CHAPTER 38

An Experimental Study of Waves on a Strongly Sheared Current Profile.

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Abstract.

This paper describes a series of observations within a combined wave-current flume. A two dimensional progressive wave train was superimposed upon a co-flowing current profile. The direction of this current was reversible, thereby allowing the formation of both a "favourable" current velocity (one in which the current is in the same direction as the wave celerity), and an "adverse" current velocity. The combined flow field was measured using laser Doppler anemometry. The nature of the current profile was modified so as to allow an investigation of the interaction resulting from both a uniform current and a sheared current profile.

In the case of a uniform current, there was no observable phase change between the surface elevation and the velocity profile. In this respect the present measurements are very different from the observations presented by Brevik (1980a). Indeed, they confirm that a description of the oscillatory motion merely requires the introduction of a Doppler shift as was suggested by Fenton (1985). In the case of a sheared current profile the oscillatory component of the wave motion is found to be strongly dependant upon the vorticity within the current profile. The analytical solution proposed by Kishida and Sobey (1988) appears to underestimate the extent of the interaction, while the numerical solution proposed by Chaplin (1990) provides a better description of the combined flow field. The departure from irrotational theory is significant. In the case of waves superimposed on a strongly sheared "adverse" current the horizontal velocity component may be as much as 80% larger than the predicted irrotational motion.

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1.0 Introduction.

The interactions resulting from the combination of waves and currents has been the subject of numerous publications. Thompson (1949) presented the first solution for waves on a linear shear current (ie. the current varies linearly in the vertical direction, but does not vary in the horizontal direction). Tsao (1959) also considered this problem, and established that the wave motion will remain irrotational provided the vorticity profile is uniform with depth. He conducted a classical Stokes' expansion, and obtained the third order expressions for the surface displacement and the velocity field in conditions of arbitrary depth. Dalrymple (1973) obtained the dispersion equation for finite amplitude waves on a bi-linear current profile. Using a stream function expansion he obtained a higher order numerical solution for waves on a linear, bi-linear, and arbitrary current profile. Further investigations of finite amplitude waves on a sheared current profile are given in Brink-Kjær and Jonsson (1975), Brink-Kjær (1976), Dalrymple (1977), and Kishida and Sobey (1988). These articles, together with the many other aspects of wave-current interaction, have been discussed in a number of review articles, of which Peregrine (1976) and Jonsson (1990) are two very good examples.

Despite the large number of theoretical solutions, the quantity of experimental data is extremely limited, and that which is available has tended to concentrate on the flow features within the near bed region (eg. Van Doorn, 1981). Brevik (1980a) investigated a number of different wave forms on an essentially uniform current profile (some shear did exist within the near bed region). Much of this work was directed towards the determination of the combined wave-current friction factor, and the reduction in wave height along the length of the experimental flume. However, he did take some measurements of the horizontal velocity component using a MIC-PAC micro-propeller. Unfortunately, the velocity measurements are limited to the case of a co-flowing "favourable" current (in the same direction as the wave motion), and because of the measuring system the velocity could only be determined within the lower half of the flow field. Nevertheless, the results are important in that there appeared to be a phase change between the velocity field and the surface profile. In two separate cases the horizontal water particle velocity was observed to lag behind the surface elevation by approximately 30^0. The magnitude of this phase change was found to be uniform with depth. In a second series of observations Brevik and Aas (1980b) considered the interaction of waves and currents over a rough bed.

Kemp and Simons (1982 & 1983) provide the only other measurements of wave-current interactions. In the first paper they considered the case of a "favourable" co-flowing current, and in the second an "adverse" co-flowing current. In both cases the experimental measurements are concentrated within the near bed region, and have been used to determine the variation in the turbulence intensity, the bed stress, and the nature of the bottom boundary layer. A limited number of measurements were taken at greater heights above the bed. However, because the current was introduced through the bed of the wave flume (similar to Brevik, 1980a) the current profile at these positions is essentially uniform with depth.

