ABSTRACT
This paper presents some results of simultaneous full-scale wave kinematic and local force measurements (field and laboratory) on a slender, vertical cylindrical pile (test-section) under shallow water conditions. The usual application of Morison's equation is taken under critical consideration due to the real flow regime near the surf zone. The data were analyzed in the time domain as well as in the amplitude and frequency domain. Two estimation methods for force coefficients were tested and compared.

INTRODUCTION
Pile-supported substructures are often needed for various coastal engineering structures. There are still many difficulties in the design of cylindrical slender structures in shallow water regions which are characterized by surf zones with an important dissipation of energy, especially under storm surge conditions. A decisive advance in determining reliable force coefficients is to measure the water particle kinematics synchronously with the surface elevations (waves) and wave pressures or wave forces in the field (Dean (/Z/)). Results of field-measurements and added two-dimensional full-scale laboratory investigations with the same test-pile used in the field enable to describe the mechanism of wave-structure interactions more sufficiently. The real wave attack in the experiments reported in this paper occurred at supercritical Reynolds number conditions.
FIELD MEASUREMENTS
In order to obtain field-data at high Reynolds numbers the measuring facilities were installed in the nearshore area of the Eastfrisian Island Norderney on the German North Sea Coast. Detailed informations can be found in Sparboom (/8/). In Fig.1 the configuration of the measuring devices which were used for this paper is drawn up schematically. Wave pressures were measured at a test-section in 16 points at even spaces on the circumference of the pile (diameter 0.7 m). In a distance of about 5.0 m from the pile the surface elevations and the velocities of the water particle motions (two components in a vertical plane and two components in a horizontal plane) were measured synchronously.

It should be noted here, that the test-section of the pile is structurally combined with a calibrated two-component strain gauge unit measuring the total wave forces acting on the test-section. These force signals were used to control the forces obtained by integrating the pressures which were measured synchronously by 16 single transducers. Representative samples of field data were recorded in some winter-seasons under different storm surge conditions at high tide water levels. Simultaneous records of wave kinematics and wave forces can be seen exemplarily in Fig.2. It could be assumed that the wave kinematics (horizontal velocities and accelerations, surface elevations) were always unaffected by the pile. The direction nearly normal to the shore was defined as mean inline direction (X-axis), because it coincided with the mean direction of the incoming waves. The direction nearly parallel to the shore was similarly defined as mean transverse direction (Y-axis).

LABORATORY FULL-SCALE TESTS
With the end of the winter-season 1982/83 the research program was changed to systematic full-scale tests in the new Large Wave Channel (Grüne and Führböter, (/4/); Führböter, (/3/)). The part of the test-pile in the field which contains the test-section was recovered and used for first laboratory investigations with respect to regularly generated waves (periods from 5.0 to 8.0 seconds, wave heights from 1.0 to 2.0 meters). The configuration of the installed sensors and transducers is given in Fig.3. Regarding the configuration in Fig.1 care was taken to realize the laboratory full-scale tests under equal conditions as used in the field. An example for simultaneous records of the first tests in the Large Wave Channel is plotted in Fig.4. The measured values of the horizontal transverse velocity components were very small. Therefore they were neglected for further calculations.

WAVE FORCE ANALYSIS IN THE TIME DOMAIN
The mostly used approach in the course of estimating the loading of vertical cylindrical structures is as follows: For a given set of wave height H and wave period T the wave kinematics u and w are calculated theoretically; with a suitably chosen pair of coefficients Cd and Cm and application of Morison's equation maximum wave forces are estimated. This method contains uncertainties arising from the validity or not of wave theories proposed for the actual sea state, particularly in the nearshore zone. With measured kinematics only the uncertainty in determining Cd and Cm remains. This problem will be subject of this paper.
Fig. 1:
Full-Scale Configuration in the Field

Fig. 2:
Simultaneous Field Records of Wave Kinematics and Wave Forces in the Test-Section Level
Fig. 3:
Full-Scale Configuration in the Laboratory

Fig. 4:
Simultaneous Laboratory Records of Wave Kinematics and Wave Forces in the Test-Section Level
Utilizing the first simultaneous field data it was found that the values of the force coefficients plotted against Reynolds and Keulegan-Carpenter numbers displayed high scatter (Sparboom (18)). The main reasons for the large amount of scatter are as follows:

- three-dimensional effects; large variability of wave kinematics direction
- interactions of wave kinematics and test-structure (test-section) due to vortex shedding
- superposed long-periodic oscillations; presence of trend values
- wave deformation near the breaker zone.

