CHAPTER 85

THEORY VERSUS EXPERIMENTS IN TWO-DIMENSIONAL SURF BEATS

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Abstract

Comparison is made between a deterministic infragravity-wave model and existing laboratory experiments. The theoretical model considers incident bichromatic waves including the effects of the accompanying incident long, bound wave, an oscillating position of the break point, and the intrusion of short-wave grouping into the surf zone, whereas frictional effects are neglected. A measure of the infragravity wave activity is the amplitude of a seaward progressing free, long wave. For this amplitude the qualitative agreement between theoretical results and experiments is excellent. Quantitatively the theory overestimates the infragravity waves by typically 50–100%. This may in part be attributed to the neglect of frictional effects.

1 Introduction

Numerous field experiments have shown that low frequency oscillations (periods of the order of several minutes) can account for a substantial part of the energy in a surf zone. The closer to the coastline, the more this feature is pronounced. These low frequency motions, which have been termed surf beats or infragravity waves have even been reported to exceed the magnitude of the breaking wind waves. It is widely recognized that surf beats are of major importance for the development of longshore bars in the two-dimensional case. Three-dimensional waves at infragravity frequencies account for more complicated ways of sediment transport and changes in coastal morphology.

Usually one distinguishes between the "trapped" and the "leaky" modes, where the trapped modes as opposed to the leaky ones are trapped to the coastline by refraction. In the present paper we confine ourselves to two dimensions so that only the leaky modes are considered. Three-dimensional results will be published elsewhere.

It is well known that groups of short waves induce long (low frequency) bound waves which are phase-locked to the short-wave envelope and travel with the group velocity. These bound waves are known to be a possible source of surf beats. Another effect of the modulation in the short waves is the resulting oscillations of the break point position. This further induces a time-varying

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set-up which also contributes to the low frequency motion.

A theoretical model which takes both of these effects into account has been developed. In this paper we concentrate on the comparison of the model results with laboratory measurements by Kostense (1984), and the underlying theory is only briefly discussed. A short theoretical description is given in Schäffer et al. (1990), and for a full report on the theory we refer to Schäffer (1990).

2 Mathematical model

The phenomenon of infragravity waves forced by short waves involves two scales in time as well as in space. Typically the timescale of the short waves is $O(10s)$ and of the infragravity waves $O(100s)$. One way to treat this problem is to separate the two scales explicitly as in a WKB-expansion. Here we have used another approach, which is perhaps less stringent, but probably more transparant, physically as well as mathematically. Regardless of the approach the same equations evolve.

From the "narrow-minded" short-wave point of view the ingragravity motion is merely a slowly varying current. In comparison with the large length scale of the infragravity wave, the water will be shallow and accordingly the current will be uniform over depth (wee neglect bottom friction). Thus we can use the depth-integrated and time-averaged conservation equations of mass and momentum for waves on a slowly varying uniform current, where the time averaging is taken over one short wave period. Upon linearizing these equations, the current can be eliminated to get the equation governing the elevation of the mean water surface $\overline{\zeta}$

$$\frac{\partial}{\partial x}\left(gh\frac{\partial\overline{\zeta}}{\partial x}\right) - \frac{\partial^2\overline{\zeta}}{\partial t^2} = -\frac{1}{\rho}\frac{\partial^2 S_{xx}}{\partial x^2} \tag{1}$$

(cf. Symonds et al., 1982, Mei and Benmoussa, 1984, and others). Here S_{xx} is the radiation stress associated with the incident short waves

$$\frac{1}{\rho g}S_{xx} = \frac{1}{2}|A|^2\left\{\frac{2c_g}{c} - \frac{1}{2}\right\} \tag{2}$$

where A is a complex amplitude allowed to have a slow variation in time as well as in space, c and c_g are phase and group velocities, respectively, h is depth, x is cross-shore coordinate, ρ is density and g is acceleration of gravity. The right hand side of (1) is responsible for the forcing of infragravity waves, and it is important how it is modelled. Thus a description of the radiation stress or essentially of the variation of A is needed. Given conservative circumstances of no dissipation, the forcing outside the surf zone follows from conservation of energy. After breaking sets in, assumptions concerning the variation of the breakpoint position and the decay of the short waves are required.

2.1 The breaking of incident waves

The model used for the breaking and shoreward decay of modulated incident waves is a combination of two simple, but basically different models.

The first one is obtained by assuming that the short-wave modulation is totally destroyed by the breaking, so that the wave height in the surf zone is solely dependent on the local water depth. This implies an oscillation of the break point position which is essentially harmonic in time. This model was used by Symonds et al. (1982) to study the effect of a time-varying position of the break point, neglecting the incident bound wave.

