CHAPTER 26

APPLICATION OF MAXIMUM ENTROPY METHOD TO THE REAL SEA DATA

Taerim Kim[1], Li-Hwa Lin[2], and Hsiang Wang[3]

Abstract

Two different versions of maximum entropy methods(MEM) were compared with two conventional methods for analyzing directional ocean wave spectra. The two MEMs were originally proposed by Lygre and Krogstad (1986) and Kobune and Hashimoto (1986), respectively, and the two conventional methods are the truncated Fourier series method(TFS) and Longuet-Higgins parametric model(LHM). The comparisons included hypothetical idealized cases and actual measured data. For the hypothetical cases, the MEM by Kobune and Hashimoto clearly performed better, particularly for dual-peaked spectra. As for the comparisons from measured data, the MEM generally yielded narrower directional spreading than the two conventional methods but all methods gave nearly identical main direction information. However, this MEM does have occasional convergence problem in real sea data analysis. The problem is removed with the aid of an approximation scheme. This modified scheme is employed in the automated directional spectral analysis of measured sea data.

Introduction

In coastal engineering applications, directional ocean wave information becomes increasingly important owing to the advancement of technology and the demand of better design information. In order to acquire more accurate information of the directional sea waves, much efforts have been devoted to the development of measuring system and the method of analyzing the data. There are several different measuring systems utilized today. For instance, a heave/pitch/roll buoy has been used in the open ocean while wave gage array or pressure transducer and bi-axial current meter are often deployed in coastal water to collect directional wave data. Since ocean waves can be treated as random signals in both time and space, the information derived from all these measuring systems are truncated partial statistical properties, such as in the

[1]Graduate Assistant,[2]Research Scientist,[3]Professor, Coastal and Oceanographic Engineering Department, University of Florida, Gainesville, FL 32611, USA.

form of moments or in the expression of a finite number of Fourier coefficients. Based on the limited information, different methods have been developed to estimate the true ocean wave properties which were often expressed as directional ocean wave spectra. The most direct method clearly is to express the directional spectrum by a finite Fourier series presentation known as truncated Fourier series (TFS) method. However, this method is found to often produce the unreasonable results of negative energy components in the directional domain. When this situation occurs, the estimate is evidently badly biased. For other methods which were developed to analyze directional spectrum, the Longuet-Higgins' parametric model(LHM) is presently the most popular one owing to its concise form and the guaranteed non-negative spectral values. However, the model always gives symmetrical single-peaked directional distribution for each frequency band. Hence, LHM is not suitable for waves with frequency bands containing multi-directional peaks. The maximum entropy directional spectrum estimator developed recently has a major improvement. The method is capable of showing both multiple peaks and asymmetric distribution in direction (Kim, *et al.*, 1993). Therefore, the MEM is particularly useful for shallow water application where waves can be very asymmetric in nature.

Two different entropy definitions have been utilized in finding the corresponding maximum entropy directional spectra. One is from Lygre and Krogstad (1986) who applied MEM under the assumption of a complex Gaussian, stationary process of directional waves and the other is from Kobune and Hashimoto (1986) regarding the directional distribution of wave spectrum as a probability density function. The maximum entropy estimator derived by Lygre and Krogstad was designated here as MEM I and the one by Kobune and Hashimoto as MEM II. Benoit (1992) compared twelve different methods for estimating the directional wave spectrum based on numerical simulations and showed that MEM II gives more reliable estimates but the computational time is rather long. Brissette *et al.* (1992) also compared several methods and pointed out that MEM I often overpredicts the energy at the distribution peaks. Kim *et al.* (1993) tested TFS, LHM, MEM I, and MEM II using three different types of target spectrum and concluded that MEM II is more reliable than the other methods compared. They also suggested an approximation scheme of MEM II to avoid the occasional problem in MEM II and, hence, to significantly reduce the computational time for the practical use. Up to the present, most of the tests for the reliability of the directional spectrum estimator were done based on either artificially simulated target spectra or just one sample set of data. However, simulation test of different directional methods may also be influenced by the target spectra chosen and, therefore, does not provide sufficient evidence for the better method.

In this paper, MEM II was compared and evaluated with two classical methods, TFS and LHM, using two sets of real time series data. The comparisons include the spectral pattern and statistical parameters of dominant frequency peak direction and mean direction, which are important for many different applications in the coastal and ocean engineering.

Methods Estimating Directional Spectrum

Mathematically, a true directional spectrum $E(\sigma,\phi)$, with σ and ϕ denoting the frequency and direction, respectively, can be expressed in terms of an infinite Fourier series as

$$E(\sigma,\phi) = \frac{A_0(\sigma)}{2} + \sum_{n=1}^{\infty}[A_n(\sigma)\cos(n\phi) + B_n(\sigma)\sin(n\phi)], \quad |\phi| \le \pi,$$

where A_0, A_n and B_n are the frequency dependent Fourier coefficients, which can be determined based on the measured or simulated sea data. Although several different techniques analyzing the directional spectrum have been developed in the past, only two classical and two versions of MEM methods are discussed here. The two classical methods are the truncated Fourier series (TFS) and the Longuet-Higgins' parametric model (LHM). The two MEM methods (MEM I and II) have different entropy definitions. To be consistent, the four methods are summarized below based upon the first five Fourier coefficients, A_0, A_1, B_1, A_2, and B_2.

