CHAPTER 34

Spectral Wave Attenuation by Bottom Friction: Theory

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ABSTRACT

Based on the linearized form of the boundary layer equations and a simple eddy viscosity formulation of shear stress, the turbulent bottom boundary layer flow is obtained for a wave motion specified by its directional spectrum. Closure is obtained by requiring the solution to reduce, in the limit, to that of a simple harmonic wave. The resulting dissipation is obtained in spectral form with a single friction factor determined from knowledge of the bottom roughness and an equivalent monochromatic wave having the same root-mean-square near-bottom orbital velocity and excursion amplitude as the specified wave spectrum. The total spectral dissipation rate is obtained by integration and compared with the average dissipation obtained from a model considering the statistics of individual waves defined by their maximum orbital velocity and zero-crossing period. The agreement between the two different evaluations of total spectral dissipation supports the validity of the spectral dissipation model.

INTRODUCTION

As waves propagate into water of finite depth the presence of a bottom manifests itself in various ways, e.g., causing shoaling, depth-refraction, and dissipation of energy. In the present context of wave attenuation by bottom friction the most important manifestation of the bottom is the establishment of a bottom boundary layer in the immediate vicinity of the bottom. Within this wave boundary layer, of thickness a few cm, the flow is strongly sheared, generally turbulent, and associated with significant dissipation of energy.

Several models for turbulent wave boundary layer flows and associated energy dissipation have been developed, e.g., Putnam and Johnson (1949), Kajiura (1964, 1968), and Jonsson (1966), to mention a few of the earliest contributions. According to these (and later contributions) the

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attenuation of periodic waves in constant but finite water depth, h, may be expressed formally through the conservation of wave energy equation

$$\frac{\partial E}{\partial t} + \frac{\partial E_f}{\partial x} = -E_d \tag{1}$$

where, according to linear wave theory,

$$\begin{aligned} E &= \tfrac{1}{2}\rho g a^2 \\ E_f &= c_g E = \tfrac{1}{2}\left[1 + \frac{2kh}{\sinh 2kh}\right]\frac{\omega}{k} E \\ \omega^2 &= kg \tanh kh \end{aligned} \tag{2}$$

and the average rate of energy dissipation by bottom friction is obtained from

$$E_d = \langle \tau_b u_\infty \rangle \simeq \tfrac{1}{4}\rho f_w u_{br} u_b^2 \tag{3}$$

in which f_w is the wave friction factor (Jonsson, 1966), $u_b = (a\omega/\sinh kh)$ is the near-bottom maximum orbital velocity, and $u_{br} = (8/3\pi)u_b$. Use of Eqs. (1) through (3) allows evaluation of the wave attenuation due to bottom friction to be performed from knowledge of the periodic wave characteristics--amplitude $a = H/2$ and radian frequency $\omega = 2\pi/T$--and the bottom roughness. For practical applications, however, real waves are not periodic and the question arises which wave amplitude (wave height) and period to choose to represent the random wave field.

An alternative approach was taken by Hasselmann and Collins (1968) who evaluated the average rate of dissipation from $\langle \tau_b u_\infty \rangle$ treating $\tau_b = \frac{1}{2}\rho f_w |u_\infty| u_\infty$ and u_∞ as random variables defined by the directional spectrum of u_∞, the near-bottom orbital velocity. A simplified version of their analysis, corresponding to the use of Eqs. (1) through (3) for each wave component with u_{br} being the orbital velocity having the same root-mean-square value as the specified spectrum, was later proposed and used by Collins (1972) to evaluate the spectral attenuation of waves due to bottom friction. While overcoming the problem of which equivalent periodic wave to choose to represent the wave spectrum, the Hasselmann-Collins approach does not provide information on how the friction factor is related to bottom roughness. Thus the friction factor, f_w, becomes a parameter to be fitted by comparison of model predictions and observations, e.g., Hsiao and Shemdin (1978).

To overcome the problem associated with available models for the prediction of attenuation of waves by bottom friction, we start from the linear equation governing the bottom boundary layer flow. A simple, time-invariant eddy viscosity formulation is used to express the turbulent shear stress and a solution for the boundary layer flow is obtained for a wave motion specified by its directional spectrum. The problem is closed by requiring the spectral representation of the bottom shear stress to reduce the known solution, e.g., Grant and Madsen (1979, 1986), in the limit of a single periodic wave. In this manner theoretical justification is obtained for the application of Eqs. (1) through (3) for each spectral wave component with u_{br}

as specified by Collins (1972). However, in contrast to Collins's study the present analysis also results in a relationship for the friction factor, f_w, in terms of spectral wave characteristics--represented by an equivalent periodic wave having the same root-mean-square near-bottom orbital velocity and excursion amplitude--and the bottom roughness.