The second model postulates a fixed initial break point position letting the short-wave modulation be fully transmitted into the surf zone. This model was used by Schäffer and Svendsen (1988) when studying the nearshore behaviour of an incident long bound wave.

A hybrid model is now obtained by combining these two. After some manipulations the new model can be shown to correspond to a time-varying breaker depth given by

$$h_b(t) = \frac{\overline{a}_b}{\gamma_0}\left(1 + \kappa\mu\delta\cos\omega t + O(\delta^2)\right) \tag{3}$$

where $\delta = a^{(2)}/a^{(1)}$ is a small modulation parameter ($a^{(1)}$ and $a^{(2)}$ are the amplitudes of the two waves constituting the wave groups), $\overline{a}_b$ is the mean amplitude at the mean break point, γ_o is the amplitude to depth ratio for vanishing short-wave modulation ($\delta = 0$) and κ is a parameter at our disposal. Furthermore, μ is a factor which accounts for the short-wave shoaling within the region where the initial breaking takes place. This shoaling effect was disregarded by Symonds et al. (1982), Schäffer and Svendsen (1988), and Schäffer et al. (1990) corresponding to the assumption of $\mu \equiv 1$. However, we may as well include it, and after some manipulations we obtain

$$\mu \equiv \frac{1}{1+\nu} \tag{4}$$

where

$$\begin{aligned} \nu &= -\left[h\sqrt{c_g}\frac{d}{dh}\frac{1}{\sqrt{c_g}}\right]_{h=\overline{h}_b} \\ &= \left[\frac{1}{2}\frac{h}{c_g}\frac{dc_g}{dh}\right]_{h=\overline{h}_b} \end{aligned} \tag{5}$$

so that $0 \lesssim \nu \leq \frac{1}{4}$ and thus $\frac{4}{5} \lesssim \mu \leq 1$. According to (5) ν can in principle be negative, but since this corresponds to breaking at intermediate water depth it will not happen in practice. The above two models appear for $\kappa = 1$ (short-wave modulation destroyed) and $\kappa = 0$ (fixed initial break point), respectively. The value of κ reflects the extent to which the modulation of the incident waves is "used" to produce an oscillating break point position and consequently the "amount" of grouping which is transmitted into the surf zone. On the basis on experimental results for monochromatic waves collected by Goda (1970), we have estimated κ to be in the range $1 < \kappa < 1.2$. From the emperical relations given by Hansen (1990), κ appears as a universal constant (i.e. independent of the steepness of the incident short waves) and we get $\kappa \equiv 1.09$, see Schäffer (1990). Results for γ_o versus the deep water short-wave steepness $\hat{a}_\infty \equiv a_\infty k_\infty$ are adapted from Goda (1970) and Hansen (1990), and they are depicted in Fig. 1 for various beach slopes h_x. An estimate of the deviation from unity of the parameter κ can be shown to equal minus the slope of these curves in the double logarithmic plot, i.e $\kappa = 1 - d(\log\gamma_o)/d(\log\hat{a}_\infty)$.

That $\kappa > 1$ actually means that the higher waves break so early that they end up being the lower ones inside the surf zone.

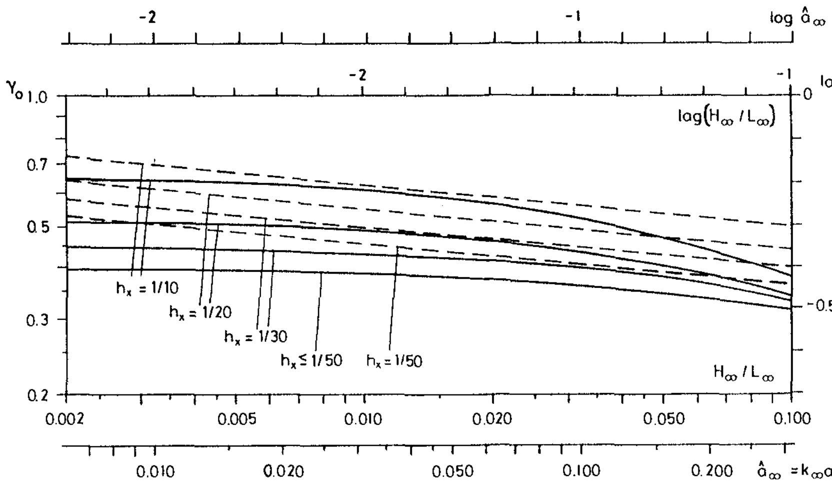


Figure 1 Index $\gamma_o \equiv a_b/h_b$ versus deep water steepness $\hat{a}_\infty$ for different bottom slopes $h_x = \tan\beta$. Results for monochromatic waves adapted from Goda (1970) (——) and from Hansen (1990) (– – –).