Fig. 5 gives an example for the time-dependent development of simultaneously measured horizontal velocity and wave force vectors (single step equal to 0.192 seconds). The plotted traces represent a record of about 40 seconds.

**Fig. 5:** Measured Vector Traces of Horizontal Velocity and Test-Section Wave Forces
In order to check the reliability of Morison's equation under shallow water conditions a statistically based method was applied to determine force coefficients. With reference to Pearcey and Bishop (/6/) the mean square method was utilized and computed. A definition sketch for the apparatus used in full-scale investigations of wave-loading can be seen in Fig. 6.

A complete analysis for the loading of cylindrical piles from the hydrodynamic considerations is beyond the scope of this paper. It is, however, instructive to recall some of the basic assumptions associated with the derivation of Morison's equation. This formula depends on the linear superposition of wave force terms as a result of water particle motions (velocity and acceleration or drag and inertia portions). One classical approach to the derivation of Morison's equation starts with the fluid power equilibrium (Lighthill (/5/)). The inertial term in Morison's equation is derived by equating the time rate of work done by the inertial force on a Newtonian fluid to the time rate of change of the fluid kinetic energy due to irrotational flow, under the assumptions of flow field uniformity (no convective accelerations) and existence of a potential function. Similarly, the drag term is derived by equating the power by drag forces to the sum of the time rate of change of kinetic energy in the wake (rotational flow) plus the power dissipation in the wake due to viscous effects. Again, uniformity of flow is assumed; additionally, this term is derived by assuming that the velocity is constant or very slowly varying. Taking into consideration the contribution to the pressure gradient due to the interaction between velocity and vorticity an additional lift force term may be derived. In this case the following assumptions are made: The vorticity is uniformly distributed; the vorticity is proportional to the velocity.
The wave force acting on a vertical, smooth slender cylinder in the horizontal plane then becomes
\[ \vec{F} = \vec{F} \text{ drag} + \vec{F} \text{ inertia} + \vec{F} \text{ lift} + \vec{F} \text{ w} \]
with \( \vec{F} \text{ w} = \) Waterline Force.

The first three terms are usually determined by integrating with respect to z the force per unit length of the submerged portion of the cylinder given in cartesian coordinates by
\[ f = \frac{1}{2} \rho D \left( (u_x - \dot{x})^2 + (u_y - \dot{y})^2 \right) \]

\[ + \frac{\rho D^2}{4} \left( \begin{array}{cc} 1 + C_{xx} & 1 + C_{xy} \\ 1 + C_{yx} & 1 + C_{yy} \end{array} \right) \left( \begin{array}{c} \ddot{u}_x \\ \ddot{u}_y \end{array} \right) \left( \begin{array}{c} C_{xx} \\ C_{xy} \\ C_{yx} \\ C_{yy} \end{array} \right) \left( \begin{array}{c} \dot{x} \\ \dot{y} \end{array} \right) \]

\[ + \frac{1}{2} C_l D \left( u_x^2 + (u_y - \dot{y})^2 \right) \left( \begin{array}{c} u_y - \dot{y} \\ u_x - \dot{x} \end{array} \right) \]

\[ f_x, f_y = \text{ wave force per unit length} \]
\[ \rho = \text{ water density} \]
\[ D = \text{ diameter of the vertical cylindrical pile} \]
\[ C_d = \text{ drag coefficient} \]
\[ C_{xx}, C_{yy} = \text{ added mass coefficients} \]
\[ C_l = \text{ lift coefficient} \]
\[ u_x, u_y = \text{ horizontal orbital velocity} \]
\[ \ddot{u}_x, \ddot{u}_y = \text{ temporal derivatives of } u_x \text{ and } u_y \]
\[ \dot{x}, \dot{y} = \text{ horizontal displacement velocity} \]
\[ \ddot{x}, \ddot{y} = \text{ horizontal displacement acceleration} \]

The reducing assumptions that pile movements are of very small value and therefore ineffective regarding force generation, and that the potential field is axial-symmetric, i.e. \( 1 + C_{xx} = 1 + C_{yy} = C_m \) (inertia coefficient), yield
\[ \left( \begin{array}{c} f_x \\ f_y \end{array} \right) = \frac{1}{2} \rho D C_d |\vec{u}| \left( \begin{array}{c} u_x \\ u_y \end{array} \right) + \frac{\rho D^2}{4} C_m \left( \frac{1}{2} \rho D C_l |\vec{u}| \left( \begin{array}{c} u_y \\ u_x \end{array} \right) \right) + \frac{1}{2} \rho D C_m \left( u_x^2 + u_y^2 \right) \]

