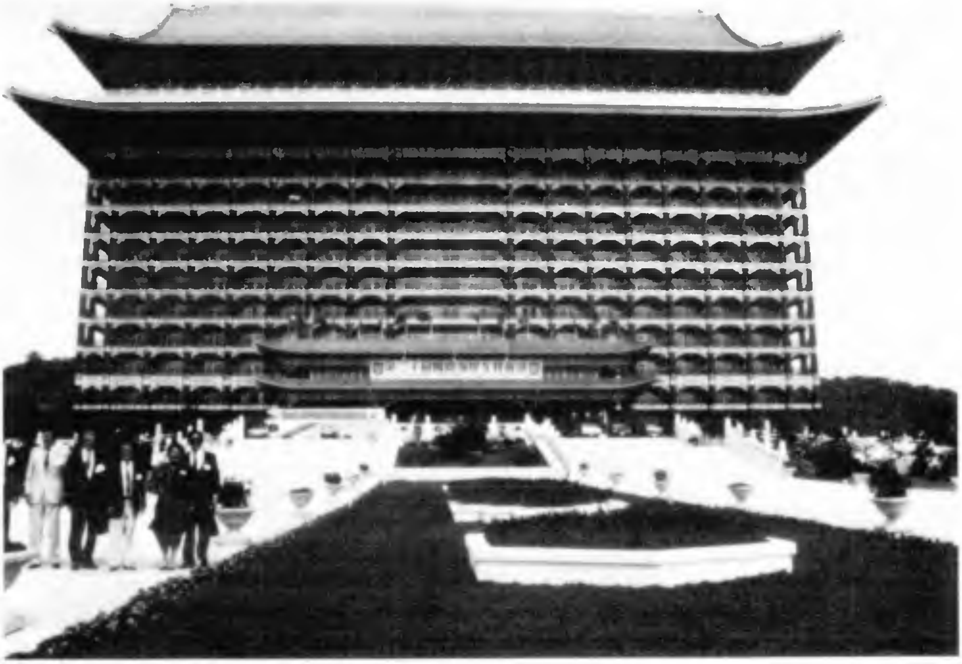

Grand Hotel, Taipei, Taiwan

PART II

COASTAL PROCESSES AND SEDIMENT TRANSPORT

Su-Ao Port—30-ton dolos, Taiwan, ROC—R.L. Wiegel

CHAPTER 70

Changes in Current Properties due to Wave Superimposing

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ABSTRACT

Changes in current properties due to wave superimposing are investigated experimentally. Variations of the mean water level gradient and the current velocity profile after wave superimposing are examined. Experimental results are discussed in relation to the energy conservation equation including the bottom friction term. It is found that changes in current properties can be well explained by increase in the time averaged bottom shear stress.

1. INTRODUCTION

Compared with studies on wave transformation due to currents, there are few studies which discussed the effect of waves on current properties. In wave-current co-existing fields, there exist mutual energy exchanges between wave and current components, so that current properties do not remain the same as in case of current only after waves are superimposed.

Phillips(1977) derived the momentum and energy conservation equations in the co-existing field; however, he did not consider frictional terms in his analysis. It should be noted that the increase in the time averaged bottom friction due to wave superimposing is the key point to represent the wave-current co-existing field. That is, the current feels larger resistance due to presence of the waves than in the current only field. Lundgren(1972) proposed an approximate theory for the reduction of the current velocity due to wave superimposing. Grant-Madsen(1979) presented an analytical model to represent the velocity field in wave-current co-existing systems.

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However, the experimental verifications on the model have not been sufficiently yet. Thus, the knowledges on the changes in current properties due to wave superimposing are not enough up to the present.

This paper investigates the water surface gradient and the velocity profile of the current in the co-existing field experimentally, and discusses the results in relation to the energy conservation equation including the bottom frictional term.

2. ENERGY CONSERVATION EQUATIONS

The energy conservation equations including the bottom friction term are obtained after Phillips' derivation. The enery balance equation for waves is given by,

$$\frac{\partial}{\partial t}\{E+\frac{M_\alpha^2}{2\rho(h+\bar{\eta})}\}+\frac{\partial}{\partial x_\alpha}\{U_\alpha E+F_\alpha-\frac{\widetilde{U}_\alpha M_\beta^2}{2\rho(h+\bar{\eta})}\}+S_{\alpha\beta}\frac{\partial U_\beta}{\partial x_\alpha}$$
$$=\widetilde{U}_\alpha\bar{\tau}_{\alpha,z=-h}-\overline{\int_{-h}^{\eta}\frac{\partial\tau_{\beta\alpha}}{\partial z}u_\alpha^+dz} \qquad (1)$$

in which,
u_α^+ : the composite velocity of the wave and the current components $u_\alpha^+=u_\alpha+U_\alpha$, u_α : the wave velocity component, U_α : the current velocity component,
E : the wave energy density
F_α : the wave energy flux,
M_α : the mass flux due to the wave motion,
$S_{\alpha\beta}$: the radiation stress,
h : the water depth,
$\bar{\eta}$: the displacement of the mean water level due to wave superimposing.