3 Method of solution

We restrict ourselves to a plane sloping beach $h = h_x x$ connected with an offshore shelf, and consider only periodic solutions. Fourier expansions of S_{xx} and $\bar{\zeta}$ turn (1) into ordinary differential equations for the different regions, which are solved by the method of variation of parameters. Solutions of different regions are matched using standard conservation considerations with regard to mass, momentum, and energy. The boundary conditions are those of finite shoreline amplitudes and absence of incident free long waves — not to be confused with the incident bound long wave which is an important part of the solution.

Special action is taken in the region of initial breaking i.e. within the inner and outer limits of the initial break point. In this region substantial forcing of infragravity wave motion takes place as first shown by Symonds et al. (1982). The solution is developed to the leading order in the short-wave modulation parameter δ. To this order it can be shown that infragravity waves are only forced at the fundamental group frequency so that no higher harmonics appear. Furthermore, a large gradient of the ingragravity wave surface elevation over the region of initial breaking is for convenience concentrated in a discontinuity at the mean break point depth $\bar{h}_b$ as can be seen in the example shown in Fig. 2.

For a detailed description of the mathematical development we refer to Schäffer (1990).

4 Results and comparison with experiments

A sample of the model results is shown in Fig. 2. The figure shows the envelope of the infragravity wave (not to be confused with the envelope of the incident

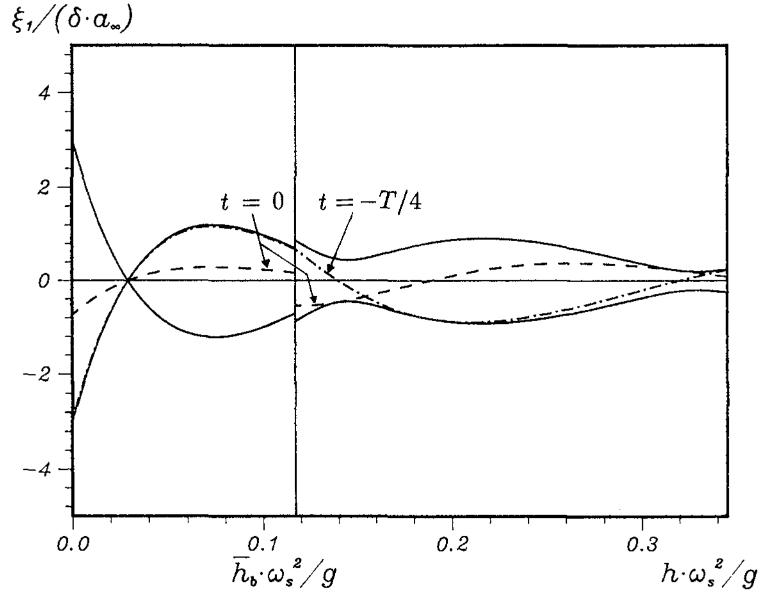


Figure 2 Sample result for $\kappa = 1.1$ of the spatial variation of the infragravity wave elevation (normalized by the modulation of the short wave amplitude in deep water) versus depth (normalized by the (inverse value of the) deep water short-wave number ω_s^2/g). T is the group period, and the mean depth at the break point is $\overline{h}_b$. The abscissa is limited by the depth of the offshore shelf. The figure corresponds to the "•" at $\chi = 5.8$ in Fig. 6a.

short waves) and the elevation at $t = 0, -T/4$ where T is the group period. The discontinuity at the mean break point position at $t = 0$ is a consequence of the substantial forcing around the mean break point position mentioned above, together with a consistent development of the solution to the leading order in the short-wave modulation parameter. The latter can be shown to be consistent with letting the spatial extent of the region of initial breaking tend toward zero leaving a discontinuity behind.

The solution shows a gradual change from an almost standing wave in the vicinity of the shoreline to an almost purely seaward progressive wave some distance offshore.