Due to the complex vorticity mechanism the sign of the lift term cannot be defined theoretically. The inline force in X-direction becomes
\[ f_x = \frac{1}{2} \rho D C_d u_x |\vec{u}| + \frac{\rho D^2}{4} C_m \ddot{u}_x \frac{1}{2} \rho D C_l u_y |\vec{u}|. \]

Neglecting the lift term we get
\[ f_x = \frac{1}{2} \rho D C_d u_x |\vec{u}| + \frac{\rho D^2}{4} C_m \ddot{u}_x. \]
For the two-dimensional case in the wave channel \( u \) in equation (3) disappears so that equation (5) gives the well-known Morison's equation

\[
f_x = \frac{1}{2} \xi D C_d \left| u_x \right| + \frac{\eta}{\xi} \xi D^2 C_m \left| u_x \right| .
\]  

(6)

For determining Morison coefficients from time-averaged field measurements the mean square method was applied (6). Equation (5) goes over in

\[
f_x^2 = A^2 \left| u_x \right|^2 + B^2 \left| \dot{u}_x \right|^2 + 2 A B \left| u_x \right| \left| \dot{u}_x \right| .
\]  

(7)

The bars indicate mean values of the squares of the measured quantities \( \left( f, u, \left| u \right|, \dot{u} \right) \) integrated over a chosen interval. Assuming linear independence between \( u \), \( \left| u \right| \) and \( \dot{u} \) the cross product can be neglected. The solution of a pair of linear equations for consecutive intervals gives the unknown coefficients \( A \) and \( B \) and also the Morison coefficients

\[
C_d = \frac{A}{\frac{1}{2} \xi D} , \quad C_m = \frac{B}{\frac{\eta}{\xi} \xi D^2} .
\]

It should be noted here, that the solution of the linear system of equations (7) will be valid if the flow is stationary and if the parameters are time-invariant.

Mean squared equation (6) becomes

\[
f_x^2 = A^2 \left| u_x \right|^2 + B^2 \left| \dot{u}_x \right|^2 + 2 A B \left| u_x \right| \left| \dot{u}_x \right| .
\]  

(8)

The mean squares corresponding to equation (4) lead to a more complicated formula

\[
f_x^2 = A^2 \left| u_x \right|^2 \left| \dot{u} \right|^2 + B^2 \left| \dot{u}_x \right|^2 + C^2 \left| \dot{u}_y \right|^2 + 2 A C \left| u_x \right| \left| \dot{u} \right| \left| \dot{u}_x \right| + 2 B C \left| \dot{u}_x \right| \left| \dot{u}_y \right| \left| \dot{u} \right| + 2 A B \left| u_x \right| \left| \dot{u}_x \right| \left| \dot{u}_y \right| \left| \dot{u} \right| .
\]  

(9)

Due to linear independence the last term can be neglected again. Because there are nonvanishing cross terms in (9), there exists no linear solution. Instead of solving equation (9) the square of the error \( E \) which is defined as the difference between the right and left hand side of equation (9) is minimized:

\[
\frac{1}{2} \sum_{i=1}^{N} E_i^2 \rightarrow \text{Min}
\]

(10)

\[ N = \text{number of averaged intervals} \]

The unknown coefficients for the feedback to equation (4) are

\[
C_d = \frac{A}{\frac{1}{2} \xi D} , \quad C_m = \frac{B}{\frac{\eta}{\xi} \xi D^2} , \quad C_1 = \frac{C}{\frac{1}{2} \xi D} .
\]

The minimization is performed by an iterative gradient technique. The essentially same method can be used to minimize the two-parametric equations (7) and (8). Two independent solutions for the parameters from the same time-averaged record can therefore be obtained and compared.
ANALYSIS IN THE AMPLITUDE AND FREQUENCY DOMAIN

Measured field data and wave channel data were analyzed using similar procedures (except for minor differences in their smoothing requirements detailed later on). The analysis was performed in the frequency domain and in the amplitude domain.

At the first stage, data were checked for stationarity. To this purpose, the original record was partitioned in equal time intervals (about 30 sec) for each of which a sample mean was calculated. Next, the number of runs about the mean value calculated for the entire record was counted and compared to the values for the run distribution. The hypothesis of stationarity was rejected at the $\alpha = 0.05$ level of significance because the number of runs counted was considerably lower than the lower bound of the acceptance region (this was especially true for the surface elevation record, less so for the horizontal orbital velocity). This implies that there is a non-negligible contribution to the surface elevation with a period at least one order of magnitude longer than the wave periods. This phenomenon may be attributed to surf beat due to wave groupiness.