Integration of the spectral wave dissipation model over all frequencies produces an expression for the total average dissipation rate associated with a wave motion specified by its frequency spectrum. This estimate of the average dissipation rate is compared with that obtained from a model which considers the joint probability of near-bottom orbital velocity and zero-crossing period derived from the near-bottom orbital velocity spectrum following Longuet-Higgins (1983). Besides resulting in comparable estimates of total average dissipation, thereby supporting the validity of the spectral dissipation model, the total dissipation rate is expressed in terms of representative surface wave characteristics, root-mean-square wave height and significant period, which greatly facilitates the approximate computation of wave attenuation by bottom friction.

SPECTRAL DISSIPATION MODEL

Governing Equation

We start by adopting the linearized boundary layer approximation for the flow above a horizontal bottom located at z = 0, i.e.,

$$\frac{\partial \vec{u}}{\partial t} = -\nabla(p/\rho) + \frac{\partial(\vec{\tau}/\rho)}{\partial z} \tag{4}$$

in which $\vec{u} = (u,v)$ and $\vec{\tau}$ are the velocity and shear stress, respectively, ∇ is the horizontal (x, y) gradient operator, and ρ is the fluid density. Realizing that the shear stress vanishes while the velocity approaches its free stream value as $z \to \infty$, we have

$$\frac{\partial \vec{u}_\infty}{\partial t} = -\nabla(p/\rho) \tag{5}$$

For small values of z, i.e., as the solid boundary is approached, the no-slip condition requires the velocity to approach zero. Thus, we obtain as $z \to 0$

$$\frac{\partial(\vec{\tau}/\rho)}{\partial z} = -\frac{\partial \vec{u}_\infty}{\partial t} = \nabla(p/\rho) \tag{6}$$

which, by integration from z = "0", where $\vec{\tau} = \vec{\tau}_b$, to $z = 0^+$ yields

$$\vec{\tau} = \vec{\tau}_b - \int_{\text{“}0\text{”}}^{0^+} \rho \frac{\partial \vec{u}_\infty}{\partial t} dz \simeq \vec{\tau}_b \tag{7}$$

i.e., a constant shear stress equal to the boundary shear stress within a region close to the solid boundary

($z \leq 0^+$). In this context it should be pointed out that "constant" refers to the spatial not the temporal variation, since τ_b clearly will vary with time.

Thus in a region very close to the solid bottom the "law of the wall" is valid. In analogy with steady turbulent flows over a rough wall we therefore expect a logarithmic velocity profile for $z \leq 0^+$. This requirement is met if we express the shear stress through the concept of a vertical turbulent eddy viscosity, ν_t, which varies linearly with distance from the bottom, i.e., if we take

$$\vec{\tau}/\rho = \nu_t \frac{\partial \vec{u}}{\partial z} = \kappa u_{*r} z \frac{\partial \vec{u}}{\partial z} \tag{8}$$

Strictly speaking we should carry the dynamic analogy with steady turbulent flows further by requiring the friction velocity u_{*r} to be $\sqrt{(\tau_b/\rho)}$ with τ_b denoting the time-varying magnitude of the bottom shear stress. However, Trowbridge and Madsen (1984a, b) considered this complication, in the context of periodic wave boundary layers, and concluded that a time-invariant eddy viscosity suffices so long as the boundary layer analysis is limited to first order in wave steepness as is the case here. For this reason we adopt the expression for the turbulent shear stress given by Eq. (8) with u_{*r} denoting a time-invariant representative friction velocity.

Incorporating Eqs. (5) and (8) in (4) and realizing that $u_\infty \neq u_\infty(z)$ we obtain

$$\frac{\partial(\vec{u}-\vec{u}_\infty)}{\partial t} = \frac{\partial}{\partial z}\left[\kappa u_{*r} z \frac{\partial(\vec{u}-\vec{u}_\infty)}{\partial z}\right] \tag{9}$$

governing the turbulent flow within the bottom boundary layer.