(1)TFS (truncated Fourier series)

The directional estimator expressed by the truncated five-term Fourier series is

$$E(\sigma,\phi) = \frac{A_0(\sigma)}{2} + \sum_{n=1}^{2}[A_n(\sigma)\cos(n\phi) + B_n(\sigma)\sin(n\phi)] \quad |\phi| \le \pi.$$

(2)LHM (Longuet-Higgins parametric model)

The parametric model proposed by Longuet-Higgins (1963) is

$$E(\sigma,\phi) = E(\sigma)\frac{2^{2s-1}}{\pi}\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}\cos^{2s}\frac{\phi-\phi_o}{2},, \quad |\phi_0| \le \pi,\ s > 0,$$

where Γ denotes the Gamma function, $s = s(\sigma)$ and $\phi = \phi_o(\sigma)$ are the directional spreading parameter and the symmetric center direction, respectively. As a first order approximation, the parameters s and ϕ_o can be determined from

$$s = \frac{C_1}{1-C_1}, \quad \phi_o = \tan^{-1}\frac{B_1}{A_1}, \quad C_1 = \frac{\sqrt{A_1^2+B_1^2}}{A_0}.$$

(3)MEM I (Maximum Entropy Approach, Method I)

By defining the entropy of directional sea as

$$M = -\int_0^{2\pi} \ln H(\sigma,\phi)\mathrm{d}\phi.$$

and maximizing M, Lygre and Krogstad(1986) showed that

$$E(\sigma,\phi) = E(\sigma)H(\sigma,\phi), \ H(\sigma,\phi) = \frac{1 - d_1c_1^* - d_2c_2^*}{2\pi|1 - d_1e^{-i\phi} - d_2e^{-2i\phi}|^2},$$

with

$$c_1 = \frac{A_1}{A_0} + i\frac{B_1}{A_0}, \quad c_2 = \frac{A_2}{A_0} + i\frac{B_2}{A_0}, \quad d_1 = \frac{(c_1 - c_2 c_1^*)}{(1 - |c_1|^2)}, \quad d_2 = c_2 - c_1 d_1,$$

where $H(\sigma,\phi)$ is the directional distribution function and the asterisk indicates a complex conjugate.

(4)MEM II (Maximum Entropy Approach, Method II)

By maximizing the entropy defined as

$$M = -\int_0^{2\pi} H(\sigma,\phi)\ln H(\sigma,\phi)\mathrm{d}\phi,$$

Kobune and Hashimoto (1986) showed that

$$H(\sigma,\phi) = \exp[-\sum_{j=0}^{4}\lambda_j(\sigma)\alpha_j(\phi)],$$

where $\alpha_0(\phi) = 1$, $\alpha_1(\phi) = \cos\phi$, $\alpha_2(\phi) = \sin\phi$, $\alpha_3(\phi) = \cos 2\phi$, $\alpha_4(\phi) = \sin 2\phi$, and λ_j's are the Lagrange's multipliers. The λ_j's are determined by iteration method solving a set of nonlinear equations:

$$\int_0^{2\pi}[\beta_i(\sigma) - \alpha_i(\phi)]\cdot\exp[-\sum_{j=1}^{4}\lambda_j(\sigma)\alpha_j(\phi)]\mathrm{d}\phi = 0, \quad i = 1,2,3,4$$

with

$$\lambda_0 = \ln\{\int_{-\pi}^{\pi}\exp[\sum_{j=1}^{4}\lambda_j(\sigma)\alpha_j(\phi)]\mathrm{d}\phi\},$$

where $\beta_1(\sigma) = A_1/A_0$, $\beta_2(\sigma) = B_1/A_0$, $\beta_3(\sigma) = A_2/A_0$, and $\beta_4(\sigma) = B_2/A_0$. It is noted here that, based upon the first five Fourier coefficients measured, the directional spreading function $H(\sigma,\phi)$ determined from both MEM I and II can have at most two directional peaks, which can be understood by taking $\partial H/\partial\phi = 0$. Using the maximum entropy technique to estimate the directional spectrum is also attractive in that the wave spectrum computed does not have to be symmetrical in direction. This certainly indicates a better approach than the conventional LHM method of which the directional distribution is modelled by a symmetrical function. On the other hand, the TFS is known to often yield a biased estimate to the directional spectrum and the computed directional spectrum may have negative energy components.

Fig. 1 displays several comparisons of four methods shown above for the simulated directional spectra which include unimodal, bimodal, and asymmetric distributions. It is seen that TFS can result in non-positive spectral densities, LHM always gives a symmetric, single-peaked distribution, and MEM I generally produces two peaks and overestimates the peak. The overall comparisons show that MEM II generates the closest estimates to the target spectra for all cases tested.