Now, letting $z(t)$ be the nonstationary surface elevation, $\eta(t)$ a random process with a stationary mean value of zero, and $G(t)$ a very slow process (henceforth called the trend of $z(t)$) compared to $\eta(t)$, we may write

$$z(t) = G(t) + \eta(t).$$ \hspace{1cm} (11)

It can then be shown (Bendat and Piersol (11)) that the expected value of $z(t)$ is $G(t)$. The mean square value of $z(t)$ is

$$E[(G + \eta)^2] = E[G^2] + E[\eta^2] + 2E[G \cdot \eta]$$ \hspace{1cm} (12)

where the last term turns out to be of negligible magnitude compared to the other two, implying that $G(t)$ and $\eta(t)$ are linearly independent; for, in this case, $E[G \cdot \eta]=E[G] \cdot E[\eta]$ and $E[\eta^2]$ is zero by definition. Equation (12) shows that the mean square value of the surface elevation would be overestimated by the amount $E[G^2]$ if the trend portion is not removed. It is by precisely this amount that the spectral density function would also be overestimated (specifically, in the lower frequency range). In fact, the trend would nullify the estimation of low frequency spectral content. It is for these reasons that it was decided to remove the trend. An appropriate estimate for $G(t)$ is given by short time averaging as follows:

$$\hat{G}(t) = \frac{1}{2T} \int_{t-T}^{t+T} z(\tau) \, d\tau.$$ \hspace{1cm} (13)

It was found that a satisfactory trade-off between bias error and random error for $G(t)$ could be achieved for $2T = 47$ sec. For a digitized $z(t)$ with sampling rate $\tau$, this operation is equivalent to the application of a finite-impulse-response digital filter with transfer function

$$H(\omega \Delta t) = \frac{\sin(\omega \tau)}{\sin(\frac{\omega \Delta t}{2})} \cdot \frac{\Delta t}{2\tau}.$$ \hspace{1cm} (14)

The transfer function being real-valued, there is no phase distortion associated with the filtering operation. Additionally, if the same
linear operation is performed on two different records (e.g. surface elevation and horizontal orbital velocity), then the transfer function and associated coherence remain unchanged. This prompted us to use the trend filter on all processed signals. Data analysis in the frequency domain concentrated on the estimation of power spectral density functions $S_{ii}(f)$, gain functions $|H_{ij}(f)|$ and coherence functions $\gamma_{ij}^2(f)$. Preliminary tests showed that there are no significant contributions above 0.6 Hz. The estimates obtained were therefore plotted up to that frequency (some up to 0.5 Hz), not up to the Nyquist frequency, this being 2.6 Hz for field measurements and 20 Hz for wave channel measurements. Since records of various length were used (15 to 20 min for the field), it was decided to keep the number of degrees of freedom for the spectral estimates practically constant at $n = 40$. This means the normalised standard error

$$\varepsilon_r = \left( \frac{1}{2} \cdot n \right)^{\frac{1}{2}} = 0.22$$

The reasons leading to the choice for $n$ are:

- a smaller number of degrees of freedom would appreciably increase the random error in the spectral estimates. For instance, $n = 22$ would give $\varepsilon_r = 0.3$.
- a larger number of degrees of freedom would not lead to a corresponding reduction of random error. For example, to achieve $\varepsilon_r = 0.1$, we would require $n = 200$ (five times more degrees of freedom). Such a disproportionate increase in the number of degrees of freedom would require increasing the resolution bandwidth (and thus increasing the bias error) or increasing the record length (and thereby violating the stationary assumption).

Frequency smoothing need not be performed on regular wave data obtained under laboratory conditions. Standard FFT procedures are used to obtain all estimates from the time records. For the power spectral density estimate $S_{uu}$ of the horizontal acceleration $\ddot{u}/\omega_t$ for field data, however, use of the well-known identity

$$\mathcal{F}\left[\frac{\ddot{u}}{\omega_t}\right] = j\omega \mathcal{F}[u]$$

was made, where $\mathcal{F}[\cdot]$ is the Fourier Transform and $j = \sqrt{-1}$.

RESULTS AND CONCLUSIONS

The application of the mean square method in two-dimensional case (equ. (8) for regular waves in the laboratory) seems to be quite satisfactory. The mean parameters $C_d$ and $C_m$ corresponding to various time-averaged intervals were plotted in Fig. 7 for the linear solution and for the minimization of the squared error. $C_m$ is almost independent from the length of intervals. The values for $C_d$ differ significantly between the estimation methods used and display considerable scatter.