Solution

To solve the governing equation we specify the free stream velocity as that associated with a directional wave spectrum, i.e.,

$$\vec{u}_\infty = \sum_n \sum_m u_{b,nm}(\cos\theta_m,\ \sin\theta_m) e^{i\omega_n t} \tag{10}$$

in which n and m denote summation over frequencies and directions, respectively. With this representation of u_∞ the velocity amplitudes $u_{b,nm}$ are related to the near-bottom orbital velocity spectrum and to the directional surface amplitude spectrum through

$$u_{b,nm} = \sqrt{2S_{u_b}(\omega_n,\theta_m)\,d\theta d\omega} = \frac{\omega_n}{\sinh k_h h}\sqrt{2S_\eta(\omega_n,\theta_m)\,d\theta d\omega} \tag{11}$$

in which ω_n and k_n are related through the linear dispersion relationship, Eq. (2).

The linearity of Eq. (9) combined with the assumed time-invariant representative friction velocity suggests a solution of the form

$$\vec{u} = \sum_n \sum_m u_{nm}(\cos\theta_m, \sin\theta_m)e^{i\omega_n t} \tag{12}$$

in which $u_{nm} = u_{nm}(z)$ represents the complex velocity component amplitudes and only the real part of Eq. (12) constitutes the solution sought.

Introducing Eqs. (10) and (12) in (9) we obtain the equation for each velocity component

$$\frac{d}{d\zeta_n}\left[\zeta_n \frac{d\tilde{u}_{nm}}{d\zeta_n}\right] - i\tilde{u}_{nm} = 0 \tag{13}$$

in which

$$\tilde{u}_{nm} = u_{nm} - u_{b,nm}$$
$$\zeta_n = z\omega_n/(\kappa u_{*r}) \tag{14}$$

The boundary conditions to be satisfied by each component are

$$\tilde{u}_{nm} \to 0 \quad \text{as} \quad \zeta_n \to \infty$$
$$\tilde{u}_{nm} = -u_{b,nm} \quad \text{for} \quad \zeta_n = \zeta_{n0} = z_0\omega_n/(\kappa u_{*r}) \tag{15}$$

where, once again, we have drawn upon the analogy with steady turbulent flows over a rough boundary by requiring the no-slip condition to be satisfied at $z = z_0 = k_b/30$, where k_b is the equivalent Nikuradse sand grain roughness of the bottom.

The solution to Eq. (13) subject to the boundary conditions stated in Eq. (15) may be written in terms of Kelvin functions of zeroth order (e.g., Hildebrand, 1976)

$$\tilde{u}_{nm} = -\frac{\ker 2\sqrt{\zeta_n} + i\,\mathrm{kei}\, 2\sqrt{\zeta_n}}{\ker 2\sqrt{\zeta_{n0}} + i\,\mathrm{kei}\, 2\sqrt{\zeta_{n0}}}\, u_{b,nm} \tag{16}$$

Invoking the limiting from of Kelvin functions for small values of their arguments (Abramowitz and Stegun, 1972, Ch. 9) we obtain from Eqs. (12), (14), and (16)

$$|\vec{u}_{nm}| = \frac{u_{b,nm}}{\left[\left[\ln\frac{\kappa u_{*r}}{z_0\omega_n} - 1.15\right]^2 + \left[\frac{\pi}{2}\right]^2\right]^{1/2}} \ln\frac{z}{z_0} \cos(\omega_n t + \phi_n) \tag{17}$$

where

$$\tan\phi_n = \frac{\pi/2}{\ln\frac{\kappa u_{*r}}{z_0\omega_n} - 1.15} \tag{18}$$

valid for small z.

Closure

While a solution for the turbulent flow in the wave boundary layer has been obtained, this solution involves the representative friction velocity u_{*r} which is yet to be specified. From the expression for the bottom shear stress, Eq. (8), we obtain with the velocity solution given by Eq. (17)

$$\frac{1}{\rho}\tau_{b,nm} = \lim_{z \to 0}\left[\kappa u_{*r} z \frac{\partial u_{nm}}{\partial z}\right]$$
$$= \frac{\kappa u_{*r}}{\left[\left[\ln\frac{\kappa u_{*r}}{z_0 \omega_n} - 1.15\right]^2 + \left[\frac{\pi}{2}\right]^2\right]^{1/2}} u_{b,nm} \qquad (19)$$