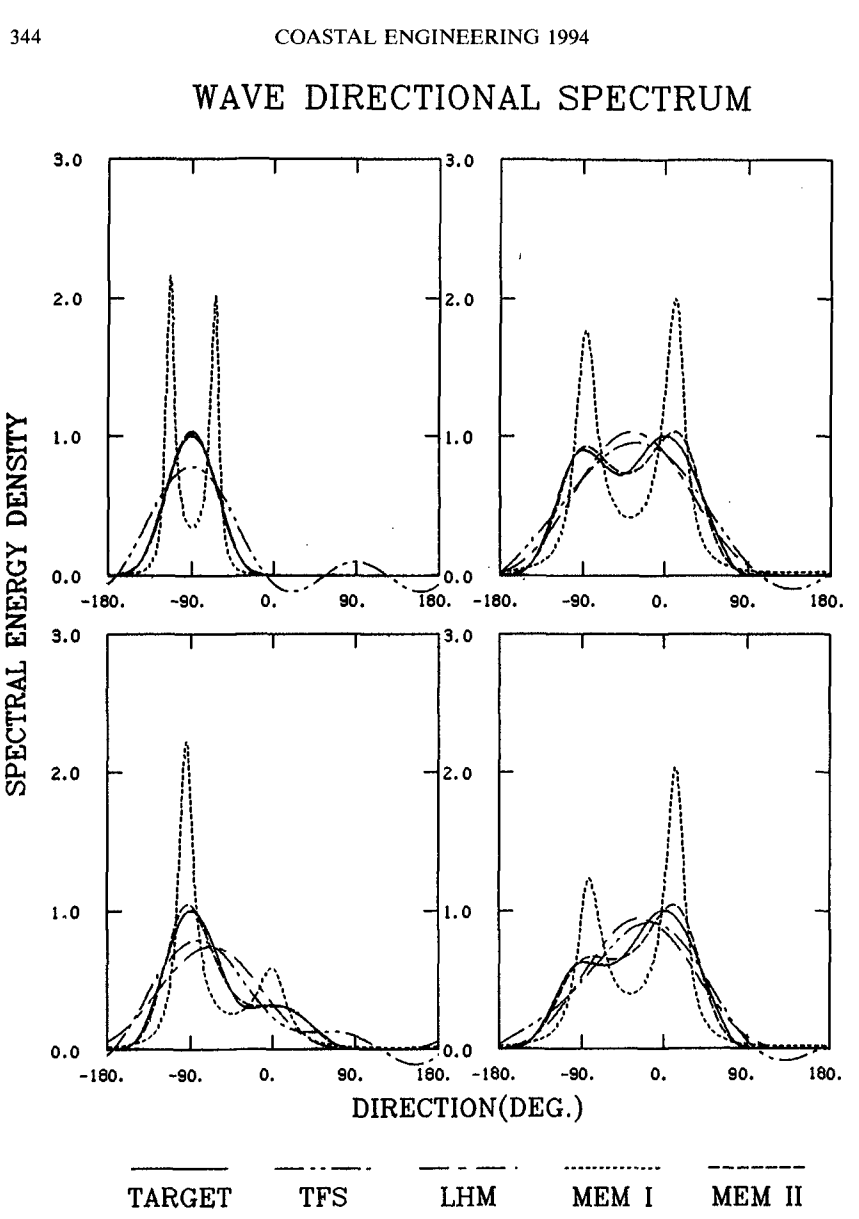

Figure 1: Comparison of simulation results with target spectra.

Application of MEM II

In general, there are no difficulties in computation of directional spectra based on TFS, LHM, and MEM I. However, when MEM II is applied, a nonconvergence problem may occur due to numerical iterations. This problem can be overcome by using an approximation scheme for solving the Lagrange's multipliers. It can be shown that by expanding the exponential term appearing in the nonlinear equations solving for λ_j's to the second order as

$$\int_0^{2\pi} [\beta_i(\sigma) - \alpha_i(\phi)] \cdot \{1 - \sum_{j=1}^{4} \lambda_j(\sigma)\alpha_j(\phi) + \frac{[\sum_{j=1}^{4} \lambda_j(\sigma)\alpha_j(\phi)]^2}{2}\} = 0,$$

an approximation of solutions of λ_i, $i = 1,2,3,4$, can be obtained as

$$\lambda_1 = 2\beta_1\beta_3 + 2\beta_2\beta_4 - 2\beta_1(1 + \sum_{i=1}^{4} \beta_i^2), \quad \lambda_2 = 2\beta_1\beta_4 - 2\beta_2\beta_3 - 2\beta_2(1 + \sum_{i=1}^{4} \beta_i^2),$$

$$\lambda_3 = \beta_1^2 - \beta_2^2 - 2\beta_3(1 + \sum_{i=1}^{4} \beta_i^2), \quad \lambda_4 = 2\beta_1\beta_2 - 2\beta_4(1 + \sum_{i=1}^{4} \beta_i^2).$$

This approximation scheme is designated as MEM AP2 in the present paper. Fig. 2 shows some numerical simulations comparing the original MEM II and MEM AP2 spectra along with the target spectra. Although the MEM AP2 is not identical to MEM II, it generally yields reasonably good result to the unimodal, bimodal, and asymmetric target spectra.

Extended MEM II

As an extension of the MEM II based on the five Fourier coefficients measured, the directional distribution function may be also estimated by combining the first and any J-th directional modes as

$$H(\sigma,\phi) = \exp[-\lambda_0 - a_1 \cos(\phi - \phi_1) - a_J \cos J(\phi - \phi_J)].$$

For example, when $J = 3$,

$$H(\sigma,\phi) = \exp[-\lambda_0 - \lambda_1 \cos(\phi) - \lambda_2 \sin(\phi) - \lambda_5 \cos(3\phi) - \lambda_6 \sin(3\phi)].$$

The solution of λ_j's from the above equation can be obtained either by iteration method or from an approximation scheme as followings:

$$\lambda_1 = -\beta_1 \frac{2\sum_{i=1}^{4} \beta_i^2 + 2.5[\sum_{i=1}^{4} \beta_i^2 - (\sum_{i=1}^{4} \beta_i^2)^2]^2}{\beta_1^2 + \beta_2^2 + 2\beta_1\beta_2\beta_4 + \beta_1^2\beta_3 - \beta_2^2\beta_3}, \quad \lambda_2 = \lambda_1\beta_2/\beta_1,$$

$$\lambda_5 = \frac{\lambda_1(3\beta_2^2 - \beta_1^2) - 4(\beta_1\beta_3 - \beta_2\beta_4)}{2(\beta_1^2 + \beta_2^2)}, \quad \lambda_6 = \frac{\lambda_2(\beta_2^2 - 3\beta_1^2) - 4(\beta_3\beta_2 + \beta_1\beta_4)}{2(\beta_1^2 + \beta_2^2)}.$$

The above approximation scheme is designated here as MEM AP3. Fig. 3 shows some comparisons between the MEM II and MEM AP3 with the target spectra. It is seen that MEM AP3 can still generate good estimate to the target spectrum and

