

## CHAPTER 44

### TRANSFORMATION OF SHALLOW WATER WAVE SPECTRA

Genowefa BENDYKOWSKA<sup>1</sup>. Gosta WERNER<sup>2</sup>.

#### ABSTRACT

Investigations are presented on some effects of non-linearity in the motion of shallow water wave spectra. The waves were generated mechanically in a laboratory wave flume with fixed bottom. Essential differences with the linear dispersion relation are found, showing vanishing dispersivity of higher frequency spectral components in strongly nonlinear spectra. The mean frequency increases with decreasing water depth. The relation of the peak frequency to the mean frequency varied in the experiments from 0.9 to 0.5, for deep to shallow water wave spectra respectively.

#### 1. INTRODUCTION

Two following spectral characteristics are analysed

- the dispersion relation of spectral components in shallow water conditions
- changes of the relation between the mean and peak frequencies of wave spectra, propagating over sloping bottom.

The experiments were performed with mechanically generated irregular wave trains, propagating in one direction, over a fixed bed. In these conditions the waves were not affected by the wind and wind generated currents / as it is in laboratory when waves are generated by wind / and did not depend on the angular energy spread nor changes of bottom topography / that always influences the wave motion in field conditions /.

The relationship between the wave number  $k$  and the frequency  $\omega$ , known as the dispersion relation:

$$\omega^2 = gk \tanh kh$$

is widely used and proves a powerful tool in analysing regular wave. However several laboratory/Romamonijariosa 1976, Mitsuyasu et al. 1979 and field Massel, 1980 / measurements on irregular wave trains, show a deviation

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1. Ph.D., Institute of Hydroengineering Polish Academy of Sciences, Kościarska 7 80-328 Gdańsk, Poland
  2. Eng. Royal Institute of Technology, S-100 44 Stockholm Sweden

of the dispersion relation from the theoretical one, especially for higher frequencies. Experimental results on wave pressure attenuation Naguszewski et al. 1985 show the same qualitative differences with the commonly used theoretical description, which includes the dispersion relation, given above.

Several theoretical studies have been published, which take into account the influence on the dispersion relation of various aspects of the nonlinearity of a random wave field /Crawford et al. 1981, Huang and Tung 1977, Massel and Chybicki 1983, Masuda et al. 1979 /. All these studies concerned deep water wave conditions.

The results of this study prove an essential departure from the linear dispersion relation of low and high frequency spectral components in a broad range of tested shallow water spectra.

It is often assumed, that for a given wave train, propagating over shallowing water, the peak frequency  $\omega_p$  and the mean frequency  $\bar{\omega}$  do not change with depth and the relation  $\omega_p/\bar{\omega} = 0.7 - 0.8$  remains constant through the surf zone / Massel 1980 /. Goda 1987, reports results of a broad statistical analysis of deep water wave trains, simulated according to JONSWAP and WALLOPS spectra, where a strong dependence of  $\omega_p/\bar{\omega}$  from the spectral width is proved. Different values of the enhancement factor  $\gamma = 1 - 20$  in JONSWAP formula followed changes of  $\omega_p/\bar{\omega} = 0.74 - 0.91$ ; different values of  $m = 3 - 20$  in WALLOPS formula gave  $\omega_p/\bar{\omega} = 0.58 - 0.95$ .

In the following a strong dependence is found of the mean frequency from the relative water depth. These changes follow the changes of the shape (width) of the spectrum and the appearance of secondary peaks at smaller depths.

## 2. EXPERIMENTS. INPUT SPECTRUM

- Two sets of experiments were performed -
- one - concerned with the measurements of the dispersion relation of spectral components in shallow water wave spectra
  - the second - concerned with measurements of wave spectra over sloping bottom, to study the transformation of spectral parameters on shallowing water depths

The wave flume of dimensions 15 m x 0.305 m x 0.62 m was build of glass. A programmable wave maker of flap type was monitored by a punched tape, read at discrete intervals of 0.02 s. The water surface elevations were measured by means of resistance gauges. Analog records were taken and stored on a tape recorder. The tape recordings were converted into digital data with sampling interval 0.05 - 0.1 s and transmitted to a computer.