In many practical cases it may, of course, be assumed that the current velocity is approximately uniform with depth. Examples of this type of behaviour would include the large scale ocean currents, and the majority of tidal flows. However, in many other instances the current velocity will vary significantly with depth leading to the creation of a vorticity profile. Perhaps the most important example of this type of behaviour is the wind driven current where the magnitude of the current velocity varies exponentially with depth, leading to a very strong shearing action within the upper layers of the flow field.

The purpose of the present investigation was to obtain experimental data describing the interaction between waves and currents throughout the depth of the flow field for a number of different current profiles. In particular, to observe the modification of the oscillatory motion in the presence of a strongly sheared current velocity, and to compare this data with the existing theoretical solutions.

2.0 Apparatus.

The experimental observations were made in the Cambridge University Engineering Department's wave flume. This facility is 0.6m wide, 18m long, and allows a maximum water depth of 0.45m. The waves were generated via a hinged paddle, located within a deepened section at one end of the wave flume. The bed conditions were smooth (covered with plate glass), and the beach slope was maintained at 1:20. At this angle the effects of beach reflection were eliminated in all but the very longest waves considered. In these cases, an additional wave absorbed was placed approximately half way up the beach slope. With this in place the reflection coefficient was never larger than 2.7%.

The re-circulating current was introduced via two loops of 50mm diameter pipework each connected to a self priming centrifugal pump. The total volume flow could be adjusted up to a maximum of $0.05m^3/s$. In the first half of the experiment the current was in the same direction as the wave celerity, and therefore constituted a co-flowing "favourable" current. In the second half, the pipework was reversed, and the case of waves on a co-flowing "adverse" current was considered. In each of these cases the nature of the current profile, U(z), was found to be dependant upon the height of the inlet and outlet pipes above the bed. To reinforce this pattern the outflow pipes, (the high pressure ends), were fitted with an appropriately shaped diffuser; while the inflow pipes (the low pressure ends), were located beneath a horizontal plate to prevent the formation of a vertical vortex and the resulting air entrainment. To further stabilise the flow field a small thickness of honeycomb, or flow straightener, was placed directly in front of both the in-flow and out-flow pipes. Figure 1 indicates the general arrangement of the experimental apparatus. This corresponds to the first case of a co-flowing current.

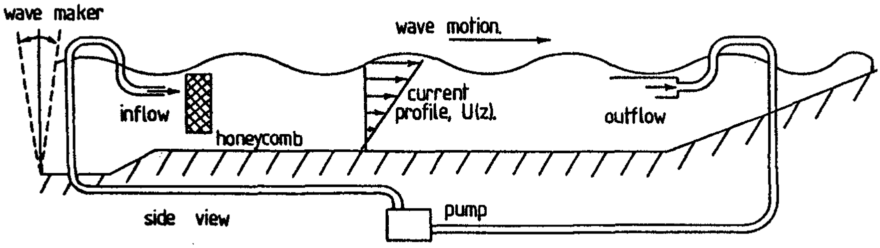


Figure 1. Experimental apparatus.

The velocity field was measured using laser Doppler anemometry. A 15mw. Helium-Neon laser was mounted above the wave flume. This was used to create a two beam system from which the horizontal component of the velocity field could be determined within a measuring volume which was estimated to be approximately $1.0mm^3$. Using this system the velocity could be determined to ±3%.

3.0 **Measuring technique.**

Unfortunately, the introduction of a current within a wave flume is not as straight forward as indicated on figure 1. There are a number of practical problems which limit the superposition of a current profile and a progressive wave train. When the pumps were initially switched on, a long wave was generated leading to a seiching motion within the wave flume. Although the wave absorber reduced the effective life span of this motion, it was still found to take approximately 45 minutes until the variation in the surface elevation could be considered negligible (< 1mm). Furthermore, the action of the current itself leads to the formation of surface ripples. If these features are allowed to develop along the length of the flume, they can reach an appreciable height, often having an amplitude in excess of 12mm. However, the addition of teepol in approximately 15 ppm. reduces the surface tension and thereby limits the formation of these features. This disturbance was never larger than 1.5mm in amplitude, and was for the most part considerably less.