A series of high quality laboratory experiments on long waves forced by short-wave groups, conducted at the Delft Hydraulics Laboratory, were reported by Kostense in 1984. Fortunately his experimental setup exactly meets the assumptions behind the present theoretical model. This applies to the bathymetry as well as the incident short-wave groups, and his measurements include the respective amplitudes of the incident, bound, long wave and the seaward progressive, free, long wave. The movement of the wave-maker paddle included second-order generation as well as active absorption of free, long waves returning to the wave maker. Consequently free, long, standing waves arising from the

Series	$a_o^{(1)}$	$a_o^{(2)}$	$\omega^{(1)}$	$\omega^{(2)}$	ω	χ
	(cm)	(cm)	(rad/s)	(rad/s)	(rad/s)	
A-1	5.5	1.1	3.062	2.145	0.917	5.81
A-2	5.5	1.1	3.062	2.296	0.766	4.05
A-3	5.5	1.1	3.065	2.456	0.609	2.55
A-4	5.5	1.1	3.077	2.618	0.459	1.44
A-5	5.5	1.1	3.063	2.755	0.308	0.65
B-1	5.5	1.1	4.295	3.372	0.923	5.57
B-2	5.5	1.1	4.065	3.293	0.772	3.93
B-3	5.5	1.1	4.070	3.455	0.615	2.48
B-4	5.5	1.1	4.071	3.609	0.462	1.39
B-5	5.5	1.1	4.070	3.762	0.308	0.62
C-1	8.0	1.6	4.294	3.522	0.772	5.33
C-2	5.5	1.1	4.065	3.293	0.772	3.93
C-3	3.5	0.7	4.295	3.523	0.772	2.65
D-1	5.5	1.1	3.065	2.456	0.609	2.55
D-2	3.5	0.7	3.065	2.456	0.609	1.76
D-3	3.0	0.6	3.065	2.456	0.609	1.55

Table 1 Short-wave characteristics for the experiments by Kostense (1984). The listed χ-values (based on $\gamma_o = 0.4$) differ a little from the ones used by Kostense, due to his neglect of short-wave shoaling.

reflection of free, long waves at the paddle were avoided, and the measurements provide an excellent test of the present theory.

Five series of experiments are reported, and the first four of these correspond to a short-wave modulation parameter of $\delta = 0.2$. This meets our assumptions of $\delta \ll 1$, considering the limited accuracy expected from the theoretical model due to the complicated physical mechanisms in question. The characteristics of the incident short waves for these four series (A–D) is given in Table 1. Here $(a_o^{(1)}, a_o^{(2)})$ and $(\omega^{(1)}, \omega^{(2)})$ are the amplitudes (over the shelf) and angular frequencies of the two wave trains, ω is the difference frequency (group frequency), and χ is a parameter defined by $\chi \equiv \omega^2 \overline{x}_b/(gh_x)$, $\overline{x}_b$ being the position of the mean break point ($x = 0$ at the shoreline). In the fifth experimental run δ equals 0.8, and no comparison is made.

The relevant parameters describing the geometry of the experimental facility are the depth $h_o = 0.5m$ on the shelf and the beach slope $h_x = 0.05$.

In series A and B the short-wave amplitudes are kept constant while the difference frequency ω is changed. The resulting amplitude of the incident, bound, long wave $|\xi_b|$ is compared with the theoretical values in Fig. 3a. The excellent agreement is indeed a manifestation of the thoroughness of the experiments. The theory of the bound, long wave is due to Longuet-Higgins and Stewart (1962, 1964). Kostense presents a comparison similar to that of Fig. 3 based on an equivalent formulation given by Ottesen Hansen (1978). Fig. 3b shows $|\xi_b|$ for series C and D for which the respective difference frequencies are kept constant, while the amplitudes of the short-wave components are changed. Again the theory and the measurements compare very well.

We now turn to the results for the amplitude of the outgoing, free, long wave $|\xi_f|$, the generation of which is far more complex. The physical mechanisms of generation involve complicated surf zone dynamics, and thus the results of the mathematical model can hardly be expected to match the measurements as closely as the results for the incident bound, long wave. One aspect of the model can be expected to give a one-sided error, and that is the neglect of frictional effects. Thus, everything else being equal we can expect the model to overestimate the long-wave activity.

Fig. 4 compares $|\xi_f|$ (at the shelf) from the theory and measurements, and the qualitative agreement is seen to be excellent. Furthermore, we recognize the expected overestimation, which for most runs is approximately 50–100%. At least part of this mismatch may be attributed to the neglect of friction.