In the measurements of the dispersion relation four gauges, spaced equally at 0.1 m, were mounted over the horizontal bottom of the flume. The beach at the end of

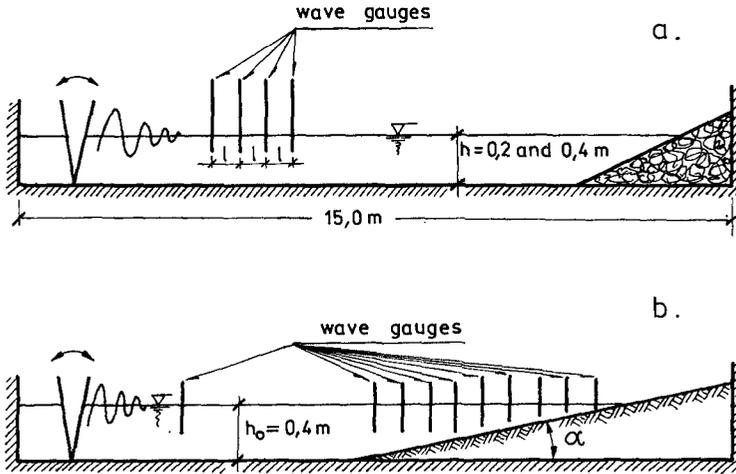


Fig. 1 Experimental scheme

the flume was a porous structure of slope 1 : 10. In the second part of the investigations an array of eleven gauges was used: one - over the deep water section and ten at sections of decreasing depth / fig. 1 /.

JONSWAP energy spectrum was generated / of peak enhancement factor  $\gamma = 1$  and 6 / consisting of 100 spectral components. The frequency of these components were distributed within cut-off frequencies

$$\omega_{\min} = 0.5 \omega_p \quad , \quad \omega_{\max} = 2.5 \omega_p$$

The frequencies of the spectral components were random values in intervals  $\Delta \omega_i = \omega_i - \omega_{i-1}$  determined by the Goda's formula

$$\omega_i = \omega_{i-1} \cdot d \quad , \quad d = \left( \frac{\omega_{\max}}{\omega_1} \right)^{\frac{1}{98}}$$

Phase angles of the components were random within  $(0, 2\pi)$

### 3. DISPERSION RELATION OF THE SPECTRAL COMPONENTS IN SHALLOW WATER WAVE SPECTRA.

#### 3.1 Range of the study.

Usual technique of cross spectral analysis was applied to two simultaneous records, taken at known distances. Wave numbers  $k(\omega_i)$  and celerities  $C(\omega_i)$  of the spectral components were determined. Only cases of coherence coefficient  $\geq 0.8$  were analysed. To describe the degree of nonlinearity of the wave spectrum a modified Stokes number was used, as a function of the parameters of the peak spectral component:

$$S = \left( \frac{2\pi}{k_p} \right)^2 \frac{H_s}{h^3} \quad , \quad H_s = 4\sqrt{m_0}$$

The effects of nonlinearity were examined by varying the peak frequency of 7 tested wave spectra in the range:  $2.45 \leq \omega_p \leq 7.98$  [rad/s] at two water depths  $h = 0.2$  m and  $0.4$  m. The Stokes' numbers varied between  $0.75$  and  $45.2$

### 3.2 Results

Examples of measured results are given in fig. 2 - 5. On fig. 2 three spectra are shown, measured at water depth  $0.2$  m