The right hand side of Eq.(1) denotes the energy dissipation due to the time averaged bottom shear stress $\bar{\tau}_{\alpha,z=-h}$ and the internal viscosity and the Reynolds stress component $\tau_{\beta\alpha}$. As the other symbols are the same as those used by Phillips, further explanations on the symbols are omitted.

The energy balance equation for the current is,

$$\frac{\partial}{\partial t}\{\frac{1}{2}\widetilde{U}_\alpha\widetilde{M}_\alpha+\frac{1}{2}\rho g(\bar{\eta}^2-h^2)\}+\frac{\partial}{\partial x_\alpha}\{\widetilde{M}_\alpha(\frac{1}{2}\widetilde{U}_\beta^2+g\bar{\eta})\}+U_\beta\frac{\partial S_{\alpha\beta}}{\partial x_\alpha}=-\widetilde{U}\bar{\tau}_{\alpha,z=-h} \qquad (2)$$

The second term of the right hand side of Eq.(1) is approximated after Jonsson's(1966) or Brevik-Aas'(1980) simplification.

$$\overline{\int_{-h}^{\eta}\frac{\partial\tau_{\beta\alpha}}{\partial z}u_\alpha^+dz}\approx(\overline{\tau_{\beta\alpha}u_\alpha^+})_{z=-h} \qquad (3)$$

The following consideration is limitted for two dimensional flow. The bottom shear stress τ_b in the streamwise direction is assumed to be presented by the following equation:

$$\tau = \frac{1}{2}\rho f_{cw}(U+\hat{u}\cos\sigma t)|U+\hat{u}\cos\sigma t| \tag{4}$$

in which, U is the depth averaged current velocity, $\hat{u}$ the velocity amplitude of the wave component at the bottom, σ the angular frequency of the waves, ρ the densityof water and f_{cw} the wave-current friction factor.

The following assumptions are introduced for the simplification of Eqs.(1) and (2).
(1) The energy density of the waves, the current velocity and the mean water level are independent of time.
(2) The current is treated as a uniform flow in depth.
(3) The ratio of the wave amplitude a to the spatial averaged mean water depth h_0 is small. This ratio is used as a small parameter in the following consideration.
(4) The spatial variation of the current is small. That is, $dU/dx=O[(a/h_0)^2]$.

Ignoring higher terms than $O(a/h_0)^2$, the energy conservation equation for the wave component is obtained as follows:

$$\frac{d}{dx}\{E(U+c_g)\} = -\frac{1}{2}\rho f_{cw}\{\overline{|U+\hat{u}\cos\sigma t|^3} - U\overline{(U+\hat{u}\cos\sigma t)|U+\hat{u}\cos\sigma t|}\} \tag{5}$$

On the otherhand, the equation for the mean current is given by,

$$\frac{d}{dx}\{\frac{\rho}{2}U^3(h+\bar{\eta})\} + \frac{3}{2}U^2\frac{dM}{dx} + \rho ghU\frac{d\bar{\eta}}{dx} + U\frac{dS_{xx}}{dx} = -\frac{1}{2}\rho f_{cw}U\overline{(U+\hat{u}\cos\sigma t)|U+\hat{u}\cos\sigma t|} \tag{6}$$

The right hand side of Eq.(5) means the energy dissipation of the wave component $(E_w)_{cw}$. The ratio of $(E_w)_{cw}$ to that without current $(E_w)_w$ is given by,

$$\frac{(E_w)_{cw}}{(E_w)_w} = \frac{f_{cw}(\overline{|U+\hat{u}\cos\sigma t|^3} - U\overline{(U+\hat{u}\cos\sigma t)|U+\hat{u}\cos\sigma t|})}{f_w\overline{|\hat{u}\cos\sigma t|^3}} = \frac{f_{cw}}{f_w}\frac{(E_w)'_{cw}}{(E_w)'_w} \tag{7}$$

Concerning the current energy dissipation, the similar expression for the ratio of with waves $(E_c)_{cw}$ to without waves $(E_c)_c$ is obtained as follows:

$$\frac{(E_c)_{cw}}{(E_c)_c} = \frac{f_{cw}U\overline{(U+\hat{u}\cos\sigma t)|U+\hat{u}\cos\sigma t|}}{f_c U^3} = \frac{f_{cw}}{f_c}\frac{(E_c)'_{cw}}{(E_c)'_c} \tag{8}$$

The further analytical considerations are possible both for $(E_w)'_{cw}/(E_w)'_w$ and $(E_c)'_{cw}/(E_c)'_c$ by dividing the condition $\hat{u} \geqq |U|$ and $\hat{u} \leqq |U|$. Fig.1 shows the relation between $(E_w)'_{cw}/(E_w)'_w$ and $|U|/\hat{u}$, and Fig.2 the variation of $(E_c)'_{cw}/(E_c)'_c$ with $\hat{u}/|U|$. It is noted from Fig.2 that the current energy dissipation in the co-existing field becomes larger than that in the current only field for the same current velocity. The velocity reduction of the current due to wave superimposing is not considered here.

