CHAPTER ONE HUNDRED SEVENTY SIX

Stability of Rubble Mound Slopes under Random Wave Attack

J.W. van der Meer* and K.W. Pilarczyk**

Abstract

The objective of the present research project is to give new practical design formulae for rubble mound slopes under random wave attack. The study is based upon a series of model tests. More than two hundred tests have been performed in order to vary systematically all the relevant variables. The main shortcomings in Hudson-type formulae have been solved as a result of the present series of investigations. Stability formulae are given which include the influence of wave period, number of waves, armour grading, spectrum shape, groupiness of waves and the permeability of the core.

Introduction

The use of coarse materials, such as gravel and natural stone for slope revetments, is very common in civil engineering. In recent years, there has been an increasing demand for reliable design formulae, to cope with the ever growing dimensions of the structures and the necessity to move into more hostile environments.

In 1981, the Netherlands Public Works Department commissioned the Delft Hydraulics Laboratory to perform a systematic study with the objective of developing design rules for both statically and dynamically stable slope revetments. The first results of this study, the design data for statically stable revetments, are given in this paper. Design criteria for dynamically stable gravel revetments have been reported earlier [5]. Research on profile development for rock material is still in progress. Reference should be made to [7] for a preliminary review of the complete study.

The Hudson formula is well known because of its simplicity. In the last decade, however, it has been found by many users to have a lot of shortcomings. It does not include the influence of the wave period and no data are available for random waves. The study of Ahrens [1] in a large wave tank showed the importance of the wave period on the stability of riprap. The tests, however, were performed with regular waves. Evaluation of Ahrens' data by Pilarczyk and den Boer [7] produced stability formulae which included the wave period.

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and Giménez-Curto [6] gave formulae for stability of rubble mound slopes under regular wave attack which also included the wave period.

An extensive investigation has been performed by Thompson and Shuttler [9] on the stability of rubble mound slopes under random waves. One of their main conclusions was, that within the scatter of the results, the erosion damage showed no clear dependence on the wave period. Reanalyzing their data, the authors, however, have found a very clear dependence on the wave period! The analysis also showed that only steep waves were used with a small range of wave periods. The work of Thompson and Shuttler has, therefore, been used as a starting point for the present research. By performing tests with longer wave periods, the dependence of erosion damage on wave period has been confirmed for a wider range of conditions. In addition the dependency on other variables has been investigated.

Governing Variables

A design formula for armour units should give a method of determining the minimum mass of individual armour units for given mass densities, required for stability as a function of all the variables involved. In the following the average mass of graded rubble is referred to as $W_{50}$ or to the nominal diameter, $D_{n50}$, where:

$$D_{n50} = \left( \frac{W_{50}}{\rho_a} \right)^{1/3}$$  \(1\)

where: $D_{n50}$ = nominal diameter (m)
$W_{50}$ = 50% value of the mass distribution curve (kg)
$\rho_a$ = mass density of stone (kg/m$^3$)

The relative mass density of the stone in water can be expressed by:

$$\Delta = \frac{\rho_a}{\rho} - 1$$  \(2\)

where: $\Delta$ = relative mass density (-)
$\rho$ = mass density of water (kg/m$^3$)

As shown by many authors there are a large number of variables affecting armour stability. The primary variables are shown in Table 1.

The wave height can be normalized by dividing the significant wave height, $H_s$, by the relative mass density and the nominal diameter. This dimensionless wave height is the same as the often used stability number $N_s$ and reduces the Hudson formula to a very simple form:

$$H_s/\Delta D_{n50} = (K_D \cot \alpha)^{1/3}$$  \(3\)

The wave period can be related to external processes, waves breaking on a slope, by the dimensionless surf similarity parameter, $\xi_z$, where:

$$\xi_z = \tan \alpha / \sqrt{2 \pi \frac{H_s}{g} \frac{T_z^2}{2}}$$  \(4\)
Table 1 Primary variables affecting armour stability