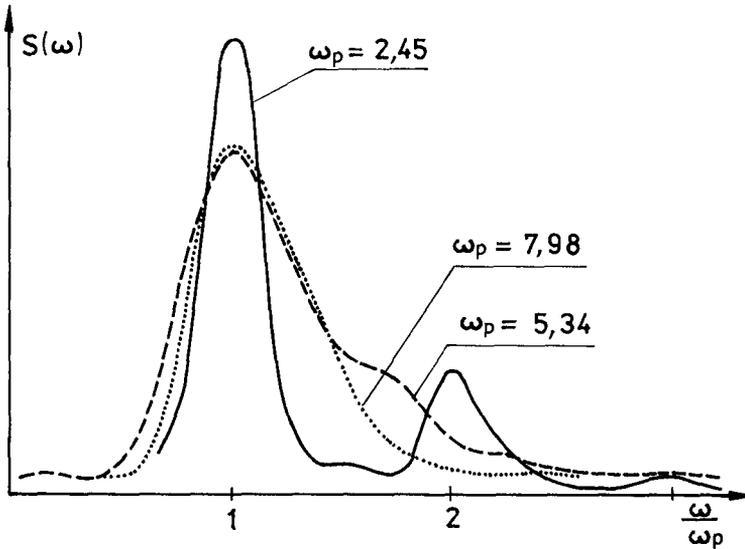


Fig. 2 Energy spectra measured.  $h = 0.2$  m

The spectrum of peak frequency  $\omega_p = 7.98$  rad/s has a shape very close to that of the input spectrum. The spectrum of the lowest peak frequency  $\omega_p = 2.45$  rad/s proves, that the deep water input spectrum transformed in this shallow water conditions into a two-peaks spectrum. The second peak, appearing at  $2\omega_p$ , is certainly due to bounded waves, that of second order Stokes type.

In fig. 3 some measured results of the wave frequency  $\omega$  against the wave number  $k$  in comparison with the theoretical curve  $\omega^2 = gk \tanh kh$  are plotted. All measured results indicate, that the prediction of the linear theory are true only in the vicinity of the peak frequency. The measured curves level out in the bands of  $2\omega_p$  and  $3\omega_p$ . As the tested wave systems were not linear, both free and bounded harmonics existed in the motion. The dominating components are those of the peak frequency, thus also their harmonics travel faster, than free waves of the same frequency. If so, the decrease of the wave numbers of the components  $2\omega_p$  and  $3\omega_p$  would be explained.

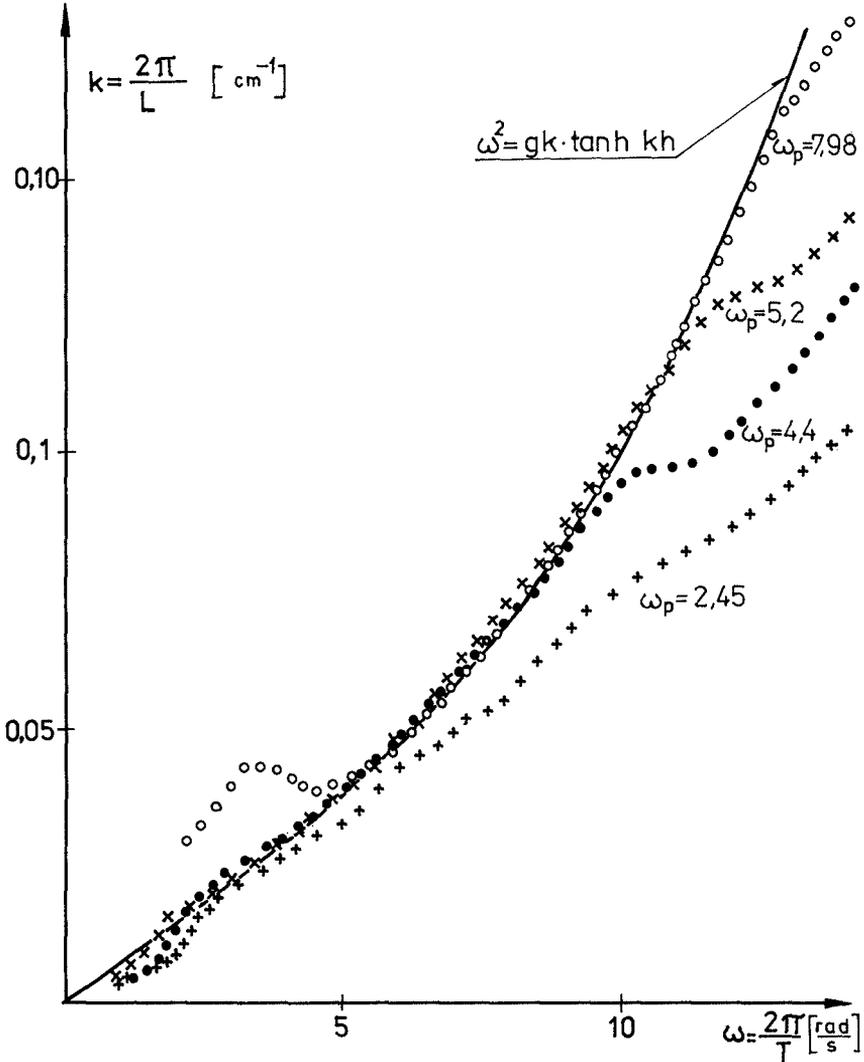


Fig. 3 Comparison of theoretical and measured dispersion relation.  $h = 0.2$  m.