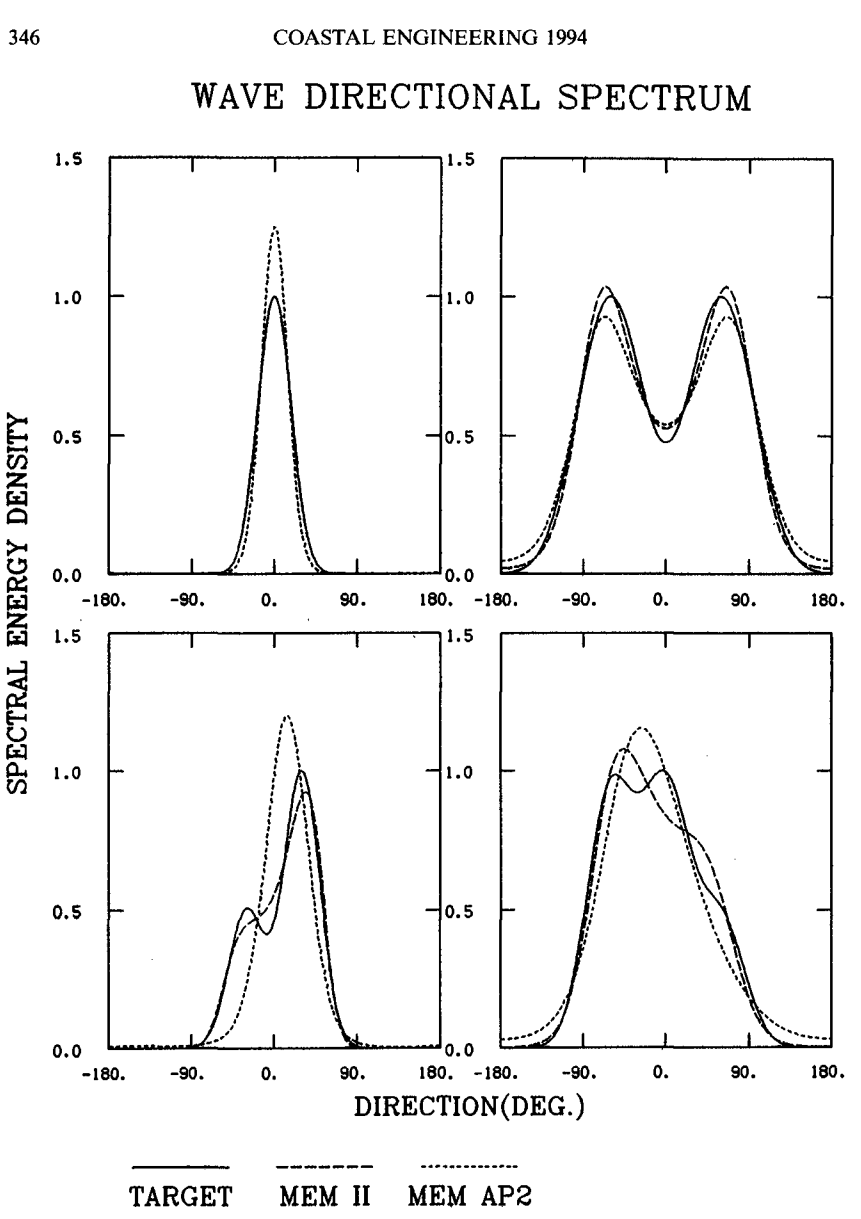

Figure 2: Comparison of the MEM II and MEM AP2 with target spectra.

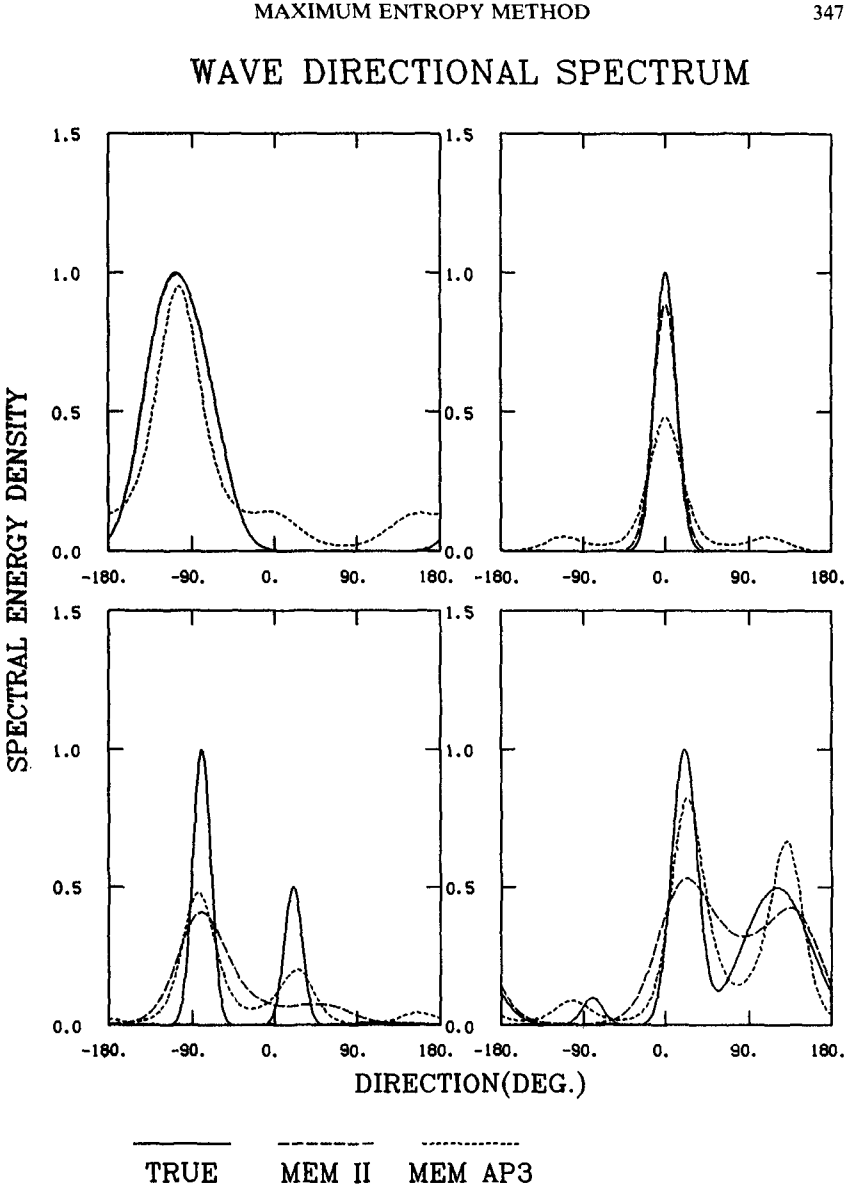

Figure 3: Comparison of the MEM II and MEM AP3 with target spectra.

sometimes show better result than MEM II depending on the target spectrum tested. However, similar to MEM II, the extended model combining the first and limited higher directional modes can result false, although small, side lobe(s) in directions.