<table>
<thead>
<tr>
<th>variable</th>
<th>symbol</th>
<th>dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>nominal diameter</td>
<td>$D_{n50}$</td>
<td>m</td>
</tr>
<tr>
<td>relative mass density</td>
<td>$\Delta$</td>
<td>-</td>
</tr>
<tr>
<td>significant wave height</td>
<td>$H_s$</td>
<td>m</td>
</tr>
<tr>
<td>average wave period</td>
<td>$T_z$</td>
<td>s</td>
</tr>
<tr>
<td>slope angle</td>
<td>$\alpha$</td>
<td>deg</td>
</tr>
<tr>
<td>damage level</td>
<td>$S_2$</td>
<td>-</td>
</tr>
<tr>
<td>number of waves</td>
<td>$N$</td>
<td>-</td>
</tr>
<tr>
<td>armour grading</td>
<td>$D_{85}/D_{15}$</td>
<td>-</td>
</tr>
<tr>
<td>spectrum shape</td>
<td>$\varepsilon_{5%,Q_p}$</td>
<td>-</td>
</tr>
<tr>
<td>groupiness of waves</td>
<td>$G.F.,J_1,J_2$</td>
<td>-</td>
</tr>
<tr>
<td>permeability of core</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Plots of $H_d/\Delta D_{n50}$ or $N_d$ versus $\varepsilon_{5\%}$ are used by many authors to show the influence of wave period on armour stability. The wave period $T_z$ can be related to internal processes as initiation of damage and transport of material, by coupling it to the nominal diameter:

$$\text{dimensionless wave period} = \frac{T_z^2}{D_{n50}}$$

(5)

This wave period is also used in the investigations of van Hijum and Pilarczyk [5], on the stability of gravel beaches. Using (5) instead of (4) means that the influence of the wave height and the wave period on stability can be treated independently.

The dimensionless damage level, $S_2$, is described by:

$$S_2 = A_2/D_{n50}^2$$

(6)

where:

$A_1 = \text{accretion above water level} \ [m^2]$
$A_2 = \text{eroded area of the profile} \ [m^2]$
$A_3 = \text{accretion below erosion area} \ [m^2]$

A physical description for $S_2$ is the number of cubical stones with a side of 1 $D_{n50}$, eroded over a width of 1 $D_{n50}$. The "no damage" criterion of Hudson and Ahrens is taken generally to be when $S_2$ is between 1 and 3 stones eroded and "failure" of the slope is assumed when $S_2$ is between 8 and 17. The exact value of $S_2$ is dependent to some extent on the slope of the revetment.

Test Equipment, Materials and Procedure

All tests were conducted in a 1.0 m wide, 1.2 m deep and 50.0 m long wave flume with test sections installed about 44 m from the
random wave generator. This wave generator is capable of performing both translatory and rotational motions by means of a hydraulic actuator, programmed by a closed loop servo-system. The command signal of this loop is obtained from a punched tape, representing a random signal with a predetermined wave energy spectrum. A new system developed by the Delft Hydraulics Laboratory was used to measure and compensate for reflected waves at the wave board. With this system standing waves and basin resonance are avoided.

For the investigation a surface profiler was developed with nine gauges placed 0.10 m apart on a computer controlled-carriage. The surface along the slope was measured every 0.040 m. Depending on the slope angle every survey consisted between 500 and 1600 data points. Successive soundings were taken at exactly the same points using the relocatability of the profiler. An average profile was calculated and plotted by computer and used for determining the erosion damage, $S_2$.

Broken stone was used for the armour layer, the main characteristics of which were: $W_{50} = 0.123$ kg; $\rho_a = 2630$ kg/m$^3$; $D_{n50} = 0.036$ m; layer thickness 0.080 m. The sieve analysis curves were straight lines on a log-linear plot, see Fig. 1. Two gradings were used with $D_{85}/D_{15} = 2.25$ (riprap) and 1.25 (uniform stones) respectively. The filter layer was defined by $D_{n50}$ (armour)/$D_{n50}$ (filter) = 4.5 and $D_{85}/D_{15} = 2.25$. The thickness of the filter layer was 0.02 m. This layer was placed directly on a slope constructed of mortar when an impermeable core was being tested. When a permeable core was tested the armour layer was placed directly on this core.