Now, in deriving the asymptotic form of u_{nm} as given by Eq. (17) it was assumed that $\zeta_{n0} = z_0\omega_n/(\kappa u_{*r}) \ll 1$. The proportionality factor between the bottom stress component amplitude, $\tau_{b,nm}/\rho$, and the orbital velocity component amplitude, $u_{b,nm}$, given by Eq. (19) is therefore dominated by the first term in the denominator, which in turn is a weak function of ω_n. We may therefore replace ω_n in Eq. (19) by a constant representative radian frequency, ω_r, and consider the ratio of bottom shear stress and orbital velocity amplitudes to be constant. With these approximations the bottom shear stress spectrum is given in terms of the near-bottom orbital velocity spectrum by

$$S_{\tau_b}(\omega,\theta) = \left[\frac{\rho\kappa u_{*r}}{\ln\frac{\kappa u_{*r}}{z_0 \omega_r} - 1.15}\right]^2 S_{u_b}(\omega,\theta) \qquad (20)$$

Integration of Eq. (20) over all frequencies and directions and denoting

$$(\tau_{br}/\rho)^2 = 2\iint S_{\tau_b/\rho}(\omega,\theta)\,d\omega d\theta$$
$$u_{br}^2 = 2\iint S_{u_b}(\omega,\theta)\,d\omega d\theta \qquad (21)$$

we obtain the following relationship

$$\tau_{br} = \frac{\rho\kappa u_{*r}}{\ln\frac{\kappa u_{*r}}{z_0\omega_r} - 1.15} u_{br} \qquad (22)$$

which, given the approximation made in the present analysis, is identical to the result obtained for a periodic wave motion (Grant and Madsen, 1986) when we take

$$u_{*r} = \sqrt{\tau_{br}/\rho} = \sqrt{f_{wr}/2}\; u_{br} \qquad (23)$$

In fact, Eq. (23) may be introduced in Eq. (22) to obtain an equation for the wave friction factor similar to that originally proposed by Jonsson (1966)

$$\frac{1}{4\sqrt{f_{wr}}} + \log_{10}\frac{1}{4\sqrt{f_{wr}}} = \log_{10}\frac{A_{br}}{k_b} - 0.17 \qquad (24)$$

with $k_b = 30z_0$ denoting the equivalent bottom roughness and

$$A_{br}^2 = \left[\frac{u_{br}}{\omega_r}\right]^2 = 2\iint \omega^{-2} S_{u_b}(\omega,\theta)\, d\omega d\theta \quad (25)$$

denoting the near-bottom orbital excursion amplitude of an equivalent periodic wave having the same root-mean-square near-bottom orbital velocity and excursion amplitude as the specified directional wave spectrum. While Eq. (24) is limited to small relative roughness, i.e., large values of A_{br}/k_b, the general formula given by Grant and Madsen (1979) may be used for large relative roughness values.

Spectral Dissipation and Wave Attenuation

Following Kajiura (1968) the average rate of dissipation of wave energy in the bottom boundary layer is given by Eq. (3), $<\tau_b u_\infty>$. Expressed in terms of the spectral components of τ_b, Eq. (19), and u_∞, Eq. (10), including random phase consideration for components of the same frequency, results in

$$\begin{aligned} E_{d,nm} &= <\tau_{b,nm}\ u_{\infty,nm}> = \tfrac{1}{2} f_{wr}\rho u_{br} u_{b,nm}^2 <\cos(\omega_n t+\phi_n)\cos(\omega_n t)> \\ &= \tfrac{1}{4} f_{wr}\rho u_{br} u_{b,nm}^2 \end{aligned} \quad (26)$$

in which the phase difference between bottom shear stress and free stream velocity--amounting to a $\cos\phi_n$ factor--is neglected to be consistent with previous approximations.

Rewriting the dissipation rate given by Eq. (26) by making use of Eq. (11), we may formally express the conservation of wave energy equation in spectral terms as

$$\begin{aligned} \frac{D\{S_\eta(\omega,\theta)\}}{Dt} &= -\frac{1}{\rho g} E_d(\omega,\theta) = -\frac{1}{2g} f_{wr} u_{br} S_{u_b}(\omega,\theta) \\ &= -\frac{1}{2g} f_{wr} u_{br} \left[\frac{\omega}{\sinh kh}\right]^2 S_\eta(\omega,\theta) \end{aligned} \quad (27)$$

with u_{br} given by Eq. (21) and f_{wr} obtained from knowledge of the equivalent Nikuradse sand grain roughness of the bottom and an equivalent periodic wave having the same root-mean-square orbital velocity and excursion amplitude as the specified directional wave spectrum.