In fig. 4 the line  $C(\omega)/C_0(\omega) = 1$  is the theoretical relation. All spectral components of  $\omega > 1.5 \omega_p$  in all tested cases travel at higher velocities, than the theoretical. The presence of bounded waves is one explanation. Another one is, that in the range  $\omega > 2.5 \omega_p$  no free wave components are generated, so only bounded and interaction waves of that frequencies exist in the wave train, both propagating faster than free waves. In the range  $\omega < \omega_p$

the measured components phase velocity is either higher / high Stokes' number / or lower / low Stokes' number / than predicts the theory. A similar result was found by Crawford et al, 1981, in their numerical study. The relationship in fig. 5 shows most evidently the influence of the nonlinearity of the motion on the dispersive behaviour of the spectral components. For cases of higher Stokes' numbers the dispersivity of components of  $\omega > \omega_p$  vanishes and all these components travel at the velocity of the peak frequency component.

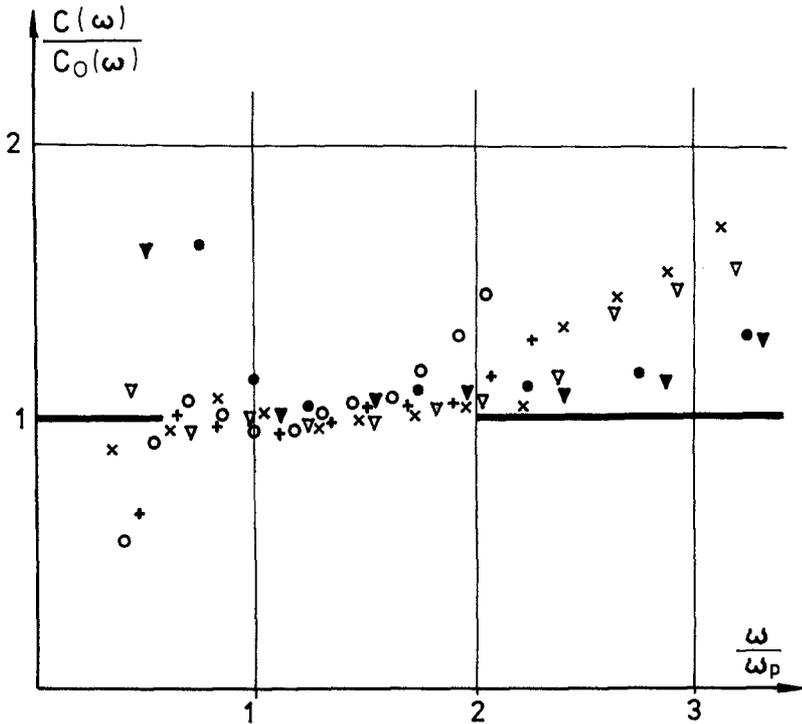


Fig. 4 Measured component phase velocity normalised by its theoretical value.  $h = 0.2$  m.

- Legend:
- $\omega_p = 2.45$  rad/s,  $S = 45$
  - ▼  $\omega_p = 2.76$  rad/s,  $S = 42$
  - ▽  $\omega_p = 4.40$  rad/s,  $S = 25$
  - x  $\omega_p = 5.34$  rad/s,  $S = 9.8$
  - +  $\omega_p = 6.74$  rad/s,  $S = 6.4$
  - $\omega_p = 7.98$  rad/s,  $S = 3.73$