Each complete test consisted of a pre-test sounding, a test of 1000 waves, an intermediate sounding, a test of 2000 more waves and a final sounding. After each complete test the armour layer was removed and rebuilt. A test series consisted generally of 5 tests with the same wave period, but different significant wave heights. Wave heights ranged from 0.05 m to 0.26 m and wave periods from 1.3 to 3.2 seconds. A water depth of 0.80 m was applied for all tests. A damage curve was drawn for $N = 1000$, and $N = 3000$, for each test series, as shown in Fig. 2. From this the $H_s/A_Dn_{50}$ value was taken for three damage levels and the surf similarity parameter, $\varepsilon_2$, given in (4) was calculated. The following three damage levels were chosen:
- start of damage (the $H_{D=0}$ for Hudson)
- intermediate damage
- failure: filter layer is visible.

\[
\cot \alpha = 3.0 \\
\frac{D_{85}}{D_{15}} = 1.25 \\
T_z = 2.25 \text{ s} \\
\bigcirc N = 1000 \\
\bullet N = 3000 \\
\ast \text{ filter layer visible} \\
\text{Pierson Moskowitz spectrum}
\]

Fig. 2 Example of damage curves

Test Programme

The test programme is shown below.

Table 2 Test programme

<table>
<thead>
<tr>
<th>slope angle cot $\alpha$</th>
<th>grading $\frac{D_{85}}{D_{15}}$</th>
<th>spectrum shape</th>
<th>permeable core</th>
<th>number of tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.25</td>
<td>PM</td>
<td>no</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>2.25</td>
<td>PM</td>
<td>no</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>2.25</td>
<td>PM</td>
<td>no</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>2.25</td>
<td>PM</td>
<td>no</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>1.25</td>
<td>PM</td>
<td>no</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>1.25</td>
<td>PM</td>
<td>no</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>2.25</td>
<td>narrow</td>
<td>no</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>2.25</td>
<td>wide</td>
<td>no</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>1.25</td>
<td>PM</td>
<td>yes</td>
<td>19</td>
</tr>
</tbody>
</table>

PM = Pierson Moskowitz spectrum

Results

The influence of the primary variables on armour stability, shown in Table 1 is discussed below and new practical stability formulae, obtained from the results, presented.
Influence of Number of Waves

In the investigation of Thompson and Shuttler [9] profiles were sounded after each 1000 waves, up to \( N = 5000 \). Analyzing their results gives the relationship between damage and number of waves shown in Fig. 3. Data points are based on 50 tests and are independent of slope angle, wave period and damage level. Since a clear relationship was found between \( S_2 \) and \( N \), it was decided to decrease the total number of waves in the present study to \( N = 3000 \) and to conduct one intermediate sounding after \( N = 1000 \). Analyzing the ratio of damage after 3000 and 1000 waves and using the damage after 3000 waves as a reference, gave a new point (*) in Fig. 3. The difference between the two investigations is small.

The relationship between \( S_2 \) and \( N \) can be described by the following formula:

\[
S_2(N) = \frac{0.014}{\sqrt{N}} * S_2(5000) \tag{7}
\]

Since (7) is a square root function it is easy to find a parameter which describes the influence of the number of waves on the damage. This parameter is \( S_2/\sqrt{N} \). The constant 0.014 becomes a part of a stability coefficient. The parameter \( S_2/\sqrt{N} \) can be used for \( N \) in the range of, approximately, 1000 to 7000. For \( N < 1000 \) a linear relationship fits the data better, see Fig. 3. It is, however, very clear that with random waves a stable profile is not found with less than...
10,000 waves. This is very different to regular waves where equilibrium is generally found within 1000 - 2000 waves.