Real Sea Data Analysis

Although MEM II shows attractive advantage in simulation test over TFS, LHM, and MEM I, it is more important to see how the method corresponds when applied to the real sea data. The effort here was to test MEM II for the measured time series sea data and compare the results with those from TFS and LHM. Two sets of real data representing two different sea states near coast were chosen for the test. One is a typical storm event of high wind and large waves. The other is for an event of combined swell and local waves due to moderate wind. Both data sets composed of two-day time series. In addition to the wave data, the wind information was also collected from the nearby coastal weather station. In most cases, the computations in MEM II converged rapidly after about five iterations. When they did not converge, the approximation scheme of MEM AP2 automatically took over the calculation.

Fig. 4 shows the computed results of directional spectra for the large wind and wave event which occurred at the Perdido Key, Florida, in the Gulf of Mexico from January 16th to 17th, 1994. The results displayed that new short waves were developed in the beginning when small wind started over the calm sea. As the wind strengthened, the waves were seen to grow steadily and, meanwhile, extend the spectral pattern toward the low frequency region. During the high wind stage, the waves appeared to have reached a state of equilibrium as the spectral pattern remained nearly stationary. As the wind gradually died out, spectra exhibited energy dissipation near the high frequency end. The spectral estimates for this case mostly have single directional peak. The estimates from TFS and MEM II occasionally yielded asymmetric distribution with two peaks, but mostly in frequency bands with little energy content. In terms of the directional dispersion, the directional spectra computed by MEM II displayed much narrower distribution than the results from TFS and LHM methods. Since the MEM II is deemed to predict better directional properties than TFS and LHM, the narrow distribution of directional spectra as estimated by MEM II shall be more representative to the real sea waves.

Fig. 5 shows the results for the event of combined swell and moderate wind waves. The measurement was taken at Cape Canaveral on the Atlantic coast from December 20th to the 21st, 1993. In this event, westbound sea swell was observed throughout the two-day time data used in analysis. At first, only small short waves in scattered directions were observed as the wind was nearly absent. Later on as a moderate southerly wind started, local waves were rapidly developed heading to the north. It was noticed that although local wind waves and existing swell have different directions, as the local wind waves grew, the directions of wind waves and swell began to merge in the middle frequency region. This result seems to suggest that interaction took place between wind waves and swell in this frequency range. Again, the general pattern of directional distribution is narrower as obtained from MEM II than the other two methods.

Ideally, if both wind waves and swell are present at sea, there may be good chance

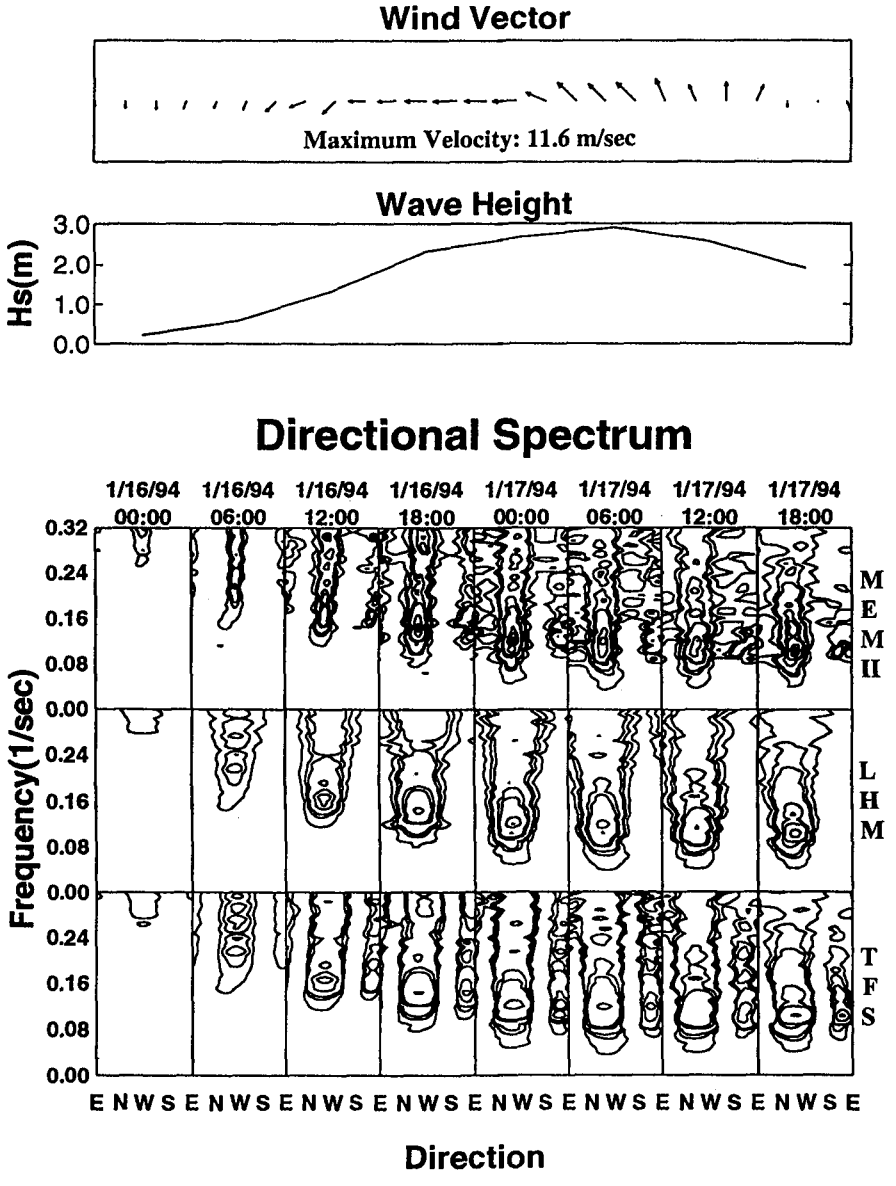

Figure 4: Comparison of MEM II, TFS, and LHM with real time series sea data measured at Perdido Key, Florida (contours in log scale).