Next, the similar calculation is performed on the ratio of the wave energy dissipation to the total energy dissipation. Fig.3 shows

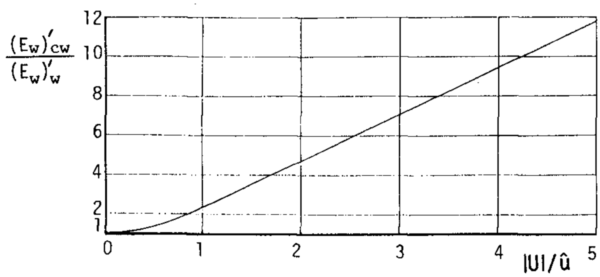

Fig. 1 Energy dissipation ratio for wave component

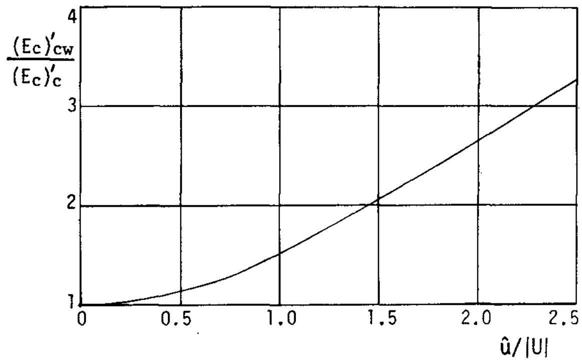

Fig. 2 Energy dissipation ratio for current component

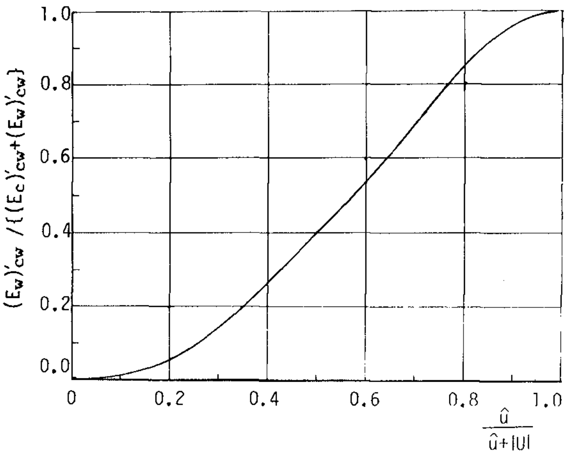

Fig. 3 Composite ratio of wave energy dissipation to total energy dissipation

the relation of the ratio with the wave-current composite ratio $\hat{u}/(\hat{u}+|U|)$. The figure indicates $(E_w)'_{cw}/\{(E_w)'_{cw}+(E_c)'_{cw}\}$ is 0.4 when the wave velocity component is equal to the current one which means that $\hat{u}/(\hat{u}+|U|)=0.5$. It implies that the energy dissipation of the current is 1.5 times of that of the wave component under the condition of $\hat{u}=|U|$.

3. VARIATION OF TIME-AVERAGED WATER SURFACE DUE TO WAVE SUPERIMPOSING

3.1 Experimental apparatus and procedure

The experiment was carried out in a 27m long, 0.5m wide and 0.7m high wave tank(Fig.4), in which circulating flow could be generated by a power pump. In order to produce large energy dissipation in the tank of limited length ,two dimensional artificial roughness elements of 12mm×12mm in cross section and 10cm intervals were added on the bottom over a distance of 16m. Six capacitance type wave gauges were equipped at 2.5m intervals. The water depth is kept 30cm constant for all the cases.

The measurements were conducted for the following three cases; waves without current, current without waves and wave-current combined flow. The signals from the wave gauges were analysed to obtain the wave damping ratio and the gradient of the mean water level. In the experiments, only those waves of a wave train which did not contain reflection effects were used. Since the contamination effect is known to be a cause of wave damping[12], special attention was paid on the cleanness of both water and the surfaces of side walls.

3.2 WAVE-CURRENT CO-EXISTING SYSTEM

Svendsen(1985) has pointed out several difficulties in generating a wave-current system without any disturbance. For example, the ratio of the width of the flume to the water depth should be large enough to avoid the side wall effect on the current component. However, some extent of the water depth is required for wave generating. Therefore, a wide wave tank and a large capasity power pump are needed for both requirements.

The changes in current properties depend on how the wave-current co-existing system is generated. In this experimental facility, the total mass of water in the wave tank and the pipe system is kept constant after wave superimposing; therefore, the increase in the time averaged bottom shear stress due to wave superimposing causes the reduction of the water discharge through the wave tank. Meanwhile, if the discharge is kept constant by a head tank, the increase in the time averaged bottom shear stress changes the water depth and the gradient of the time averaged water surface.