Influence of Wave Period and Slope Angle

The extent of damage depends on the slope angle. More stones have to be displaced for gentler slopes before the failure criterion is reached. From the investigations the lower and upper damage levels were determined as shown in Table 3.

Table 3 Lower and upper damage levels

<table>
<thead>
<tr>
<th>cotα</th>
<th>start of damage</th>
<th>failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>S₂ = 2</td>
<td>S₂ = 8</td>
</tr>
<tr>
<td>3</td>
<td>S₂ = 2</td>
<td>S₂ = 12</td>
</tr>
<tr>
<td>4</td>
<td>S₂ = 3</td>
<td>S₂ = 17</td>
</tr>
<tr>
<td>6</td>
<td>S₂ = 3</td>
<td>S₂ = 17</td>
</tr>
</tbody>
</table>

The H₅/H₁₅₀ and ξ₂ values for the three damage levels have been plotted for each slope angle and for N = 3000 in Figs. 4 and 5. Although there is a little scatter the influence of the wave period is very clear, especially for the gentler slopes. The influence of the wave period is evidently larger for breaking waves (ξ₂ < 2.5 - 3.5) than for non-breaking waves (ξ₂ > 2.5 - 3.5).

Influence of Armour Layer Grading

For slopes with cotα = 3.0 and cotα = 4.0 the tests were repeated with different grading of the armour. The wide grading with, D₈₅/D₁₅ = 2.25 (riprap), was replaced by a narrow grading with D₈₅/D₁₅ = 1.25 (uniform stones). Test results are plotted on Figs. 4b and 5a (open symbols). No difference in damage was found for the two gradings. It can be concluded that the grading of the armour within the range tested has no influence on the stability and that the armour layer can be described by the nominal diameter, D₅₀₅₀, only. This conclusion is in contrast to the use of different Kp values for riprap (Kp = 2.2) and for uniform stones (Kp = 3.2). It should be stated that the difference between the Hudson formula and the work of Ahrens [1] can not be explained by the different grading of the armour. Probably, this difference is due to the difference in core permeability of the two investigations.

Influence of Spectrum Shape and Groupiness of Waves

The main part of the present series of tests was conducted with a Pierson Moskowitz (PM) spectrum. The test series with a slope angle with cotα = 3.0 were performed with both a very narrow spectrum and a wide spectrum. Although the last word has not yet been written about the description of spectrum shape and groupiness of waves the following parameters give a reasonable idea.

\[ \varepsilon_{5\%} = (1 - m²/m_{0.4})^{0.5} \]
Fig. 4a Cot $\alpha = 2.0$

Fig. 4b Cot $\alpha = 3.0$

Fig. 5a Cot $\alpha = 4.0$

Fig. 5b Cot $\alpha = 6.0$
\[ Q_p = \frac{2}{m_0} \int_{0}^{\infty} f[S(f)]^2 \, df \]  

(9)

GF = groupiness factor = \((m_{0,\text{group}})^{0.5}/m_0\)  

(10)

\[ J_1(A) = \text{mean length of wave group above level A} \]  

(11)

\[ J_2(A) = \text{mean total length of wave group (Goda [3])} \]  

(12)

where: \(\varepsilon_{5\%}, Q_p\) = spectral width parameters  

\(m_n\) = nth spectral moment of the energy density spectrum  

\(f\) = frequency  

\(S(f)\) = energy density as function of \(f\)  

\(m_{0,\text{group}}\) = spectral moment of the SIWEH spectrum (Funke and Mansard [2])

Parameter values for the narrow, PM and wide spectra mentioned above are given in Table 4.