EVALUATION OF TOTAL DISSIPATION

Spectral Model

From the model of spectral dissipation derived in the preceding section we may obtain the average rate of dissipation of energy for the entire wave field by integration of $E_d(\omega,\theta)$, given by Eq. (27), over frequency and direction. Performing this integration and recalling the definition of u_{br}, Eq. (21), the total average dissipation rate may be expressed as

$$E_{d1} = \iint E_d(\omega,\theta)\, d\omega d\theta = \tfrac{1}{4}\rho f_{wr} u_{br}^3 \quad (28)$$

While quite simple is appearance, the evaluation of Eq. (21) is somewhat cumbersome since it requires evaluation of quantities that depend on spectral characteristics of the near bottom orbital motion. It would be far simpler if the total average dissipation rate were expressed in terms of wave characteristics derived from the surface amplitude spectrum. We therefore define a characteristic periodic wave with the same root-mean-square amplitude as the wave motion specified by its directional spectrum, i.e.,

$$a_c^2 = 2\iint S_\eta(\omega,\theta)\,d\omega d\theta \tag{29}$$

and a period defined by

$$\omega_c = \frac{2\pi}{T_c} = \frac{m_1}{m_0} \tag{30}$$

where m_j denotes the j'th spectral moment

$$m_j = \iint \omega^j S_\eta(\omega,\theta)\,d\omega d\theta \tag{31}$$

Denoting the near-bottom velocity associated with this characteristic wave by u_{bc} we have

$$u_{bc} = \frac{a_c\omega_c}{\sinh k_c h} = A_{bc}\omega_c \tag{32}$$

where $k_c = 2\pi/L_c$ is the wave number corresponding to the characteristic wave period, i.e.,

$$\omega_c^2 = k_c g \tanh k_c h \tag{33}$$

Introducing the characteristic wave parameters in the expression for the energy dissipation, Eq. (28), we obtain

$$E_{dl} = \left\{\frac{f_{wr}}{f_{wc}}\left[\frac{u_{br}}{u_{bc}}\right]^3\right\}\frac{1}{4}\rho f_{wc} u_{bc}^3 = C_l E_{dc} \tag{34}$$

where E_{dc} is the average rate of dissipation predicted for the characteristic equivalent periodic wave defined by Eqs. (29) and (30).

Values of C_l have been computed for JONSWAP spectra with different peak-enhancement values (γ = 1, 3.3, 7) in different water depths, defined by h/L_c, for a range of different relative bottom roughness, A_{bc}/k_b. The results, shown in Figure 1 as dashed curves, indicate that the value of C_l depends on spectral peakedness and relative water depth with C_l increasing as the spectrum becomes broader (γ = 1) and as the relative depth increases. This behavior reflects the increased importance of the low-frequency part of the surface amplitude spectrum in determining the near-bottom orbital velocity characteristics. Within the range of relative bottom roughness tested, $1 \lesssim A_{bc}/k_b \lesssim 1000$, the resulting value of C_l was found to be essentially independent of this parameter.