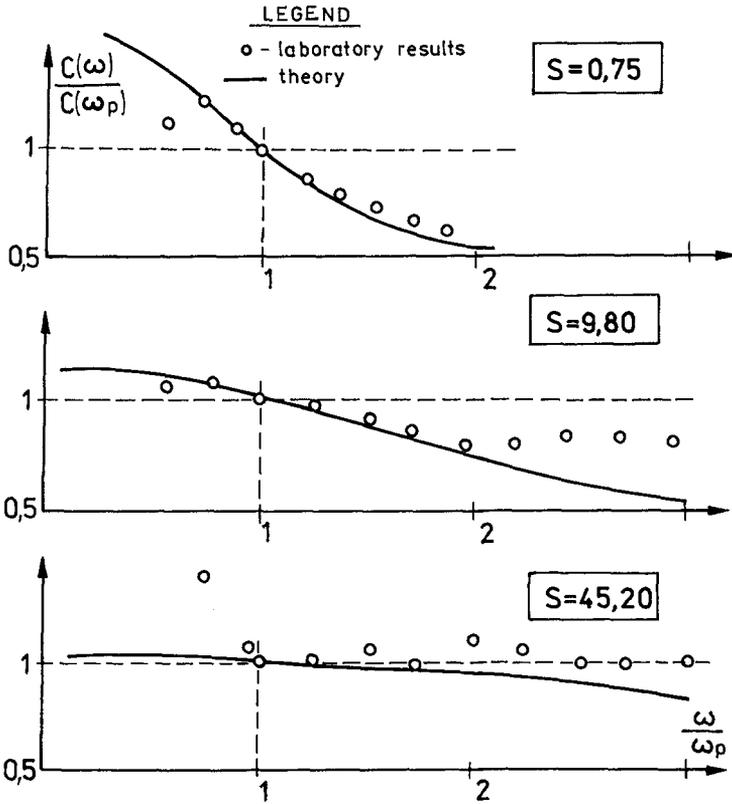


Fig. 5 Measured component phase velocity normalised by the peak component phase velocity.

4. SPECTRAL CHARACTERISTICS OF WAVE SPECTRA OVER SLOPING BOTTOM.

4.1. Range of the study.

The experiments were performed for four different wave spectra of peak frequencies  $\omega_p = 6,25, 7,87$  and  $12,56$  while two wave trains of different energies were tested for  $\omega_p = 7,87$  rad/s. Wave spectra were measured over two slopes  $\gamma = 0,04$  and  $0,06$  with  $h = 0,4$  m in the deep water section of the flume. The mean frequencies and the width of the spectra were determined from spectral analysis:

$$\bar{\omega}^2 = \frac{m_2}{m_0} , \quad \varepsilon^2 = 1 - \frac{m_2}{m_0 \cdot m_4} , \quad m_n = \int_0^\infty \omega^n S(\omega) d\omega$$

4.2 Results.

Results, concerning the changes of the mean frequency of the spectrum, when propagating from deep water  $\bar{\omega}_0$  / over decreasing depth  $\bar{\omega}$  / are shown in fig 6. Results, obtained for both tested slopes are plotted together. They prove an increase of the mean frequency when the water depth decreases.

Changes of the relation between the peak  $\omega_p$  and the mean frequency  $\bar{\omega}$  of the spectrum as a function of the relative water depth  $k_{p_0} \cdot h$  ( $k_{p_0}$  - wave number of the deep water peak frequency component  $k_{p_0} = \omega_p^2/g$ ) are given in fig. 7. The peak frequency didn't change from deep water  $/h = 0.4m/$  up to the shallowest water depth  $/h = 0.05 m/$ , except the case of highest peak frequency tested  $/\omega_p = 12.56 \text{ rad/s} /$ , when a shift of about 10 % toward the lower frequency band occurred for both tested slopes. The relation  $\omega_p/\bar{\omega}$  varies from 0.9 on deep water, to 0.5 for very small relative depth. Those changes are due to the changes of the shape of the spectrum. The values of the spectral width given in table 1 show a consequent increase of  $\varepsilon^2$  with decreasing water depth. An example of energy spectra measured at different water depth presented on fig. 8, explains, that the