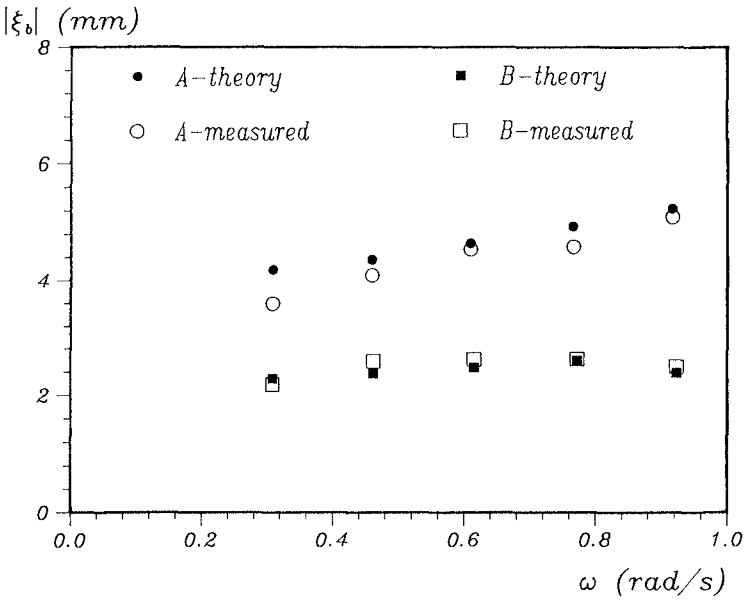
The results for $|\xi_f|$ are repeated in Fig. 5 with $\chi \equiv \omega^2 \overline{x}_b/(gh_x)$ as the abscissa. Note that the extraordinary trend of a straight line for the measurements of series D is reproduced by the theory. The measurements of series A, B, and C all show a convex trend as also given by the theoretical results.

In order to compare with the theory of Symonds et al. (1982) we have used Green's law to assign the computed as well as the measured values of $|\xi_f|$ to their values $|\xi_f(\overline{x}_b)|$ at the break point. Furthermore, $|\xi_f(\overline{x}_b)|$ was normalized by the variation about the mean of the stationary shoreline set-up for infinitely long groups, denoted $\Delta\overline{\zeta}$. The results are shown in Fig. 6a together with the theoretical curve of Symonds et al., which is seen to be inadequate. Here $(\gamma_o, \kappa) = (0.4, 1.0)$ was used. Fig. 6b is equivalent to Fig. 6a, only the results are based on the dashed curve for $h_x = 1/20$ in Fig. 1, which gives γ_o-values ranging from 0.47 to 0.53 and $\kappa = 1.09$. The general picture is seen to be the same. (Note that the compression of the abcissa χ in Fig. 6b as compared with Fig. 6a (which is due to the larger γ_o-values used) affects the measured and computed results equally much, and it has nothing to do with how well they compare).

Fig. 7a is adapted from Kostense, and it shows $|\xi_f|$ versus $|\xi_b|$. As noted by Kostense, the correlation is very poor (as could be expected). This also applies to our theoretical results as shown in Fig. 7b.

The theoretical $|\xi_f|$-values of this section were calculated with the assumptions of full long-wave reflection from the coastline. Corrections for partial reflection can be introduced by requiring the maximum shoreline amplitude of the elevation of the standing infragravity wave to be $|\xi_1(0)|_{max} = gh_x^2/\omega^2$. This limit is valid for a free standing long wave on a plane sloping beach as indicated by the solution of the nonlinear shallow water equations by Carrier and Greenspan (1958) (see Meyer and Taylor, 1972). However, when these modifications are incorporated, the only correction is a 10% reduction of theoretical $|\xi_f|$-value in the first run of series A, the rest of the runs being unchanged. Thus no conclusions are changed.

Kostense mentions three possible reasons for the limited validity of the theory by Symonds et al. These are the neglect of the incident, long, bound wave, the preclusion of short-wave grouping inside the surf zone, and the assumption of full long-wave reflection at the shoreline. The present model accounts for all these effects, and particularly inclusion of the incident, long, bound wave is of great importance. The next step towards a correct mathematical model should be the inclusion of frictional effects such as turbulence in the surf zone and bottom friction.