The generation of a two dimensional current was rather more difficult to achieve. After a large number of experimental tests it was concluded that the honeycomb placed directly in front of the inflow and outflow pipes was critical in the development of an appropriate current profile. If the flow straightener was removed, or was of insufficient thickness, then the flow field would develop a three dimensional component, and in many cases become unstable within the central portion of the wave flume. Figure 2 shows the variation in the current profile across the tank width with the honeycomb in place. Although the magnitude of the current velocity does appear to be slightly larger along the centre line, the general form of the current profile is surprisingly consistent. In figure 3 the time variation in the current profile is shown throughout the duration of a test run. The current profile appears to be relatively unchanged even after a period of 8 hours. These measurements were obtained with a 75mm thickness of honeycomb as indicated on figure 1.

While the addition of honeycomb allows the development of a suitable current profile, its presence within the wave flume produces an important source of wave reflections. A problem which is further complicated by the horizontal plate located above the inflow pipes. As a result of these difficulties it became apparent that the creation of a reasonable current profile, and a continuous progressive wave train was not possible using the present arrangement. However, it was possible to achieve the required current profile and superimpose the effects of 5 or 6 gravity waves before the flow field is disrupted by the presence of the reflected waves. The measuring technique thus developed along the following lines. The required current profile was established within the wave flume. The long wave seiching was allowed to dissipate before the commencement of the experimental run. The wave maker was switched on, but the sampling procedure was delayed until the first two or three waves had passed the measuring section. At this point the wave profile will have achieved a regular form. The combined velocity field is then sampled for the duration of three or four wave cycles, but ceases before the reflected components start to disrupt the established order. The resulting pattern is shown on figure 4.

For this technique to provide information about the vertical variation in the horizontal velocity, it must be repeatedly applied at a number of different locations in the same vertical section. If the resulting data is to be

Figure 2. The variation in the current profile across the width (W) of the wave flume.

▲—▲—▲ y=0.25W.
■—■—■ y=0.50W.
●—●—● y=0.75W.

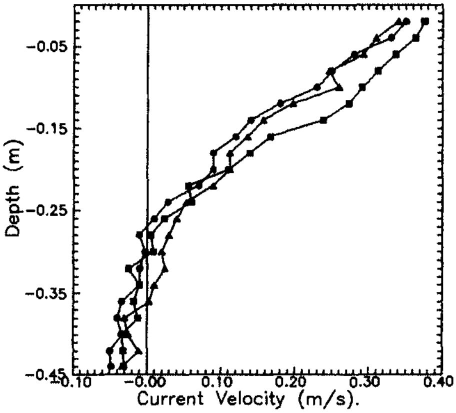
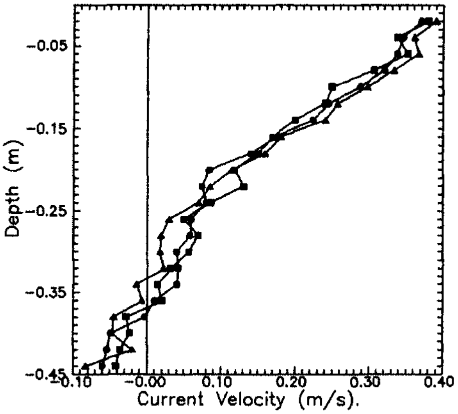


Figure 3. The variation in the current profile with time t.

▲—▲—▲ t=1.0hr.
■—■—■ t=4.0hr.
●—●—● t=8.0hr.