In the present wave-current system, the head raised by the power pump is balanced with the head losses due to the frictions on the

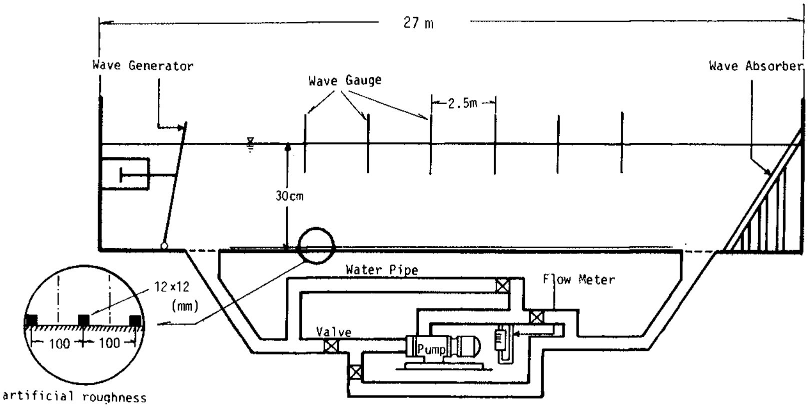

Fig. 4 Experimental apparatus

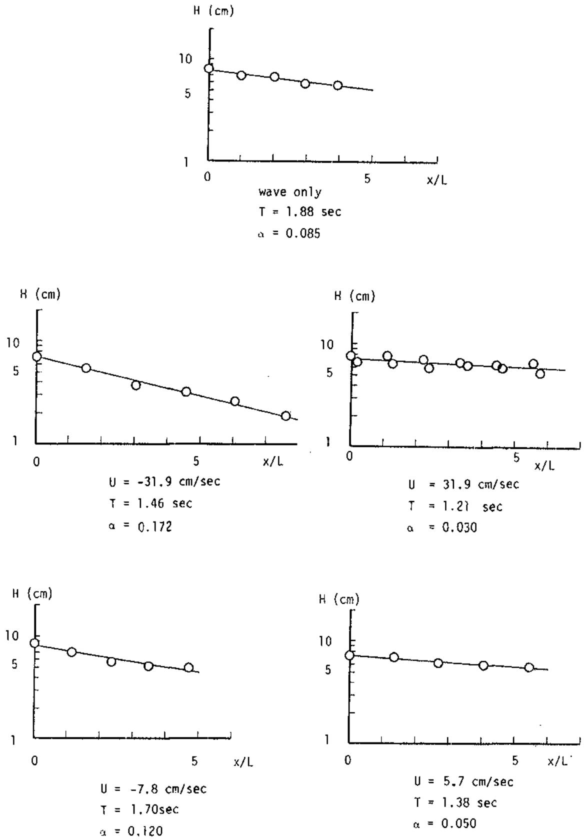

Fig. 5 Wave attenuation profiles

bottom, on the side walls, on the surface of the pipe system, etc. The reason why the large bottom roughness was equipped on the bottom is to make the other losses small compared with the loss by the bottom friction. It is natural to consider that the driving power of the pump is kept constant for both the case of current without waves and the case with waves. Therefore, the both cases are considered to be comparable systems dominated by the same driving power of the pump and the respective properties of the bottom shear stress. Consequently, it is possible to discuss the changes of the current properties due to wave superimposing by comparing the both cases.

3.3 Experimental Results

Preliminary experiments were conducted to investigate the wave damping properties in the fields with and without current. Some examples are shown in Fig.5. It is found that the waves attenuate exponentially even in the wave-current co-existing field, and the damping becomes significant in the opposite current cases. The wave decay modulus α was calculated for the each run by the least square method. The results on α are shown in Fig.6, in which β is defined as $\sqrt{\sigma/2\nu}$ (ν : kinematic viscosity) and L is the wave length. Meanwhile, the water surface variation in the current only field was also measured as a preliminary experiment. The results are shown in Fig.7, in which Δh is the relative water level to that at x=5m. It is noted that the variation of the water surface in the upper stream region than the position x=10m seems to be expressed by a straight line.

The results on the time averaged water surface with and without waves are shown in Fig.8. It is found that the slopes of the water surface become steep after wave superimposing regardless of the current direction. The gradients of the straight lines $d(h+\bar{\eta})/dx$ are determined, then the properties of $d\bar{\eta}/dx$ are discussed as follows. Fig.9 shows the results on $d\bar{\eta}/dx$ with the wave-current composite ratio $\hat{u}/(\hat{u}+|U|)$ as an abscissa. The results indicate that $d\bar{\eta}/dx$ increases with increase in $\hat{u}/(\hat{u}+|U|)$. As shown in Eq.(6), not only the $d\bar{\eta}/dx$ term but also the following terms ; the gradients of the kinematic energy, the radiation stress and the mass flux are related to the energy conservation for the current component. Since dM/dx and dS_{xx}/dx are related to the wave height variation dH/dx , dH/dx becomes one of the factors to contribute to the energy conservation for the current. The ratio of the current energy dissipation in the co-existing field $(E_c)_{cw}$ to that in the current only field $(E_c)_c$ can be calculated on the basis of the left hand side of Eq.(6) from the experimental data of the water surface gradient $d(h+\bar{\eta})/dx$ and the decay modulus α. The results are shown in Fig.10. It is found that the results of $(E_c)_{cw}/(E_c)_c$ are well arranged by the wave-current composite ratio $\hat{u}/|U|$. As the wave component dominates in the composite velocity, $(E_c)_{cw}/(E_c)_c$ becomes large due to the increase in the time averaged bottom friction.

