CHAPTER 56

PERFORMANCE OF A SPECTRAL WIND-WAVE MODEL IN SHALLOW WATER

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Abstract

A full spectral third-generation wave prediction model has been extended with formulations for surf breaking and nonlinear triad interactions and a first assessment of its performance against shallow water wave data has been examined. The formulation for wave energy loss by surf-breaking in shallow water is based on the expression of Battjes and Janssen (1978), which has heuristically been modified to predict the energy loss per spectral component. The source term for nonlinear triad interactions was taken from Abreu et al. (1992). Results of the extended model have been compared against laboratory and field data. The results of the model computations indicate that surf breaking and triad interactions are important processes in the coastal zone. Surf breaking is mainly responsible for the decay of wave energy, whereas triad interactions are mainly responsible for changes in the mean wave period. The applicability of the Abreu formulation is limited and needs further attention.

Introduction

The modelling of wind waves in shallow water is important for many coastal engineering applications in the nearshore zone. Especially the prediction of both the significant wave height and the mean wave period is still difficult with the presently available wave process formulations. A major problem is that most of the interactions between waves, bottom and currents are nonlinear and poorly understood. This is particularly true for directionally spread random waves in areas with a varying bottom topography.

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A number of model classes exists to compute wave conditions in the coastal zone (cf. Hamm et al., 1993). Commonly used models are the spectral and probabilistic models because they are relatively efficient to use, partly because they neglect diffraction effects. Such a model is also the topic of this paper. More advanced models solve Boussinesq type equations in the time domain. An advantage of such models is that they are able to model processes in a more attractive, more physical way, and also the interactions between the different processes. A disadvantage, however, is that they are rather time-consuming in comparison with spectral models.

Depending on the dimensionless water depth kd, different physical effects are important. In deep water ($kd > 1$) the waves are mainly influenced by three physical processes: wave growth by wind, dissipation of energy by white-capping and non-linear quadruplet wave-wave interactions. In water of intermediate depth ($kd \approx 1$) additional effects become important such as bottom friction and depth- and current refraction. In shallow water ($kd < 1$) also the effects of surf breaking, triad interactions and the effect of waves on currents become noticeable.

The concept of modelling the wave field in terms of the wave spectrum was introduced by Gelci et al. (1957). Since then, many spectral wave models have been developed which are usually classified in terms of their generation, which has mainly do to with the treatment of the nonlinear quadruplet wave-wave interactions and the degrees of freedom of the spectral representation of the wave field.

The first generation of spectral wave models described the evolution of the wave field in terms of parameterized spectra using simple rules, with that implicitly incorporating the effects of nonlinear wave-wave interactions. Spectral models of the second generation incorporated some effects of nonlinear interactions, but they still put limitations to the spectral shape. Only by the development of the discrete interaction approximation for nonlinear quadruplet interactions (Hasselmann et al., 1985) it became possible to develop models of the third generation. Such models explicitly compute all physical effects and they do not impose limitations to the spectral shape. The first full spectral wave model has been developed by the WAM group (WAMDI, 1988). The concept of the WAM model has been extended by Tolman (1991) to account for the effect of instationary current and water level fields. The WAM model can be applied in areas with deep or intermediate water depths, but not in shallow water because it lacks descriptions for typical shallow water processes such as surf breaking and nonlinear triad interactions.

A second generation model for shallow water was described by Holthuijsen et al. (1989). This model, which is parametric in frequency and discrete in direction, includes a formulation for surf breaking but not one for triad interactions. The formulation for wave breaking has been verified against laboratory experiments by Dingemans et al. (1986). Since this model uses one characteristic frequency per direction sector it is not always able to predict the change of the mean wave period in areas with a varying bottom topography. The only way to properly predict the

change of the mean wave period with spectral models is by using a full spectral third-generation wave model which includes all physical processes affecting the waves in shallow water.

The above physical processes have various effects on the mean wave period in shallow water. The main effect of bottom friction is that it reduces wave energy in the lower frequencies and they will decrease the mean wave period, whereas quadruplet wave-wave interactions increase the mean wave period (cf. Young and van Vledder, 1993). Little is known about the spectral modelling of source terms for energy dissipation in shallow water due to breaking waves. The well-known Battjes-Janssen (1978) model predicts the rate of change of the total amount of wave energy, but no information is given about its spectral distribution. Recent experiments by Beji and Battjes (1993), however, indicate that the wave breaking process does not change the shape of the spectrum. Instead, nonlinear triad wave-wave interactions change the spectral shape by the generation of both lower and higher harmonic components. Recently a model was presented by Abreu et al. (1992) which models these triad wave-wave interactions in the spectral domain. Although this model is based on inconsistent assumptions, it is nonetheless considered as a first step in the development of a spectral source term for triad wave-wave interactions.

The purpose of this paper is to further extend the concept of spectral wave modelling in shallow water by introducing the newly available modelling techniques for energy dissipation by breaking waves and nonlinear triad wave-wave interactions, and to give a first assessment of these processes against laboratory and field measurements. In addition, the relative importance of various physical processes in shallow water is assessed.