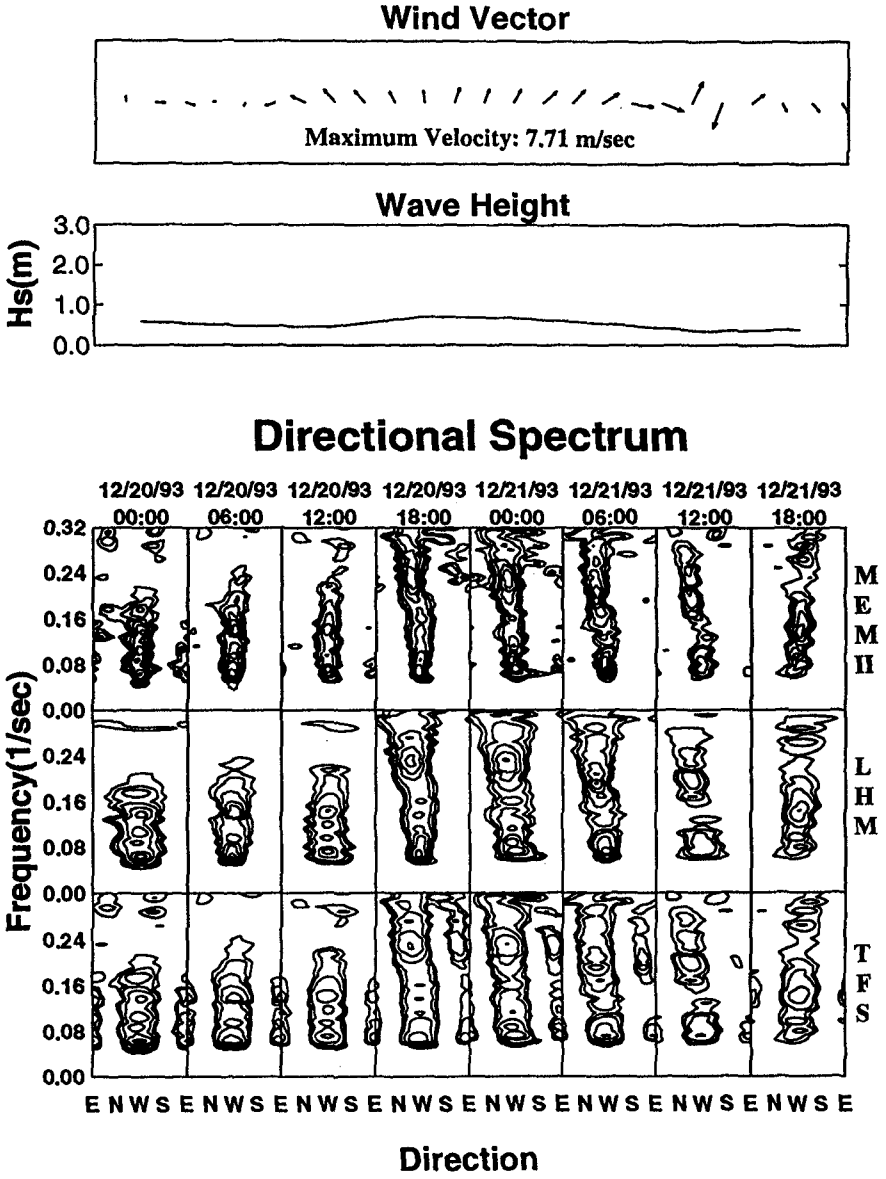

Figure 5: Comparison of MEM II, TFS, and LHM with real time series sea data measured at Cape Canaveral, Florida (contours in log scale).

to have two peaks in directional distribution at the same frequency. However, based on the results of real sea data from MEM II, two directional peaks at the same frequency were rare events, excluding the cases with false secondary peak which usually occurred in the opposite direction of the main peak direction. One possible explanation is the interaction between wind wave and swell components in the same frequency band. However, outside the main energy content range where spectral energy is relatively low, a few cases with two directional peaks were found. A few examples with the presence of directional spectra with dual peaks or asymmetric distribution from the combined wind sea and swell event are shown in Fig. 6.

In order to compare the different methods more specifically for real sea data analysis, three statistical parameters, namely, the peak direction, the mean direction, and the standard deviation at the dominant frequency, which corresponds to the largest spectral energy in frequency domain, were computed. Figs. 7 and 8 show the three parameters computed for the two sets of real sea data tested earlier. For the peak direction at the dominant frequency, all the LHM, TFS, and MEM II results are almost identical. For the mean direction at the dominant frequency, the LHM gives the same direction as the peak direction but both the TFS and MEM II show different directions from the peak directions because of the asymmetry of the directional distribution. In terms of standard deviation, the MEM II exhibits much narrower distribution than both LHM and TFS. In other words, wave energy is more concentrated around the main peak direction as resulted by MEM II.

Conclusions

Four different methods analyzing directional wave spectrum were compared using numerical simulation and actual measured sea data. The four methods include the Truncated Fourier series(TFS), the Longuet-Higgins parametric model(LHM), two different maximum entropy methods (MEM I and II) that utilize different definitions of entropy. The numerical simulation consisted of a variety of target spectra with different properties. And the test results showed that the maximum entropy method with the entropy defined as a statistical probability density function, named here as MEM II, is clearly performed better than the other methods in estimating the target spectra. From the real sea data analysis, it is also concluded that MEM II is most suitable as the method could differentiate dual peaks in the same frequency component and detect the evolution of directional interactions of each frequency component.

The specific findings from the study were summarized below:

(1) For the four different methods compared as candidates for analyzing the measured directional waves, the LHM is restricted to a symmetrical single peak distribution, the TFS has the disadvantage producing negative energy component, the MEM I often overestimates the peak, and the MEM II may have a convergence problem.

(2) For applications to both simulated and real sea data, the MEM II is considered superior to the other methods compared in the paper. The convergence problem of the MEM II in numerical iterations can be overcome by using an approximation scheme.