**Table 4 Spectral parameters and groupiness of waves**

<table>
<thead>
<tr>
<th>parameter</th>
<th>narrow spectrum</th>
<th>PM spectrum</th>
<th>wide spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_{5%})</td>
<td>0.10 ± 8%</td>
<td>0.39 ± 3%</td>
<td>0.59 ± 5%</td>
</tr>
<tr>
<td>(Q_p)</td>
<td>13.4 ± 14%</td>
<td>2.60 ± 3%</td>
<td>1.47 ± 11%</td>
</tr>
<tr>
<td>GF</td>
<td>0.99 ± 8%</td>
<td>0.77 ± 6%</td>
<td>0.72 ± 8%</td>
</tr>
<tr>
<td>(J_1(H))</td>
<td>5.43 ± 12%</td>
<td>2.60 ± 3%</td>
<td>2.16 ± 6%</td>
</tr>
<tr>
<td>(J_2(H))</td>
<td>12.6 ± 13%</td>
<td>5.60 ± 3%</td>
<td>4.76 ± 3%</td>
</tr>
<tr>
<td>(J_1(H_s))</td>
<td>2.95 ± 6%</td>
<td>1.54 ± 5%</td>
<td>1.35 ± 5%</td>
</tr>
<tr>
<td>(J_2(H_s))</td>
<td>22.0 ± 6%</td>
<td>11.3 ± 6%</td>
<td>10.3 ± 6%</td>
</tr>
</tbody>
</table>

The difference in width of the three spectra is clear. A strong wave groupiness was present for the narrow spectrum. Some groupiness existed for both the PM and wide spectrum but the difference between the GF factors is small. Although the width of the wide spectrum is much larger than for the PM spectrum (\(\varepsilon_{5\%} = 0.6\) and 0.4 respectively), the groupiness of waves was almost the same (0.72 and 0.77 respectively).

Test results for the narrow and wide spectrum are shown, for \(N = 3000\), in Fig. 6.

Using the average wave period, \(T_z\), for calculating \(\xi_z\) (4) gives good agreement between the test results for a narrow and a wide spectrum. It can be stated, therefore, that stability is not influenced by the spectrum shape or by the groupiness of waves. This conclusion was also reached for the profile development of gravel beaches, by van Hijum and Pilarczyk [5]. Wave runup and rundown, however, depend strongly on the groupiness of waves.
Fig. 6 Influence of spectrum shape and wave groupiness on stability

Fig. 7 Influence of core permeability
Influence of Core Permeability

Tests were performed on a slope with $\cot \alpha = 3.0$ with a permeable core. The armour layer was constructed directly on the core. The relative dimensions of the core were $D_{n50} \text{ (armour)} / D_{n50} \text{ (core)} = 3.2$ and $D_{85}/D_{15} = 1.5$. Test results are shown in Fig. 7.

The difference in results with an impermeable core (dotted lines) is appreciable which means that its influence on the stability is large. This was also found by Hedal [4] in 1960 for regular waves. Although stability is higher with a permeable core the same trend for the influence of wave period is shown.

New Practical Stability Formulae

The list of variables given in Table 1 can be shortened using the results described above. The influence of the number of waves is given by $S_2 / \sqrt{N}$. Armour grading, spectrum shape and wave groupiness have no influence on the stability and can, therefore, be deleted. The stability of rubble mound slopes can then be described by the following dimensionless variables:

$$\frac{H}{\Delta D_{n50}}; \xi_z \text{ or } \sqrt{\xi_z^2 / D_{n50}}; \cot \alpha; \frac{S_2}{\sqrt{N}}; \text{ permeability}$$

Stability formulae are given below for an impermeable core (lower boundary). The permeability of the core can be included in the formulae by adjusting coefficients. There is distinct difference between breaking and non-breaking waves, see Figs. 4 and 5, and different formulae have, therefore, to be applied for $\xi_z < 2.5 - 3.5$ and for $\xi_z > 2.5 - 3.5$. Non-linear regression analysis gives the following stability formulae for an impermeable core.