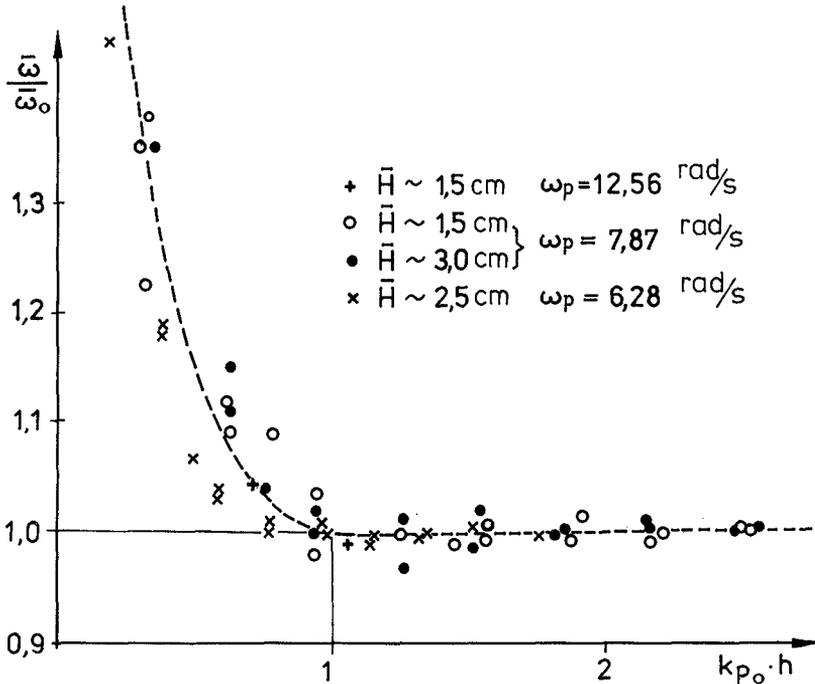


Fig. 6 Changes of mean frequency. Experimental results.  
 $\gamma = 0.04$  and  $0.06$

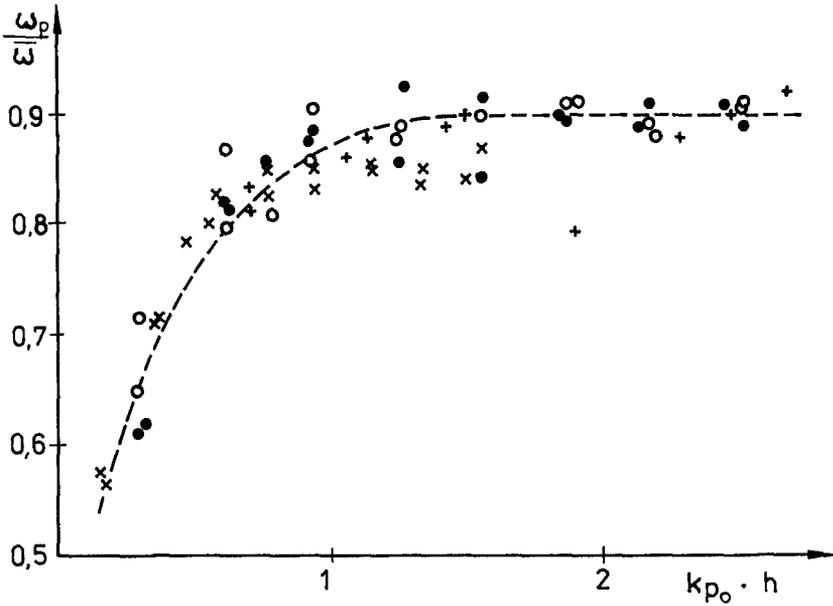


Fig. 7 Relation of peak the frequency to the mean frequency. Experimental results.  $\gamma = 0.04$  and  $0.06$ . Legend as on fig. 6

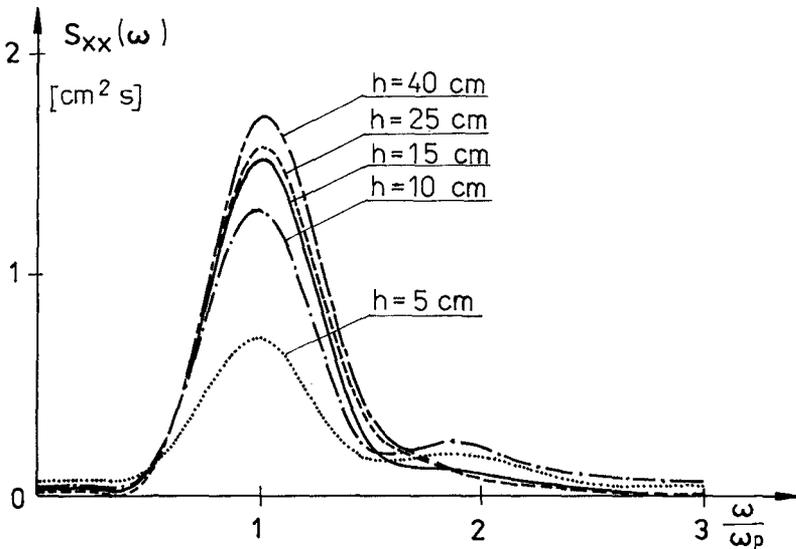


Fig. 8 Changes of energy spectra over sloping bottom  $\gamma = 0.06$ ,  $\omega_p = 6.28$  rad/s.

increase of the spectral width is due to the increasing nonlinearity of the wave motion on smaller depth / see fig. 2 for comparison /.