Feeding back a suitable pair of force coefficients ($C_d = 1.0; C_m = 1.5; Dean's reliability ratio r = 0.8 to 1.8$) to equation (6) and operating with measured kinematics, the force was obtained in comparison to the real measured force (Fig. 8). A quite good agreement of measured and calculated force plots can be observed. The goodness of mean square based estimations was also verified by the gain and coherence functions.
between measured and calculated forces (Fig.9). The gain function shows that a proportional relation exists in the range of high coherence; the value of $|H_{xy}|$ is nearly unity.

Utilizing mean square method in field data processing equations (7) and (9) were computed. Fig.10 displays results for two- and three-parametrical cases. It should be noted, that the lift term in equation (9) is superposed in inline direction because the present velocity in transverse direction generates this lift force, theoretically. Again, the coefficient $C_m$ is nearly independent from the length of intervals and from the estimation methods. Values of $C_d$ are somewhat smaller in the three-parametrical case. Feeding back suitable pairs of estimated force coefficients to equation (5) with the simultaneously measured kinema-
Fig. 9: Frequency Characteristics of Measured and Calculated Forces-Laboratory (Regular Waves)

Fig. 10: Calculated Force Coefficients from Field Data
tics, time records of calculated forces were obtained. These time records matched poorly with the measured force. Inclusion of the lift term in equation (4) didn't give any better fit. These comparisons indicate that Morison's equation may be deemed adequate in predicting the inline force for simple flow structures (harmonic unidirectional waves and no current) as encountered under laboratory conditions; this is not the case for complex flow regimes as found in the nearshore area, in particular predicting maximum forces. This should not be surprising, since the assumptions associated with the derivation of Morison's equation are valid to a much higher degree for laboratory conditions than in the real irregular sea.

On the other hand it can be noted that the power (zero order moment) of the calculated force spectrum ($C_d = 0.8; C_m = 1.4; r = 1.2$ to $2.0$) has nearly the same value as that of the measured force (Fig.11). For lower frequencies the calculated forces are somewhat overestimated and in the range of higher frequencies they are underestimated. The amount of total energy within a 16 minute record is fairly good represented by recalculations with Morison's equation using measured kinematics and mean square estimated coefficients. This was expected, because Morison's equation is derived from power equilibrium considerations. Due to rapid changes in the flow (Sarpkaya (/7/)) inline force components at higher frequencies than those considered by Morison's equation occurred in the field. The interaction between transverse and inline forces, further influenced by nearshore currents, might be responsible for the higher frequency contributions to the inline force spectrum.

Fig.11: Frequency Characteristics of Measured and Calculated Forces-Field (Irregular Sea State)
Frequency analysis of field data, performed as described in the previous section, resulted in power spectra for the surface elevation, measured wave kinematics and measured forces, resolved in quasi-inline and quasi-transverse directions. Representative plots for several processed tests are given in Fig.12. Contrary to reported data obtained from deepwater offshore structures (Pearcey and Bishop /6/), it can be readily ascertained that the quasi-transverse force spectrum has a comparable amount of spectral content to the quasi-inline force spectrum. This prompted some additional investigations regarding the directionality of force calculating the force spectra for several pertinent directions. Fig.13 illustrates the directionality of force power spectra associated with a cylindrical nearshore pile (test-section) under the influence of storm surge conditions. At first, it may seem surprising that the spectral content (which is best summarised by the $m_0$ parameter) is maximum about $45^\circ$ from the mean incident wave direction, falling off gradually from this maximum to both sides and reaching a minimum at $-45^\circ$ from the mean wave direction; the ratio of the maximum to the minimum value being 1.6 to 1.0. If, however, we consider the slowly varying velocity component established by the trend removal procedure, then the disparity between mean wave direction and "mean force direction" may be accounted for. This gradually varying current $V$ (magnitude 0.4 m/s to 0.7 m/s), moving obliquely against wave propagation direction, conspires with the wave-induced velocities to produce an asymmetric flow pattern. An idealized model derived for this flow situation suggests that the clockwise-directed vortex is consistently reenforced to the detriment of the anti-clockwise vortex. This should result in a force component in "mean force direction".

The observed effect due to complex interactions in the surf zone might explain reported failures of nearshore structures which often demonstrated material fatigue nearly transverse to the mean incident wave direction. In conclusion, only two-dimensional full-scale laboratory investigations with respect to lift force interactions will not suffice to determine the magnitude of these forces as measured nearshore at storm surge conditions.
Fig. 12: Power Spectral Density of Surface Elevation, Wave Kinematics and Wave Forces
Fig. 13: Multi-Directional Wave Force Spectra Versus Frequency
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