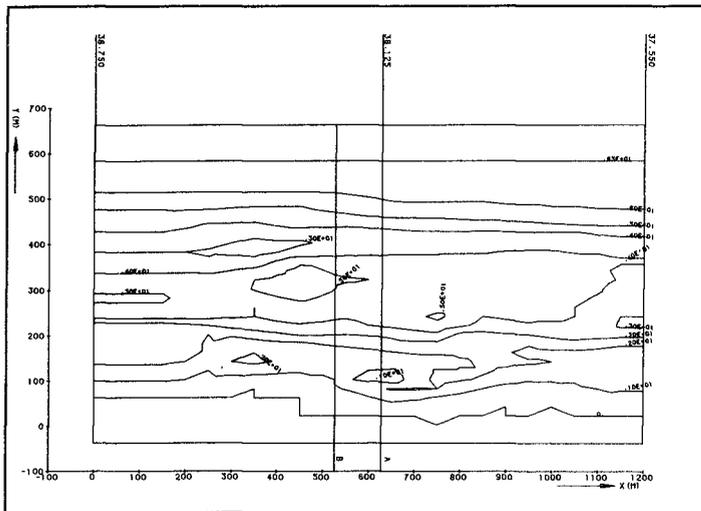


Fig. 13 Geometrical schematization

bottom topography and hydraulic conditions. During both campaigns data on wave heights, currents, vertical tide, suspended sand concentration, wind velocity and -direction, seawater temperature and bottom topography were gathered for conditions ranging from calm weather to 9 Beaufort. Here the COMOR system was applied to study the sensitivity of the measuring data to the "overall" nearshore conditions of the project site. The area covered in the COMOR model is  $700 \times 1200 \text{ m}^2$ . As can be seen in Figure 13 the depth contours are fairly straight and parallel. In the coastal profile a bar can be distinguished at a distance of approximately 400 m offshore.

The wave, current and sediment transport modules in COMOR are identical to those described for the first application. Figure 14 shows the wave field, current pattern and bottom changes resulting from deep

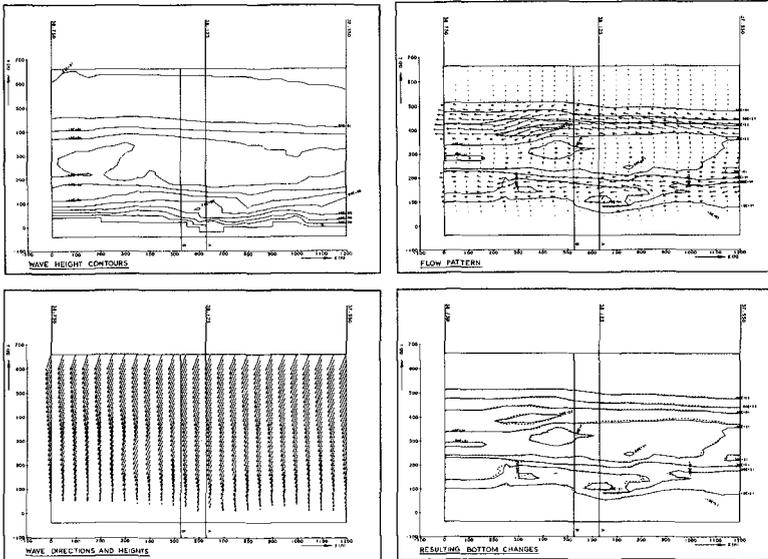


Fig. 14 Computed wave field, current pattern and bottom changes

water waves of North-westerly direction, with a significant wave height  $H_s = 3.88 \text{ m}$  and a peak frequency of  $T_p = 8.5 \text{ s}$ . The current pattern shows two longshore current lanes, one on the bar and the second current close to the shore. Note that the mild variation in the bottom topography has a pronounced influence on the current pattern. It is expected that the influence of the bottom topography on the current pattern will reduce if convection is included. The bottom topography appears to migrate in current direction due to gradients in sediment transport.

Although the analysis of the results is by no means completed, the COMOR model has already contributed considerably to the understanding of the nearshore morphology.

Tidal current and sediment transport analysis at IJmuiden, the Netherlands

For the onset of a study of morphological changes under tidal conditions, the geometry of the port of IJmuiden has been selected. IJmuiden is the gateway to Amsterdam (Figure 15). The tidal period for IJmuiden is on average 12.37 hours. The maximum velocity at a depth of 10 m is in the order of 0.6 m/s. The bottom roughness has been estimated at 0.2 m.

For the computation of the flow field a model for the solution of the Navier-Stokes equations has been used. This model has been developed in cooperation with the Laboratoire National d'Hydraulique, Chatou, France. For details, see Officier, Vreugdenhil and Wind (1985). The turbulent viscosity field is obtained by solving the  $k-\epsilon$  equations. The inflow boundary condition has been assumed to vary sinusoidally with time. The variation in mean water level has not been taken into account (rigid lid approximation) as it is not expected to have a dominant effect on the development of the flow pattern.