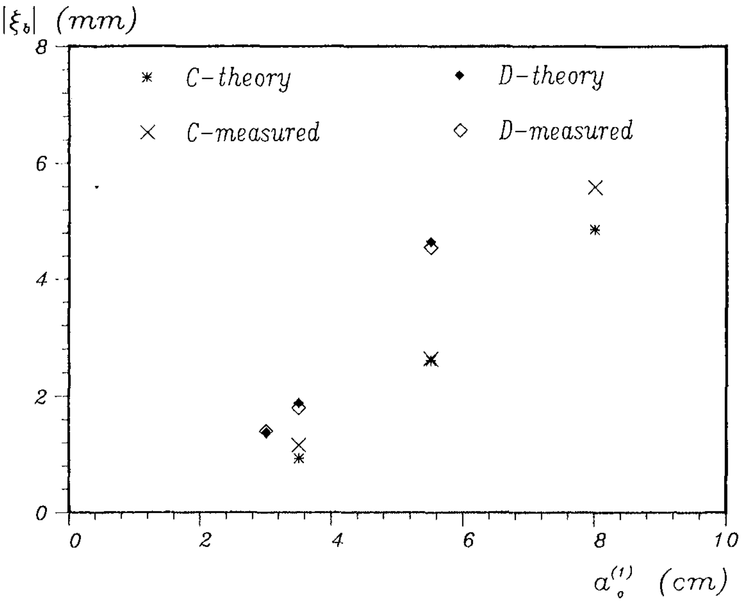
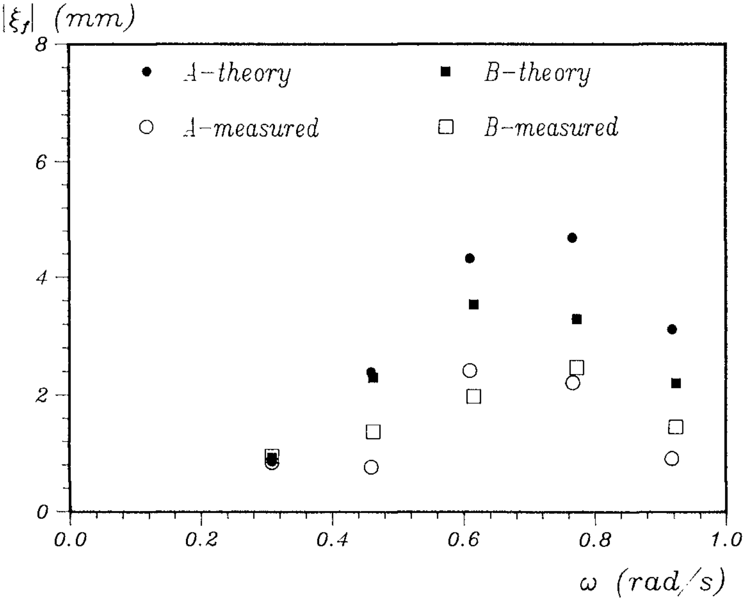


Figure 3 Amplitude of incident bound, long wave $|\xi_b|$ versus (a) the difference frequency ω (series A and B), and (b) the amplitude of the largest short-wave component $a_o^{(1)}$ (series C and D).



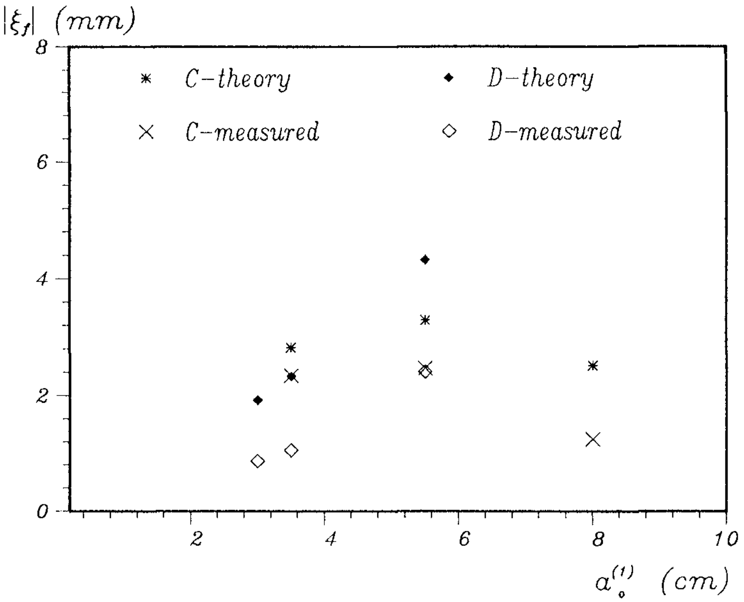


Figure 4 Amplitude of outgoing free, long wave $|\xi_f|$ versus (a) the difference frequency ω (series A and B), and (b) the amplitude of the largest short-wave component $a_o^{(1)}$ (series C and D). $(\gamma_o, \kappa) = (0.4, 1.0)$.