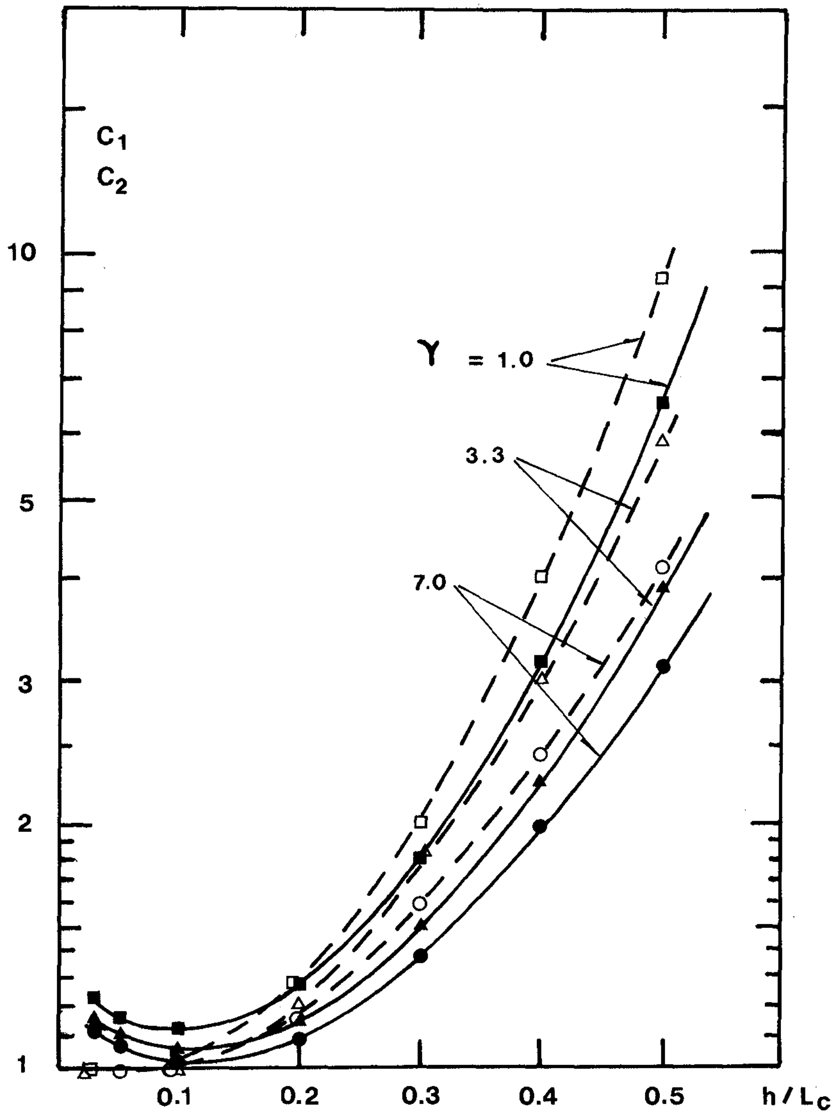

Figure 1. Variation of constants C_1, Eq. (34), and C_2 Eq. (42), for the evaluation of total bottom dissipation for a random wave field with relative water depth h/L_c and spectral peakedness γ.

Individual Wave Model

Longuet-Higgins (1983) derived the joint probability distribution of zero-crossing wave heights and periods for a narrow banded surface spectrum. Replacing the surface amplitude spectrum by the near-bottom orbital velocity spectrum

$$S_{u_b}(\omega) = \left[\frac{\omega}{\sinh kh}\right]^2 S_\eta(\omega) \quad (35)$$

the formulae derived by Longuet-Higgins (1983) may be adopted directly to obtain the joint probability of maximum near-bottom orbital velocity and period

$$p(R,\tau) = \frac{2}{\nu\sqrt{\pi}}\left[\frac{R^2}{\tau^2}\right] e^{-R^2[1+(1-1/\tau)^2/\nu^2]} \cdot L(\nu) \quad (36)$$

where

$$R = \frac{u_b}{\sqrt{2M_0}} = \frac{u_b}{u_{br}}$$

$$\tau = \frac{M_1}{2\pi M_0}T = \frac{T}{T_1} \quad (37)$$

denote the normalized maximum orbital velocity, u_b, and period, T, respectively, and

$$1/L(\nu) = [1 + (1 + \nu^2)^{-1/2}]/2$$

$$\nu^2 = \frac{M_0M_2 - M_1^2}{M_1^2} \quad (38)$$

with M_j denoting the j-th moment of the near-bottom orbital velocity spectrum

$$M_j = \int \omega^j S_{u_b}(\omega)\,d\omega \quad (39)$$

Assuming each individual wave, defined by u_b and T, to be simple harmonic a friction factor f_w can be calculated for each wave from knowledge of the bottom roughness. Thus, the dissipation of energy associated with a single wave, D, may be written as the product of the average dissipation rate, cf Eq. (26), and the wave period T, i.e.,

$$D = \tfrac{1}{4}\rho f_w u_b^3 \cdot T = \left[\frac{u_{br}}{u_{bc}}\right]^3 \left[\frac{f_w}{f_{wc}}\left[\frac{u_b}{u_{br}}\right]^3 \frac{T}{T_1}\right] \tfrac{1}{4}\rho f_{wc} u_{bc}^3 T_1$$

$$= \left[\frac{u_{br}}{u_{bc}}\right]^2 \left[\frac{f_w}{f_{wc}} R^3 \tau\right] \tfrac{1}{4}\rho f_{wc} u_{bc}^3 T_1 \quad (40)$$