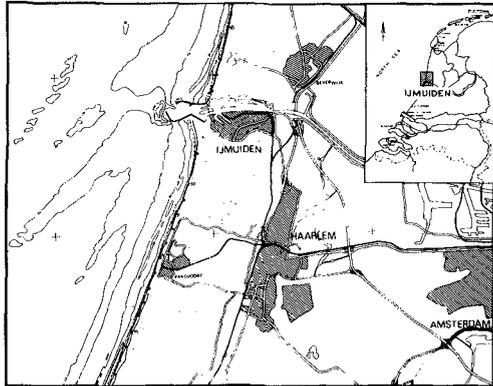


Fig. 15 Site location at IJmuiden

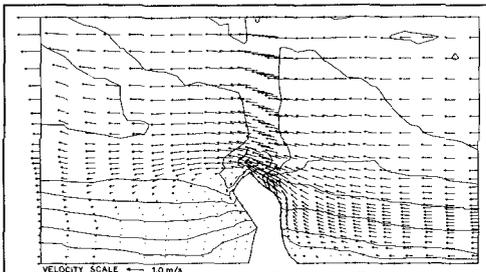


Fig. 16 Computed flow field

The flow field shown in Figure 16 results 3.30 hours after slack tide. Near the harbour entrance a flow contraction can be noticed, while downstream of the harbour moles an eddy is visible. A preliminary comparison of the computed flow field with float tracks indicates similar characteristics. For a more detailed comparison field measurements are a prerequisite. It will be clear

that the flow field is by no means stationary. According to a simple rule of thumb the length of the eddy parallel to the shore should be, for a stationary condition, in the order of six times the length of the harbour mole. Due to the tide reversal the growth of the eddy is stopped and the eddy moves towards deeper water. The time dependency of the flow field has implications for the sediment transport. For instance any morphologic study for IJmuiden based on stationary current fields seems not to be warranted.

A first estimate of the sediment transport can be obtained if a sediment transport formula is combined with the flow field. The resulting sediment transport rate is formed by the small difference between the large sediment transport quantities during the ebb and flood cycle. In nature the resulting sediment transport is among others also determined by the net tidal flow and the rate of suspension of the sediment. These effects are under investigation. As a preliminary result in Figure 17 the change in bottom topography is shown if the sediment transport is calculated using the Engelund-Hansen transport formula. The artificial time scale for the change in bottom topography is 24 hours. It is interesting to note that the gully in front of the harbour is moving in current direction. The wave like character of the bottom change is inherent to the local sediment transport concept.

Although the analysis of the tidal morphology is just started, the model performance is positive. It appeared that the Navier-Stokes current model could model could easily be included resulting in predictions of the evolution of the eddies near the port entrance. The effect of evolution of the flow field on the morphology is presently under investigation.

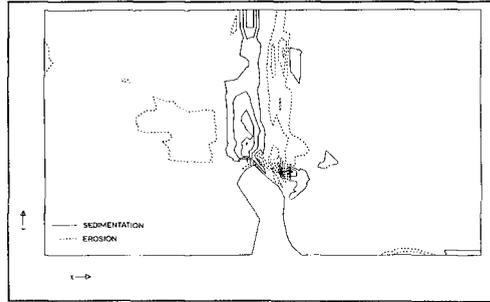


Fig. 17 Computed change in bottom topography

## 5 EVALUATION

Despite the accumulation of a substantial knowledge of the individual morphological processes, the simulation of coastal morphological evolution by mathematical modelling is still in a early stage of development. It is realized that a considerable amount of knowledge, insight and experience remains to be gathered before COMOR may be used as a fully independent quantitatively predictive model. For instance:

- the validity of the depth averaged description of the flow is not satisfactorily verified for coastal currents;
- the transport formulae presently applied in coastal morphology are rather inaccurate;
- little is understood of the time-dependent interaction of waves, currents and transports on the one hand and bottom changes on the other hand;
- the criteria imposed on the schematization of the wave climate and tidal motion contain a great deal of arbitrariness;
- the verification of the compound morphological model is still lacking, hampered as it is by insufficient experimental information.

The aforementioned aspects may suggest that the application of the COMOR model for coastal engineering advisory purposes is at present rather questionable. However the way COMOR is applied so far is

not essentially different from the procedures which are widely used in desk calculations for isolated points of the field. So far the practical applications of COMOR concern only initial morphological changes. The reliability of the model is determined to a high extent by the reliability of the constituent models. In most cases these models have been verified individually, so the reliability of their results can be estimated. Nevertheless the final computational results of the COMOR model must be interpreted with caution. This interpretation is primarily a matter of physical insight and awareness of the model limitations. The results of COMOR must be considered as a support of advice just as the traditional computation methods which are already used in desk studies for years. Applied in this way, COMOR models for initial changes appear to be quite useful in coastal advisory practice, especially in comparative studies (investigation of alternative plans, response to an abrupt change in conditions, etc.) or as a qualitative support of other well calibrated tools (coastline models, scale models, etc.).

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