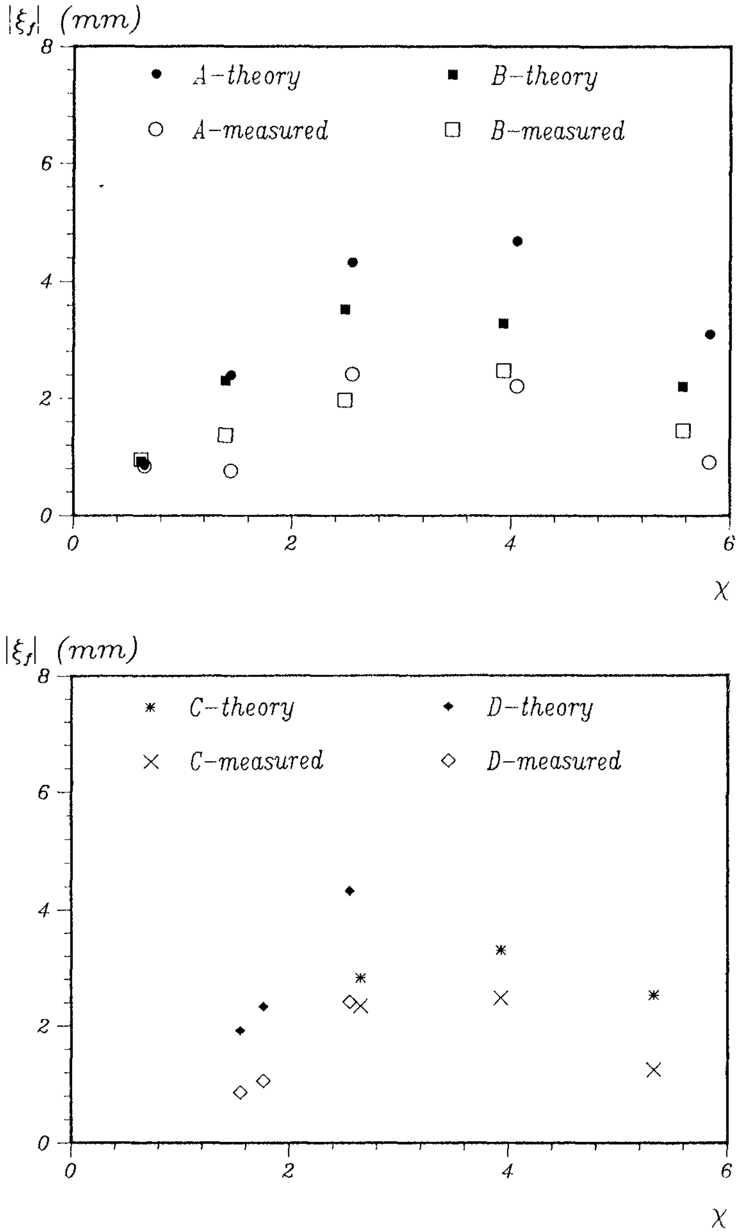
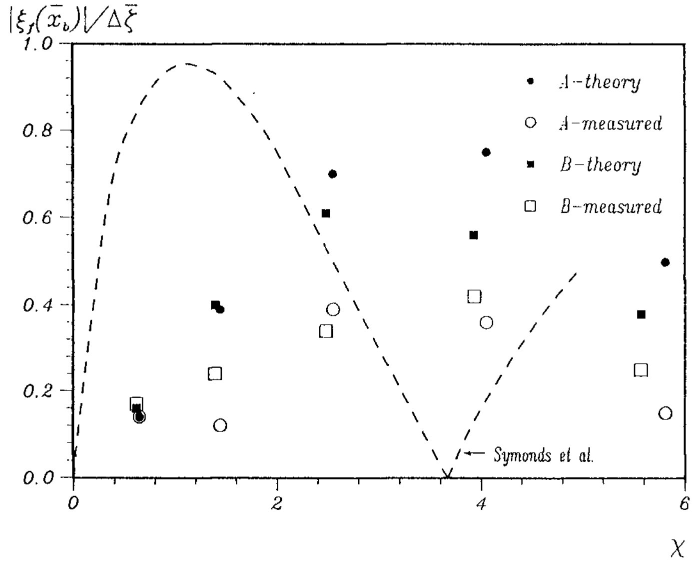


Figure 5 Amplitude of outgoing free, long wave $|\xi_f|$ versus χ for (a) series A and B, and (b) series C and D. $(\gamma_o, \kappa) = (0.4, 1.0)$.