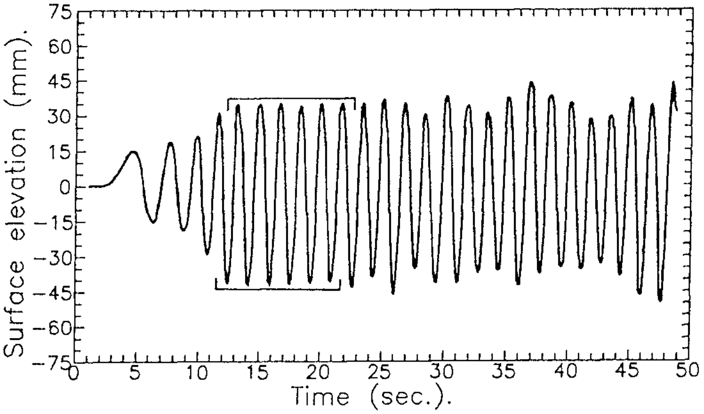


Figure 4. The development of the surface profile.

representative of the combined wave-current motion, then it is essential that the wave maker is capable of generating the same wave form throughout the duration of a test run (approximately 8 hours). To ensure that this requirement did not cause any difficulties, the wave amplitude was continuously monitored. The variation was generally found to be small (<3%). However, this variation did become more significant as the wave length was reduced. In an attempt to average out these fluctuations extreme waves, or those waves in which the surface amplitude differed from the mean value by more than ±5%, were neglected. Furthermore, the measured kinematics at any one depth were based on an average of five bursts, each containing three complete wave lengths.

In addition to the repeatability requirement, the wave motion should reduce to the irrotational solution proposed by Stokes (1880) in the absence of a current. Figure 5 shows the variation in the horizontal velocity at a number of different depths for the case U(z)=0. Although the current velocity was zero, the experimental arrangement is as indicated on figure 1. Both the flow straightener and the pipework were still located within the wave flume. The agreement with the third order irrotational solution is remarkably good.

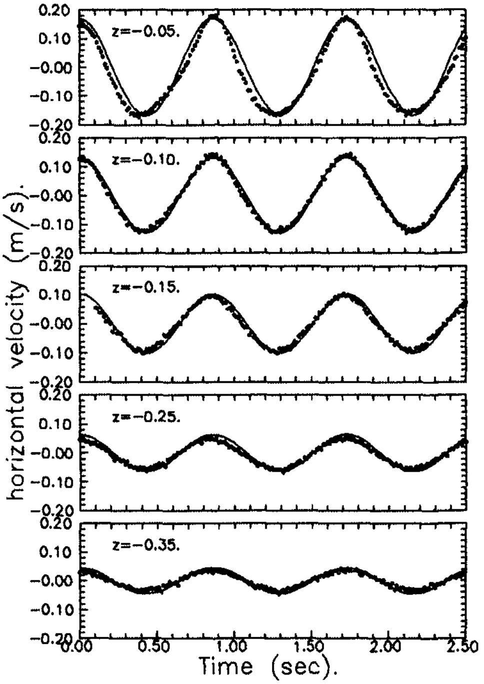


Figure 5. Horizontal velocity component (Waves only).

4.0 Experimental results.

A total of six different types of interaction were considered, the experimental details of which are given in table 1.

Case	Wave form.	Current profile.
(1)	a=35.1mm, h=0.35m, T=1.412s.	Uniform, favourable.
(2)	a=35.7mm, h=0.45m, T=0.877s.	Uniform, adverse.
(3)	a=22.5mm, h=0.45m, T=0.869s.	Linear, favourable.
(4)	a=31.5mm, h=0.35m, T=1.418s.	Linear, favourable.
(5)	a=45.5mm, h=0.45m, T=0.998s.	Linear, adverse.
(6)	a=61.5mm, h=0.35m, T=1.420s.	Linear, adverse.

Table 1. Experimental parameters.