For breaking waves: $\xi_z < 2.5 - 3.5$

$$\frac{H}{\Delta D_{n50}} = 4.4 \left( \frac{S_2}{\sqrt{N}} \right)^{0.22} \left( \frac{\xi_z}{\xi_z} \right)^{-0.54}$$  \hspace{1cm} (13)

For non-breaking waves: $\xi_z > 2.5 - 3.5$ and $\cot \alpha < 3$

$$\frac{H}{\Delta D_{n50}} = 1.25 \sqrt{\cot \alpha} \left( \frac{S_2}{\sqrt{N}} \right)^{\frac{1}{6}} \left( \frac{\xi_z}{\xi_z} \right)^{0.1}$$ \hspace{1cm} (14)

For non-breaking waves: $\xi_z > 2.5 - 3.5$ and $\cot \alpha > 3$

$$\frac{H}{\Delta D_{n50}} = 1.25 \sqrt{3} \left( \frac{S_2}{\sqrt{N}} \right)^{\frac{1}{6}} \frac{\xi_z}{\xi_z}^{0.1}$$ \hspace{1cm} (15)

Eqs. (13), (14) and (15) become straight lines on log-log paper as shown in Fig. 8. $H/\Delta D_{n50}$ is plotted in this figure against $\xi_z$ and lines for several damage levels, $S_2$, are given for $N = 3000$.

$\xi_z^{0.54}$ and $\xi_z^{0.1}$ give the slope for the lines and also the influence of the wave period.
The damage level to be taken for design depends on the wave conditions, but must always be within the limits mentioned in Table 3. For storms with a recurrence interval of a few years $S_2$ can be taken close to the lower limit ($S_2 = 2$ - 3), for very severe storms an intermediate or even the upper limit should be chosen. Figures can be drawn with (13) (14) and (15) for any number of waves required. The stability formulae are also plotted in Figs. 4 and 5, and show good agreement with the test results.

Another way of describing the influence of the wave period is to use the variable $\sqrt{g \frac{T^2}{D \text{n50}}}$ instead of $\xi_z$. The following simple power function has been found for breaking waves:

\[
H_s / \Delta D \text{n50} \times \sqrt{g \frac{T^2}{D \text{n50}}} \times \tan \alpha = 31 \left( \frac{S_2}{\sqrt{N}} \right)^{1/3}
\]  

(16)

The correlation coefficient, in this case, is 0.90. The formula is based on the actual test results but with the restriction $S_2 > 2$. 225 data points were used for the regression analysis. Fig. 9 gives (16) for all test results. 156 data points could be used from Thompson and Shuttler [9] and 171 data points from the present investigation. The results of four different slope angles and two gradings are summarized in the figure.

The following relationships between wave height and damage for constant wave period, slope angle and number of waves can be determined from (14), (15) and (16).

\[
S_2 = A H_s^{3/6}
\]

for breaking waves  

\[
S_2 = A H_s
\]

for non-breaking waves  

where: $A = a$ constant

A third and sixth power function are found. From (17) and (18) it can be concluded that for non-breaking waves the curve is steeper and the damage more progressive. This is also clear in Fig. 4a.

Although it seems that (13) is very different from (16), equation (13) can be rewritten in almost the same form as (16):

\[
(H_s / \Delta D \text{n50})^{1.35} \times \sqrt{g \frac{T^2}{D \text{n50}}} \times \tan \alpha = 39 \left( \frac{S_2}{\sqrt{N}} \right)^{0.41}
\]

(19)

Using this formula the relationship between wave height and damage becomes:

\[
S_2 = A H_s^{3/3}
\]

(20)

which is almost the same as found in (17). The agreement between (13) and (16) is good.

Higher stability was found for the permeable core, but with the same trend for the influence of the wave period, see Fig. 7.
Fig. 8 Stability formulae for $N = 3000$ and an impermeable core

![Graph showing stability formulae with various lines representing different conditions.]