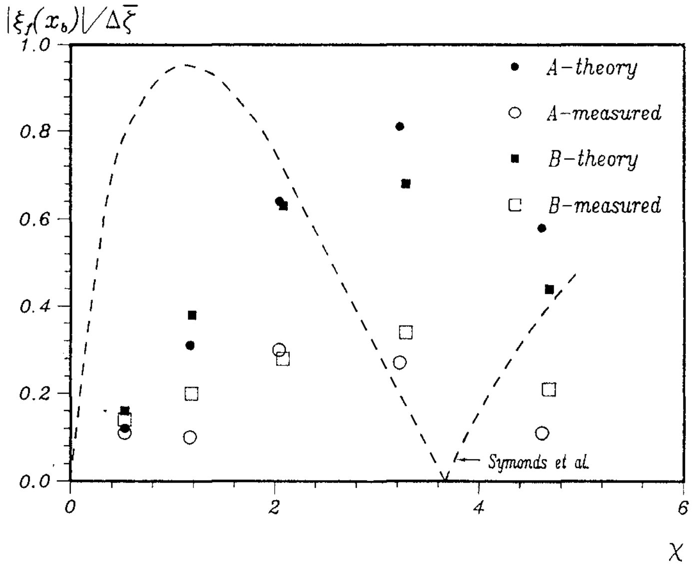


Figure 6 Normalized amplitude of outgoing free, long wave $|\xi_f(\bar{x}_b)|/\Delta\bar{\zeta}$ at the mean initial break point position versus χ for series A, B, C, and D. Comparison with the theory of Symonds et al. (1982) (– – –). (a) $(\gamma_o, \kappa) = (0.4, 1.0)$ and (b) (γ_o, κ)=(0.47–0.53,1.09) as given by the dashed curve for $h_x = 1/20$ in Fig. 1.

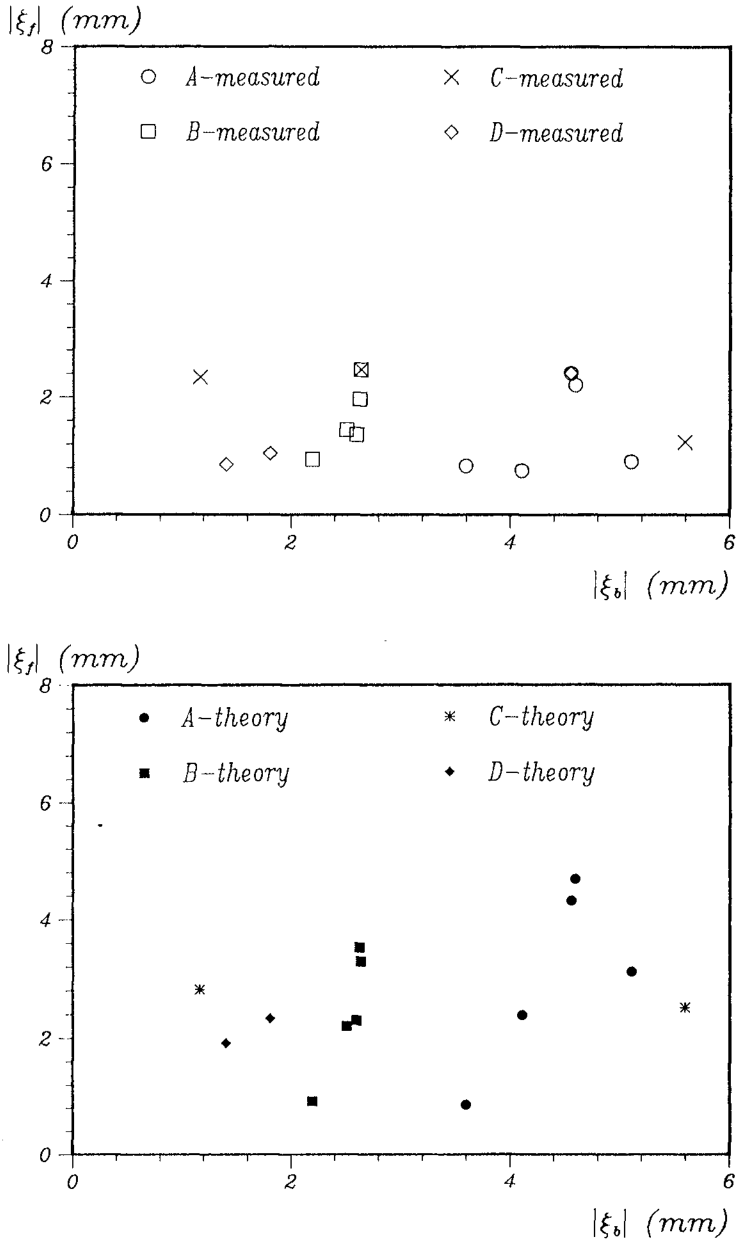


Figure 7 Amplitude of outgoing free, long wave $|\xi_f|$ versus amplitude of incident, bound, long wave $|\xi_b|$ for series A, B, C, and D. (a) measurements, and (b) theory for $(\gamma_o, \kappa) = (0.4, 1.0)$.

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