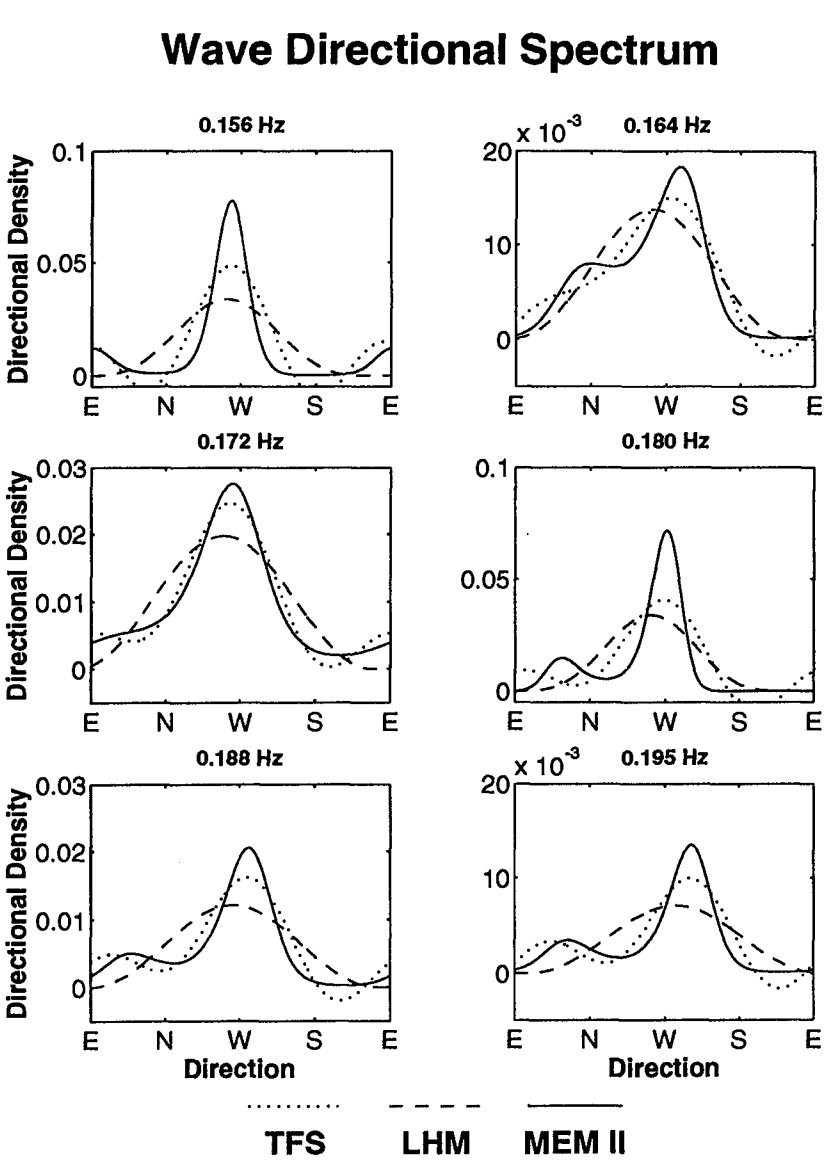

Figure 6: Examples of bimodal and asymmetric directional spectra computed by MEM II along with those from LHM and TFS, based on measured real sea data.

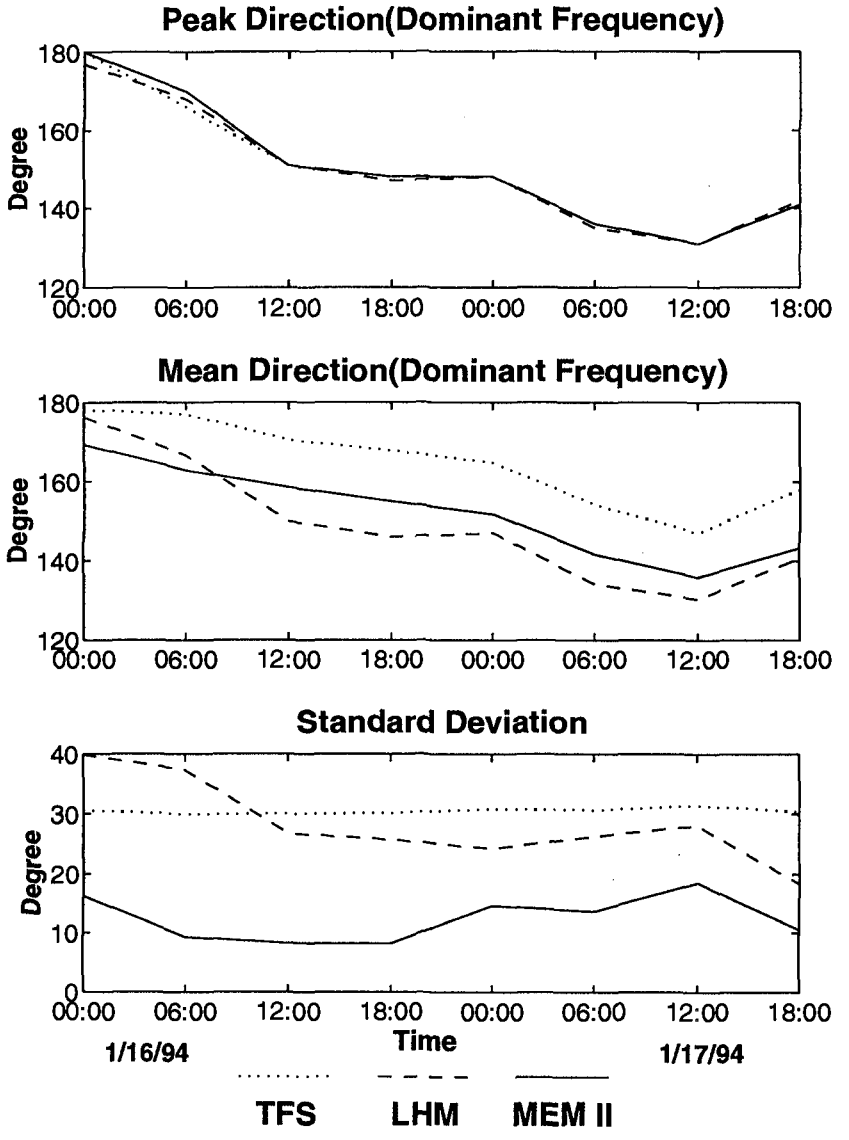

Figure 7: Comparison of dominant frequency peak and mean directions with standard deviations computed based on Perdido Key data.