Fig. 9 shows a comparison of the experimental curve with field data measured in the coastal zone of the Black Sea. The measurements were done at six wave conditions, of peak frequencies varying between 0.72 to 1.59 rad/s, at depths 1.5 - 3 m. It should be mentioned, that for the field data the mean frequency was determined as  $\bar{\omega} = \frac{2k_p}{T_z}$ , where  $T_z$  is the average zero-up crossing period.

Table 1. Spectrum width parameter measured in the experiments

slope	$\omega_{p0}$ rad/s	$\xi^2$	
		h = 40 cm	h = 5 cm
0.06	12.56	0.210	0.379
	7.87	0.329	0.403
	7.87	0.393	0.631
	6.28	0.338	0.628
0.04	12.56	0.228	0.403
	7.87	0.382	0.560
	7.87	0.417	0.573
	6.28	0.437	0.639

## 5. CONCLUDING REMARKS.

- The relative water depth of the peak frequency component  $k_p \cdot h$  proves to be good measure of the nonlinearity of a shallow water wave spectrum. Presence of bounded harmonics and waves of interaction between free spectral components are supposed to account for all shallow water effects observed in the experiments.
- An essential departure from the theoretical dispersion relation is proved in the experiments. The spectral components of the peak frequency band follow the relation in all tested cases. Components of frequency  $\omega > \omega_p$  propagate faster, than predicted theoretically and for strongly nonlinear wave trains the dispersivity of those components almost vanishes. Components of frequency  $\omega < \omega_p$  propagate either slower / moderate nonlinearity / or faster / strong nonlinearity /, than predicts the theory.
- The mean frequency  $\bar{\omega}$  increases with decreasing water depth /  $k_p \cdot h < 1$  /. The relation between the peak and the mean frequencies decreases with decreasing relative water depth in a range  $0.5 < \omega_p / \bar{\omega} < 0.9$ .
- The spectral characteristics of the peak frequency components are the most reliable parameters in a simple estimation of a spectrum. The parameter of the mean frequency / zero-crossing period / has a poor physical meaning in shallow water wave spectra.

- The very simple laboratory model, applied in this experiment, proves a good tool in studying general features of shallow water wave spectra. The comparison with other field data and results of different laboratory technics and numerical simulations / Crawford et al. 1981, Goda 1987, Mitsuyasu et al. 1979, Ramamonijarisoa 1976 / gives a satisfactory qualitative agreement.

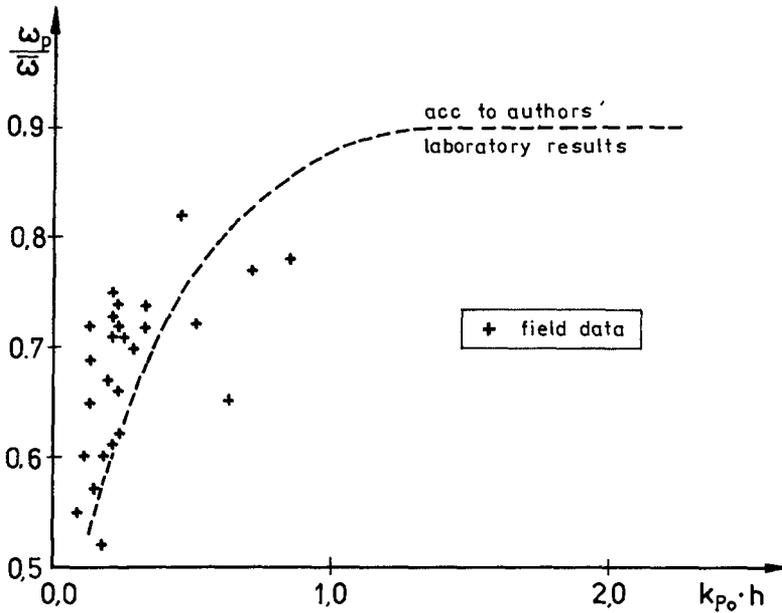


Fig. 9 Field data compared with experimental curve.

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