To obtain the average rate of energy dissipation we sum the contribution of N individual random waves and divide by the time necessary for N waves to pass. This time is NT_2, where

$$T_2 = 2\pi\left[\frac{M_0}{M_2}\right]^{1/2} \tag{41}$$

is the mean zero-crossing period of the near-bottom waves. Since the number of waves within an area around u_b (or R) and T (or τ) relative to the total number of waves is given by the joint probability density function, Eq. (36), we obtain the average rate of energy dissipation from

$$E_{d2} = \left[\left[\frac{u_{br}}{u_{bc}}\right]^3 \frac{T_1}{T_2} \int_0^\infty \int_0^\infty \frac{f_w}{f_{wc}} R^3 \tau p(R,\tau)\, dR d\tau\right] E_{dc} = C_2 E_{dc} \tag{42}$$

Again we take the JONSWAP spectrum with $\gamma = 1$, 3.3, and 7 for the surface amplitude spectrum, transform it into a near-bottom orbital velocity spectrum for different values of h/L_c, and evaluate the constant C_2 in Eq. (42) for a range of relative bottom roughness, $1 \lesssim A_{bc}/k_b \lesssim 1000$. Again, the value of C_2 was found insensitive to the value of relative bottom roughness while varying, as shown by the full lines in Figure 1, with relative water depth and spectral peakedness, in much the same manner as found for C_1. Over the important range of relative water depths where bottom frictional attenuation is expected to be most pronounced, $h/L_c < 0.25$, the excellent agreement between C_1 and C_2 supports the validity of the simple model for spectral wave dissipation derived here.

SUMMARY AND CONCLUSIONS

Based on a simple formulation of the turbulent flow in a bottom boundary layer a solution is obtained corresponding to a wave motion specified by its near-bottom orbital velocity spectrum. From the solution the spectral wave dissipation is obtained in the form

$$E_d(\omega,\theta) = \tfrac{1}{2}\rho f_{wr} u_{br} S_{u_b}(\omega,\theta)$$

in which

$$u_{br} = \sqrt{2\iint S_{u_b}(\omega,\theta)\, d\omega d\theta}$$

and f_{wr} may be obtained from any of the many available wave friction factor relationships from knowledge of the equivalent Nikuradse roughness of the bottom, k_b, for an equivalent periodic wave of near-bottom orbital velocity u_{br} and radian frequency, ω_r, defined by

$$A_{br} = \sqrt{2\iint \omega^{-2} S_{u_b}(\omega,\theta)\, d\omega d\theta} = u_{br}/\omega_r$$

In passing it is noted that the analysis of turbulent wave-current bottom boundary layers, e.g., Grant and Madsen (1979, 1986), may be extended to waves specified by their directional spectrum by an identical procedure to the one employed here for a pure wave bottom boundary layer. The result of such an analysis, Madsen (in prep.), shows that available theories for wave-current interaction, which assume a periodic wave motion, may be used also to approximate waves specified by their directional spectrum when the spectral wave is represented by the equivalent periodic wave, defined by u_{br} and A_{br}, propagating in the mean wave direction.

In the important range of intermediate to shallow water depths, the total average dissipation rate predicted by the spectral model is shown to agree with the prediction afforded by a model which considers the statistics of individual waves, defined by their near-bottom orbital velocity characteristics. Besides supporting the validity of the spectral dissipation model the formulae and the results presented in Figure 1 for the overall dissipation experienced by random waves may serve as a simple tool for the prediction of wave attenuation by bottom friction. This use of the results presented here is facilitated by the average dissipation rate experienced by random waves being expressed in terms of a characteristic equivalent periodic wave defined in terms of the surface amplitude spectrum. In this context it should be noted that the characteristic equivalent wave is defined with the root-mean-square wave height, not the significant wave height!

The practical limitation of the present results is that they require a priori knowledge of the bottom roughness, k_b. Experimental results directed towards overcoming this limitation are presented in a companion paper by Madsen and Rosengaus (1988).

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