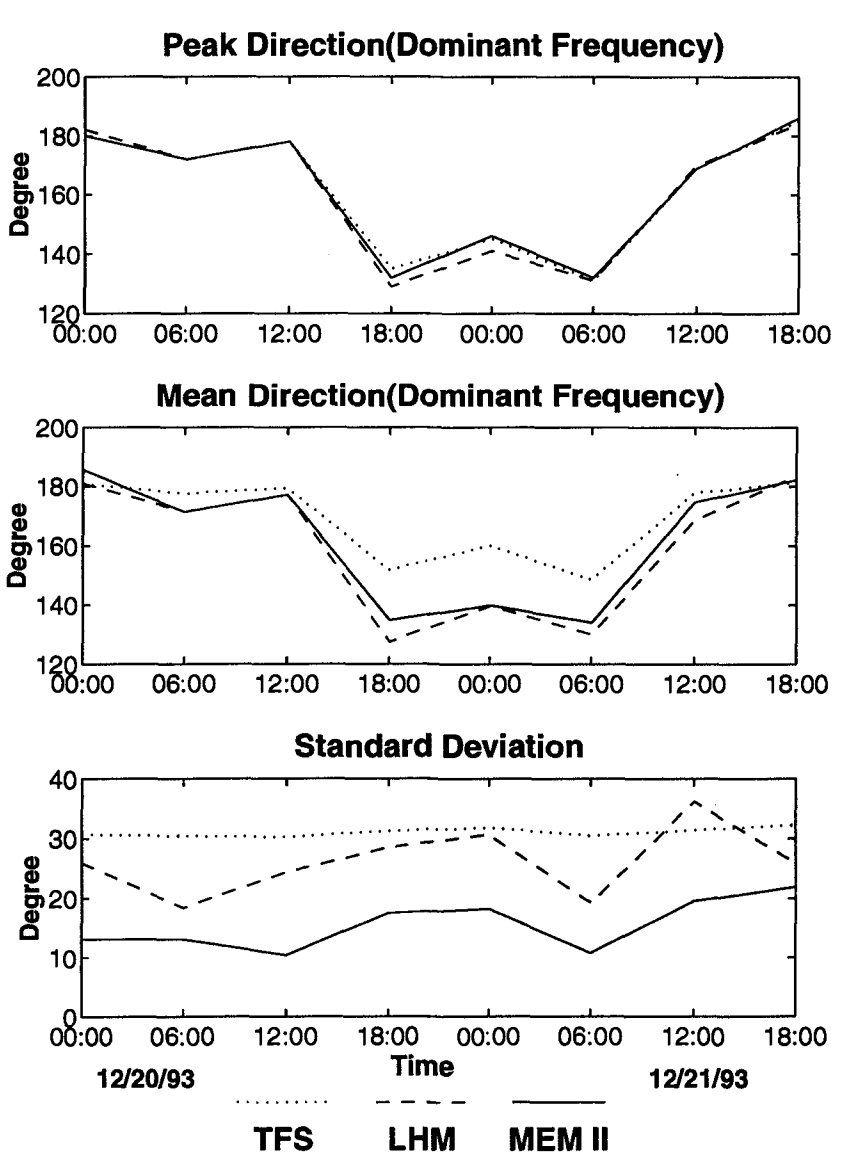

Figure 8: Comparison of dominant frequency peak and mean directions with standard deviations computed based on Cape Canaveral data.

(3) Applying TFS, LHM, and MEM II to the real sea time series data shows similar patterns of directional spectrum. The TFS, LHM, and MEM II are all seen to result almost identical peak direction for the dominant frequency component. However, they all yield different mean directions for dominant frequency component. This is because LHM produces symmetrical directional distribution whereas TFS and MEM II give asymmetrical distribution of directional spectrum. The MEM II, in general, produces narrower directional distribution than the other two methods.

(4) Even with the presence of both local wind waves and swell, it is generally rare to have two distinguished directional peaks at the same frequency. It appears that in the mid frequency range that contains most of the wave energy, the directional components from wind waves and swell tend to merge.

(5) With the aid of an approximation scheme, the MEM II can be programmed for practical applications such as automated directional spectrum analysis from real time data with significant reduction of computational time.

References

Benoit, M., 1992, "Practical Comparative Performance Survey of Methods Used for Estimating Directional Wave Spectra from Heave-Pitch-Role Data.",Proc. 23rd ICCE, ASCE, pp.62-75.

Brissette, F.P. and Tsanis, I.K.,1992, "Estimation of Wave Directional Spectra from Pitch-Roll Buoy Data.", J. WPCOE, ASCE, Vol. 120, No. 1., pp 93-115.

Krogstad, H.E., 1989, "Reliability and Resolution of Directional Wave Spectra from Heave, Pitch, and Roll Data Buoys", Directional Ocean Wave Spectra, The Johns Hopkins University Press, Baltimore and London, pp.66-71.

Longuet-Higgins, M. S., Cartwright, D. E., and Smith, N. D., 1963, "Observations of the Directional Spectrum of Sea Waves Using The Motion of a Floating Buoy", in Ocean Wave Spectra, Prentice Hall, Englewood Cliffs, N. J., pp.111-136.

Kim, T., Lin, L. W., and Wang, H., 1993, "Comparisons of Directional Wave Analysis Methods.", Ocean Wave Measurement and Analysis, Proc. of the Second International Symposium, New Orleans, pp.554-568.

Kobune,K., and Hashimoto, N., 1986, "Estimation of Directional Spectra from the Maximum Entropy principle", in Proc. 5th International Offshore Mechanics and Arctic Engineering (OMAE) Symposium, Tokyo, pp.80-85.

Lygre, A., and Krogstad, H. E., 1986, "Maximum Entropy Estimation of the Directional Distribution in Ocean Wave Spectra", J. Phys. Oceanogr. 16, pp.2052-2060.