Meanwhile, $(E_c)_{cw}/(E_c)_c$ can be also computed by the right hand side of Eq.(8). The friction factors f_{cw} and f_c are estimated by the

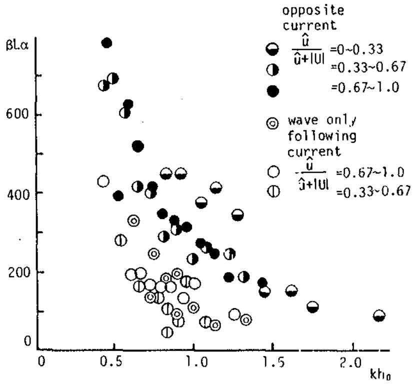

Fig. 6 Wave decay modulus

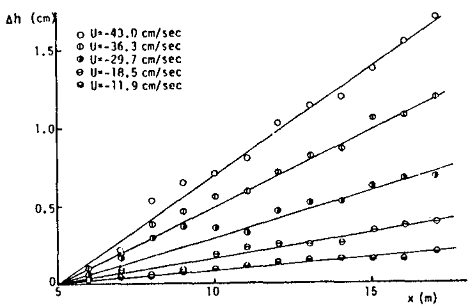

Fig. 7 Water surface profiles without waves

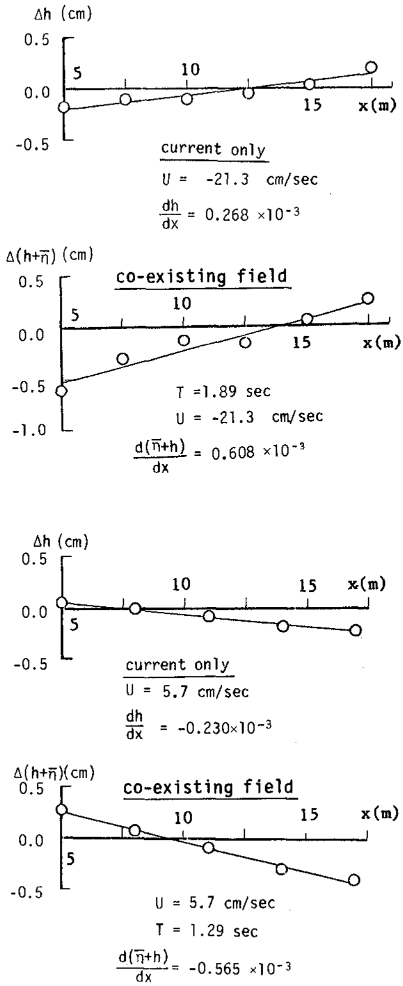

Fig. 8 Mean water level with and without waves

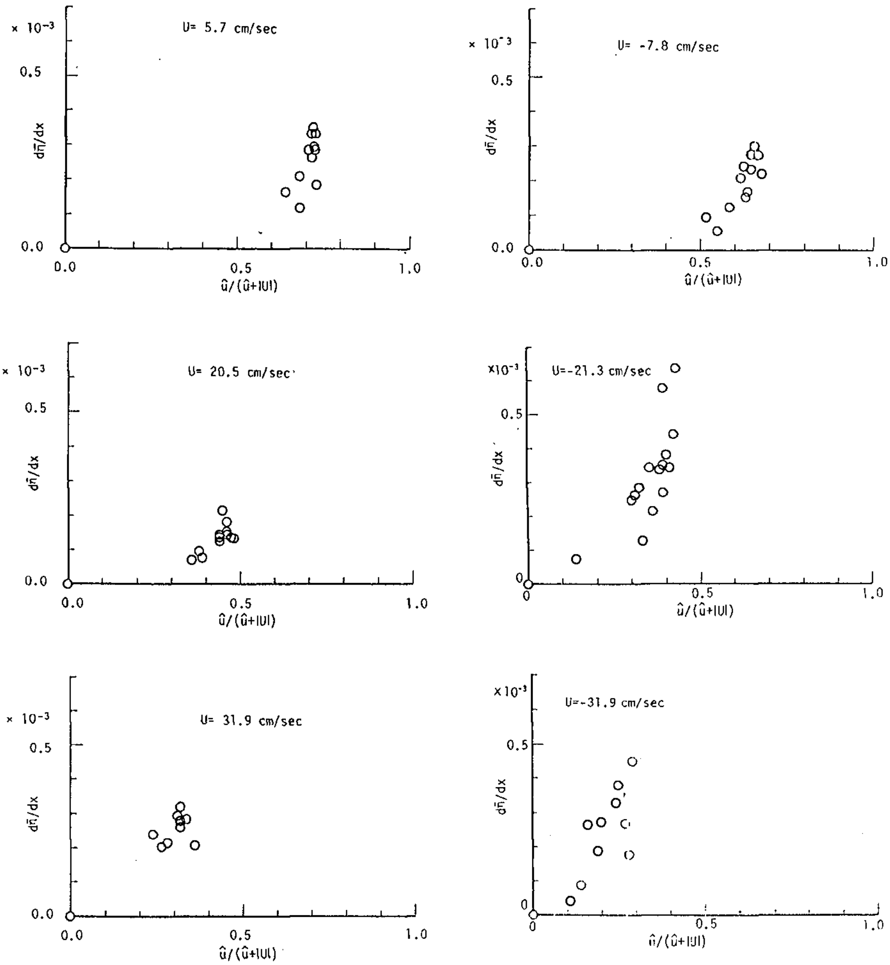

Fig. 9 Variations of gradient of mean water level with wave-current composite ratio

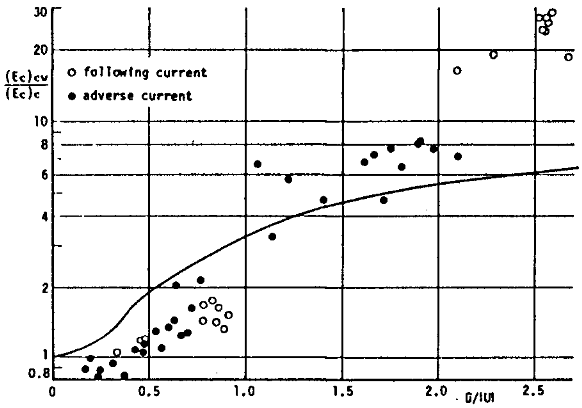

Fig. 10 Energy dissipation ratio for current component

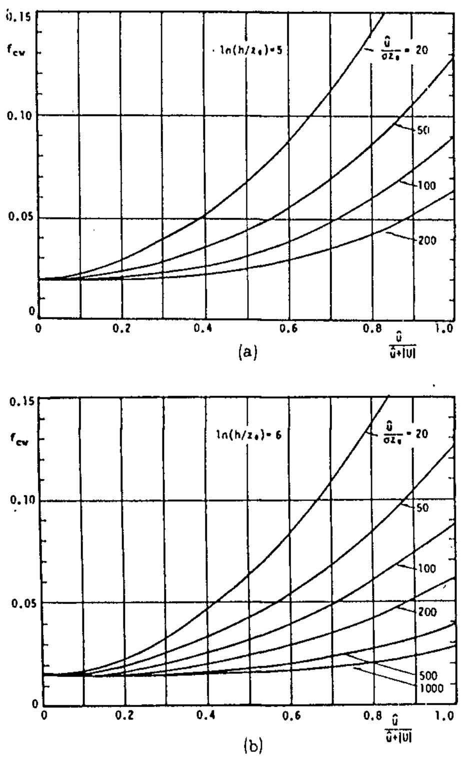

Fig. 11 Friction factors in wave-current co-existing fields

turbulent boundary layer theory in the co-existing field[1]. For fully rough turbulent flow, the friction factor f_{cw} is a function of h/z_0, $\hat{u}/\sigma z_0$ and $\hat{u}/(\hat{u}+|U|)$. Fig.11 shows an example of the properties on the friction factor f_{cw}. The curve in Fig.10 is the calculated result of $(E_c)_{cw}/(E_c)_c$ where f_c and f_{cw} are estimated under the condition of $ln(h/z_0)=4$ and $\hat{u}/\sigma z_0=10$ which are the average values of the present experimental conditions. The current velocity in the numerator of Eq.(8) should be replaced by $\overline{U}_{mod}$ which can be estimated by Eq.(12). Although the present calculation of $(E_c)_{cw}/(E_c)_c$ is only rough estimation, it agrees fairly well with the experimental results for the range of $\hat{u}/|U|\leq 2$.

4. DEFORMATION OF VERTICAL VELOCITY PROFILE OF CURRENT COMPONENT DUE TO WAVE SUPERIMPOSING

4.1 Experimental procedure

The experiment was carried out in the same wave tank mentioned in 3.1. Two dimensional roughness elements of 2mmx2mm in cross section were added on the bottom at 15mm intervals. A laser-doppler anemometer was used to measure the velocities both in the wave-current co-existing field and in the current only field. All the currents used in the test were in the opposite direction to the wave propagation. The test condition is shown in Table 1.