The first two cases correspond to a preliminary set of measurements in which the current profile was essentially uniform with depth. Case (1) considers the effects of a "favourable" current velocity (U_{mean}=0.108m/s),

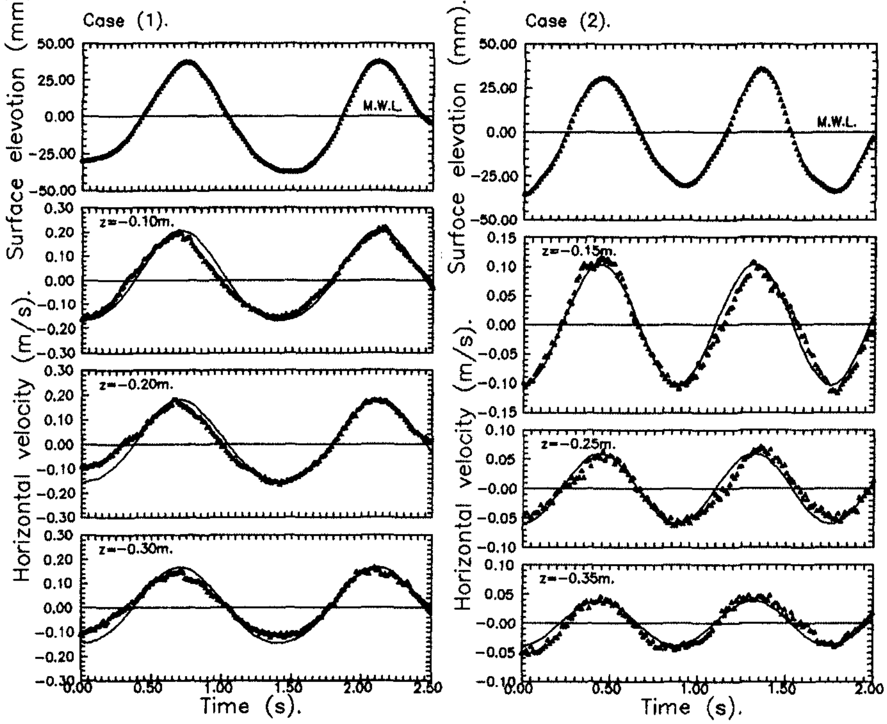


Figure 6. Waves on a uniform current.
——————— Fenton (1985).

and case (2) the effects of an "adverse" current velocity (U_{mean}=-0.120m/s). In each case the only significant shearing motion occurred within a relatively thin layer directly above the bed. The oscillatory motion resulting from this type of wave-current interaction is shown on figures 6a and 6b respectfully. In both cases, the observed kinematics are shown to be in phase with the surface elevation. Furthermore, the magnitude of the oscillation appears to be in agreement with Fenton's (1985) Doppler shifted solution.

The remaining experimental results all concern the interaction with a sheared current profile. In cases (3) and (4) a "favourable" current profile with positive shear is considered (ie the velocity increases with height above the bed), while in cases (5) and (6) an "adverse" current with negative shear is considered. In each case the variation in the current velocity with depth is approximately linear. The importance of this assumption will be considered in section 5. One interesting feature of each current profile is the reversal in the current

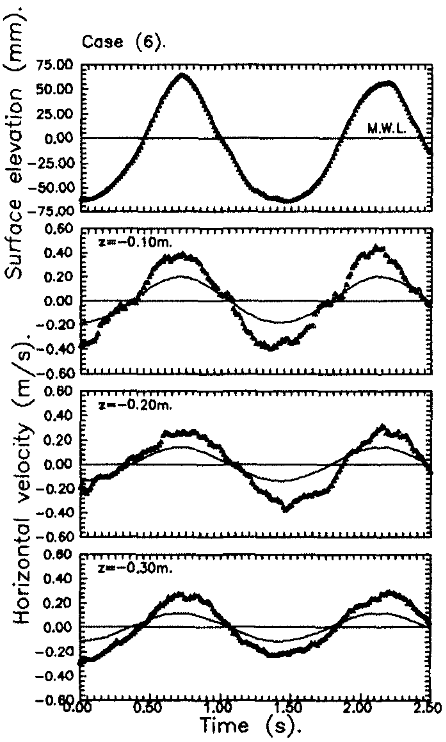


Figure 7. Waves on a linear shear current.
—————— Fenton (1985).

velocity within the near bed region. This would appear to correspond to a large scale longitudinal circulation along the length of the wave flume. It was consistently observed throughout the working section (figure 2.), and was small in comparison with the magnitude of the surface current ($U_{z=0}$).