Fig. 9 Stability formulae and test results for breaking waves ($f_0 \times 1.5$)

156 data points from Thompson and Shuttler
171 data points from the present investigation
makes it possible to use the same stability formulae (13) to (16) but with adjusted coefficients. For a permeable core the formulae become:

\[ H_s / \Delta D_{n50} = 5.8 \left( \frac{S_z}{\sqrt{N}} \right)^{0.22} \frac{0.54}{\xi_z} \]  
\[ (13a) \]

\[ H_s / \Delta D_{n50} = 1.65 \sqrt{\cot \alpha} \left( \frac{S_z}{\sqrt{N}} \right)^{1/6} \frac{0.1}{\xi_z} \]  
\[ (14a) \]

\[ H_s / \Delta D_{n50} = 1.65 \sqrt[3]{3} \left( \frac{S_z}{\sqrt{N}} \right)^{1/6} \frac{0.1}{\xi_z} \]  
\[ (15a) \]

\[ H_s / \Delta D_{n50} \times \sqrt{\frac{T_z^2}{D_{n50}}} \times \tan \alpha = 49 \left( \frac{S_z}{\sqrt{N}} \right)^{1/3} \]  
\[ (16a) \]

The increase of stability for (13a), (14a) and (15a), is 32% and for (16a) 58%. Since there is no known simple parameter which describes the permeability of a structure it is left to the engineers judgement to use (13) to (16) or (13a) to (16a) or even intermediate values. The stability formulae (13a) to (16a) are shown on Figs. 7 and 9 and correspond well with the test results. Further investigation is still necessary with other values of core permeability and with other slope angles in order to solve this problem.

Conclusions

1. Based on more than two hundred tests and on the work of Thompson and Shuttler [9] practical design formulae have been developed for rubble mound slopes under random wave attack.

2. The stability, in a dimensionless form, has been determined using
   - the significant wave height: \( H_s / \Delta D_{n50} \)
   - the average wave period: \( \xi_z \) or \( \sqrt{\frac{T_z^2}{D_{n50}}} \)
   - the slope angle: \( \cot \alpha \)
   - the damage as function of number of waves: \( S_z / \sqrt{N} \)
   - the permeability of the core.

3. Within the conditions tested the following had no influence on the stability:
   - the grading of the armour
   - the spectrum shape
   - the groupiness of waves.

References

HIJUM E. van and PILARCZYK, K.W., Gravel beaches: equilibrium profile and longshore transport of coarse material under regular and irregular wave attack. Delft Hydraulics Laboratory, Publication No. 274, July 1982.


Notation

- $H_s/\Delta D_{n50}$ = dimensionless wave height
- $\xi_z = \tan^{-1}\left(\frac{2nH_s}{\sqrt{gT_z^2}}\right)$ = surf similarity parameter
- $\frac{\sqrt{gT_z^2}}{D_{n50}}$ = dimensionless wave period
- $S_2/N$ = dimensionless damage as function of number of waves
- $A_2$ = area of erosion profile ($m^2$)
- $D_{n50}$ = nominal diameter; $D_{n50} = (W_{50}/\rho_a)^{1/3}$ ($m$)
- $D_{15}$ = 15% value of sieve curve ($m$)
- $D_{85}$ = 85% value of sieve curve ($m$)
- $G_F$ = groupiness factor (-)
- $H_s$ = significant wave height (m)
- $K_D$ = stability coefficient (-)
- $N$ = number of waves (-)
- $N_s$ = stability number; $N_s = H_s/\Delta D_{n50}$ (-)
- $Q_p$ = parameter for spectral width (-)
- $S_2$ = damage level; $S_2 = A_2/D_{n50}^2$ (-)
- $T_z$ = average wave period (s)
- $W_{50}$ = 50% value of mass distribution curve (kg)
- $g$ = acceleration due to gravity ($m/s^2$)
- $\bar{J}_1$ = mean length of wave group (-)
- $\bar{J}_2$ = mean total length of wave group (-)
- $\alpha$ = slope angle (degr)
- $\Delta$ = relative mass density; $\Delta = \rho_a/\rho_l$ (-)
- $\epsilon_{5%}$ = variable for spectral width (-)
- $\rho$ = mass density of water ($kg/m^3$)
- $\rho_a$ = mass density of stone ($kg/m^3$)