In addition to the present experimental results, the data by Bakker-van Doorn(1980) and Kemp-Simons(1982) obtained under the similar experimental conditions are used for the analysis.

4.2 Results and Discussion

Fig.12 shows one example of the current profiles measured by authors(1984), and Fig.13 shows that by Bakker-van Doorn(1980). In order to discuss the effect of wave superimposing on current profile quantitatively, several characteristics should be introduced. These characteristics indicated schematically in Fig.12, are defined as follows.

Table 1 Experimental conditions

		$\overline{U}$ (cm/sec)	H (cm)	T (sec)	h (cm)
CASE-I	wave only	0	8.52	1.67	30
	current only	-18.6	——		
	co-existing	-17.4	7.74		
CASE-II	wave only	0	7.99	1.67	30
	current only	-31.6	——		
	co-existing	-25.9	7.49		
CASE-III	wave only	0	8.22	1.67	30
	current only	-42.3	——		
	co-existing	-36.7	6.30		
CASE-IV	current only	-34.9	——	1.67	25
	co-existing	-36.0	7.13		

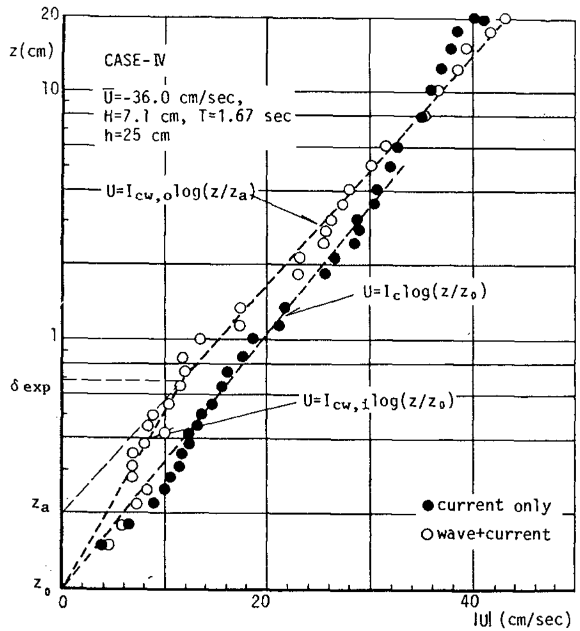

Fig. 12 Velocity profiles of current without and with waves (authors; 1984)[1]

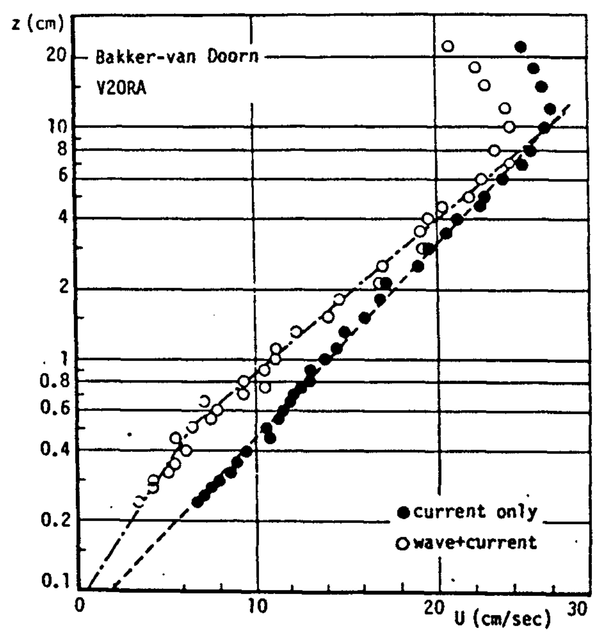

Fig. 13 Velocity profiles of current without and with waves (Bakker-van Doorn; 1980)[2]

The familiar logarithmic current profile can be found in the current only field. The gradient is defined as I_c and the zero intercept of the logarithmic profile as z_0. Meanwhile, the current velocity in the co-existing field increases more slowly with z in the region of $z \leqq \delta_{exp}$; however, in the range of $z > \delta_{exp}$ the gradient is found to be larger than that of the current only field I_c. The gradients of the current profile in the co-existing field are defined as $I_{cw,i}$ for $z \leqq \delta_{exp}$ and $I_{cw,o}$ for $z \geqq \delta_{exp}$. The zero intercept is obtained by extending the upper velocity profile. The height is named z_a, which means the apparent bottom roughness when waves superimpose on the current.

In the existing analytical model, $I_{cw,o}$ has been treated to be equal to I_c and it has been considered that the gradient of the upper current profile is unchanged and the profile only shifts towards smaller velocity after wave superimposing. However, both of authors' and Bakker-van Doorn's data show that $I_{cw,o}$ is always larger than I_c. This property can be explained by the increase in the time averaged bottom shear stress due to wave superimposing.