In each of the above cases the oscillatory motions were again found to be in phase with the surface elevation. Figure 7 concerns the flow conditions in case (6) and shows the characteristics of the surface elevation together with the time variation in the horizontal velocity at a number of different depths (z=-0.1m, -0.2m, and -0.3m). The amplitude of the oscillatory motion for cases (3)-(6) are shown on figures 8a-8d.

5.0 Comparisons with other work.

The experimental data appertaining to waves on a uniform current (Cases 1 and 2) differs from previous measurements (Brevik, 1980a) in that there is no observable phase change between the velocity profile and the surface elevation. In this respect, the present measurements appear to be in agreement with the existing analytical solutions (Fenton, 1985) and the recent numerical simulations (Chaplin, 1990). The accumulated evidence therefore suggests that the phase change observed by Brevik (1980a) does not represent a true wave-current interaction. A possible explanation of these results may well be found in the nature of the velocity measurements. Indeed, Brevik comments on the suitability of a horizontal micro propeller in the presence of a substantial vertical velocity component, ie. at significant heights above the bed. Wave reflection may also have contributed to the creation of the observed phase change. This latter point is at least partially substantiated by the asymmetrical nature of the measured surface profile.

The amplitude of the oscillatory motion produced by waves on a uniform current (figures 6a and 6b) shows good agreement with the Doppler shifted solution proposed by Fenton (1985). This clearly indicates that provided the magnitude of the surface drift velocity is taken into account within a modified dispersion equation, the measured characteristics of the velocity field for waves on a uniform current may be described on the basis of a modified Stokes' (1880) expansion.

In contrast, the measurements shown in figures 7 and 8 cannot be explained by the introduction of a simple Doppler shift. The shearing motion within the current profile produces an associated vorticity profile which must be taken into account when describing the combined flow field. However, in the case of a linear shear current the vorticity profile is constant with depth.

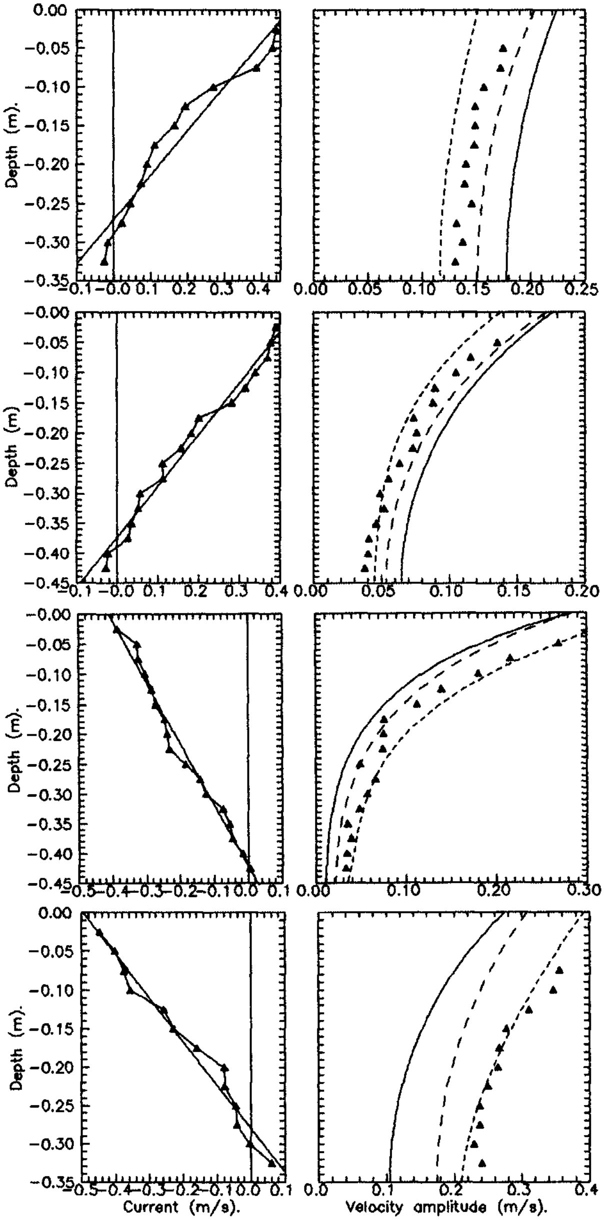


Figure 8a–8d. Waves on a linear shear current.