In the following, the properties of $I_{cw,i}$, $I_{cw,o}$, δ_{exp} and z_a are discussed. These characteristics are obtained from the Authors', Bakker-van Doorn's and Kemp-Simons' data. Figs. 14, 15, 16 show the relations between these characteristics and the wave current composite ratio $\hat{u}/|U|$. These properties may depend on the other parameters such as $\hat{u}/\sigma z_0$, h/z_0 etc.. However, the dependence on $\hat{u}/|U|$ is only discussed here.

The variation of δ_{exp}/z_0 with $\hat{u}/|U|$ can be expressed by the following equation:

$$\delta_{exp}/z_0 = 1 + 7\,\hat{u}/|U| \tag{9}$$

The following relation between z_a/z_0 and $\hat{u}/|U|$ is found from Fig.15:

$$\frac{z_a}{z_0} = 1 + 1.85\,(\hat{u}/|U|)^2 \tag{10}$$

Meanwhile, the ralation between $I_{cw,o}/I_c$ and $\hat{u}/|U|$ can be expressed in the following equation:

$$I_{cw,o}/I_c = 1 + 0.22\,(\hat{u}/|U|)^{1.4} \tag{11}$$

The reduction of the depth averaged current velocity due to wave superimposing is discussed as follows. Since the velocity in the region $z \leqq \delta_{exp}$ is small enough to be disregarded, the ratio of the depth averaged velocity in the co-existing field $\overline{U}_{mod}$ to that in the current only field $\overline{U}$ is given by,

$$\frac{\overline{U}_{mod}}{\overline{U}} \simeq \frac{I_{cw,o}}{I_c}\,\frac{\ln(\frac{h}{z_a})-1}{\ln(\frac{h}{z_0})-1} = \frac{I_{cw,o}}{I_c}\left\{1-\frac{\ln(z_a/z_0)}{\ln(h/z_0)-1}\right\} \tag{12}$$

Invoking Eqs.(10) and (11), $\overline{U}_{mod}/\overline{U}$ can be calculated as a function of $\hat{u}/|U|$ and h/z_0.

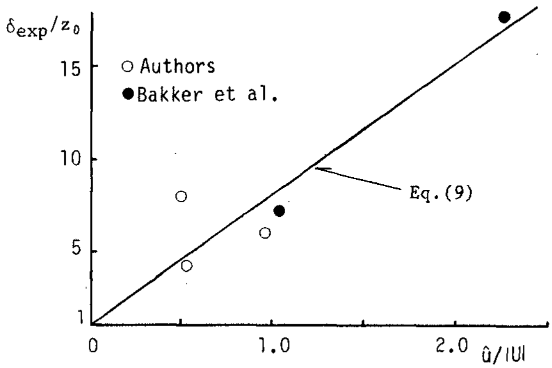

Fig. 14 δ_{exp}/z_0 vs $\hat{u}/|U|$

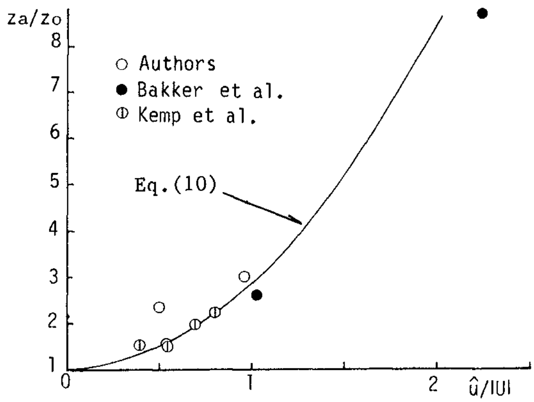

Fig. 15 z_a/z_0 vs $\hat{u}/|U|$

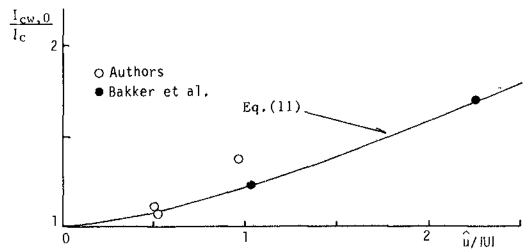

Fig. 16 $I_{cw,o}/I_c$ vs $\hat{u}/|U|$

5. Conclusions

This study investigates changes in current properties due to wave superimposing. The variations of the gradient of the mean water level and the current velocity profile in the co-existing field are examined in relation to the consideration on the energy conservation equation including the bottom stress term. The main results obtained in this study are as follows:

1) The gradient of the time averaged water level becomes steep after wave superimposing due to increase in the time averaged bottom shear stress.
2) The experimental results on the energy dissipation for the current component can be well arranged by the wave-current composite ratio. The estimated energy dissipation based on the energy conservation equation agrees fairly well with the experimental results.
3) Several characteristics representing current profiles with and without waves are introduced. Arranging the existing measurements of the current profiles, the relations between the characteristics and the wave-current composite ratio are obtained. The gradient of current profile after wave superimposing above the wave boundary layer is found to be larger than that without waves.

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