———— Doppler shifted solution (Fenton, 1985).

— — — — Linear shear solution (Kishida et al,1988).

-------- Numerical solution (Chaplin, 1990).

Tsao (1959) considered this situation and concluded that the waves would remain irrotational, though different from Stokes' (1880) classical expansion. Kishida and Sobey (1988) extended this argument and obtained a 3rd. order solution for waves on a linear shear current. The results of this theory are shown on figures 8a-8d. The numerical solution proposed by Chaplin (1990) is also shown on figures 8a-8d. This solution allows a "best fit" cubic approximation of the current profile to interact with the required wave motion. There are no prior assumptions regarding the nature of the vorticity profile, and convergence is achieved after the summation of a large number of terms within a stream function expansion.

It is clear from figure 8 that the solutions proposed by Kishida and Sobey (1988) and Chaplin (1990) both predict the nature of the observed departure from the Doppler shifted solution (Fenton, 1985). The interaction with a co-flowing "favourable" current having positive shear is shown to produce a reduction in the amplitude of the horizontal velocity component; while the interaction with a co-flowing "adverse" current having negative shear is shown to increase the amplitude of the horizontal velocity component.

The analytical solution proposed by Kishida and Sobey (1988) appears to underestimate the extent of these interactions. This is particularly noticeable in the case of an "adverse" current having negative shear. In case (6) the observed velocity amplitude was as much as 81% larger than the Doppler shifted solution (Fenton, 1985), and 42% larger than the 3rd. order solution for waves on a linear shear current (Kishida and Sobey, 1988). Although this case corresponds to the steepest wave form investigated, it is, according to the classification code proposed by Dean (1970), a third order wave. As a result, the apparent discrepancies between the solution proposed by Kishida and Sobey (1988) and the experimental observations is rather surprising. One possible explanation for this observation is that the wave-current interaction increases the non-linearity of the wave form by enhancing the relative contribution of the higher harmonics. As a result, the wave energy is transferred to the higher harmonics, and consequently the traditional classification codes used for progressive gravity waves may be less appropriate in the presence of a significant wave-current interaction. The numerical solution shown on Figures 8a-8d (Chaplin, 1990) appears to support this view. By incorporating the full effects of the higher harmonics within a stream function expansion it provides a very good description of the experimental data in all cases.

6.0 Concluding remarks.

The present observations have shown the importance of the interaction resulting from a combination of waves and currents. In many practical cases it may, of course, be assumed that the current velocity is approximately uniform with depth. Under these conditions the horizontal velocity component remains in phase with the surface elevation, and the Doppler shifted solution proposed by Fenton (1985) is shown to provide a good description of the velocity field.

In many other instances the current velocity will vary significantly with depth. This leads to the creation of a vorticity distribution which must be taken into account when defining the characteristics of the oscillatory motion. Under these conditions the introduction of a simple Doppler shift within the dispersion equation is no longer sufficient to define the flow field. The third order analytical solution proposed by Kishida and Sobey (1988) is shown to predict the general effects of the vorticity profile. However, in the case of a strongly sheared "adverse" current profile it appears to underestimate the extent of the resulting interaction, thereby hinting at the increasing importance of the higher harmonics. In contrast, the numerical solution proposed by Chaplin (1990) is shown to provide a good description of the flow field in all cases.

The modification of the wave induced flow field in the presence of a strongly sheared "adverse" current profile is of particular importance, and undoubtedly warrants further study. The effect of this type of interaction is to significantly increase the amplitude of the oscillatory motion. Indeed, the effect is so pronounced that an interaction of this type may, under some circumstances, represent a possible "loading criteria" for the design of coastal and near-shore structures.

7.0 Acknowledgements.

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