The present study has been carried out within the framework of the HYDRA project, which is aimed at the determination of hydraulic boundary conditions along the Dutch coast. For the present study DELFT HYDRAULICS's third-generation wave prediction model PHIDIAS has been used.

2 The PHIDIAS wave model

The third-generation wave model PHIDIAS has been developed by DELFT HYDRAULICS for application on oceanic, shelf-sea and coastal zone scales, and for application in deep, intermediate depth and shallow water. In addition, it has successfully been implemented in a real-time data assimilation system. The PHIDIAS model can be applied in areas with a spatially-varying bottom topography and time-dependent current fields.

The PHIDIAS model is based on a spectral description of the sea surface in terms of wave action density, which is a convenient description of the wave field in the presence of currents. Wave action density is considered in the PHIDIAS model as a

function of the location x and y, wave number k and direction θ. The PHIDIAS model solves the time-dependent action balance equation (cf. Hasselmann et al., 1973):

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(\dot{x}N) + \frac{\partial}{\partial y}(\dot{y}N) + \frac{\partial}{\partial k}(\dot{k}N) + \frac{\partial}{\partial \theta}(\dot{\theta}N) = S \tag{1}$$

in which $N = N(x,y,k,\theta;t)$ is the action density defined as the energy density divided by the relative frequency σ. The dot-terms in Eq. (1) are the spectral velocities which follow from linear wave theory (cf. Mei, 1983):

$$\dot{x} = c_{g,x} + U_x \tag{2}$$

$$\dot{y} = c_{g,y} + U_y \tag{3}$$

$$\dot{k} = -\left[\frac{\partial \sigma}{\partial d}\frac{\partial d}{\partial s} + \vec{k}\cdot\frac{\partial \vec{U}}{\partial s}\right] \tag{4}$$

$$\dot{\theta} = -\frac{1}{k}\left[\frac{\partial \sigma}{\partial d}\frac{\partial d}{\partial n} + \vec{k}\cdot\frac{\partial \vec{U}}{\partial n}\right] \tag{5}$$

where d is the local water depth, $c_{g,x}$ and $c_{g,y}$ the x- and y-components of the group velocity, U_x and U_y the velocity components of the ambient flow field, and $\vec{s}$ and $\vec{n}$ the components of unit vectors in the direction of and the direction perpendicular to the direction θ of a wave component.

The source term S on the right-hand side of equation (1) contains expressions of all physical processes that affect the action density of a spectral wave component. For deep water applications the source term S contains state-of-the art expressions for wave growth by the action of the wind (Snyder et al., 1981), dissipation by white-capping (Komen et al., 1984) and nonlinear quadruplet interactions computed with the discrete interaction approximation of Hasselmann et al. (1985). In intermediate water depth applications, the above deep water source terms are scaled as described in WAMDI (1988) and supplemented with state-of-the art formulations for bottom friction (Hasselmann et al, 1973; Madsen et al., 1988). For shallow water, the PHIDIAS model also uses source terms for surf breaking and triad wave-wave interactions. The expressions for surf breaking and triad interactions in a full spectral wave model are recently developed and need some explanation.

A method incorporating the dissipation of wave energy by breaking waves was given by Young (1988). In this method a limit is imposed on the total wave energy and the excess of wave energy is removed from the lowest energy containing waves. This is a rather coarse method because it only removes energy from the lowest

frequencies and because it is not formulated in terms of a source term, i.e. a dissipation rate.

A successful model for computing the energy dissipation in random waves by wave breaking in shallow water was given by Battjes and Janssen (1978). This dissipation model is formulated in terms of the rate of change of the total wave energy E_{tot}:

$$\frac{\partial E_{tot}}{\partial t} \approx -\frac{1}{4}\alpha f_p Q_b H_{\max}^2 \tag{6}$$

where α is a factor of about 1, f_p the peak frequency, Q_b a measure for the fraction of breaking waves and $H_{\max}$ a maximum wave height. The parameter Q_b is computed from :

$$\frac{1 - Q_b}{\ln(Q_b)} = \left\{\frac{H_{rms}}{H_{\max}}\right\}^2 \tag{7}$$

in which H_{rms} is the root mean square wave height, which can be computed from the total wave variance σ^2 according to:

$$H_{rms} = \sqrt{8}\,\sigma \tag{8}$$

In the Battjes-Janssen model the maximum wave height is computed according to a combined steepness and depth-limited breaking criterion (Battjes and Stive, 1985):

$$H_{\max} = \frac{\gamma_1}{k_p}\tanh\left(\frac{\gamma_2}{\gamma_1}k_p h\right) \tag{9}$$

in which γ_1 and γ_2 are coefficients and where k_p is the peak wave number. In this expression the coefficient γ_1 controls the breaking on wave steepness and γ_2 the depth-limited wave breaking. For application in a full spectral wave model three adaptions are needed of the original Battjes-Janssen model. Firstly, an assumption has to be made about the spectral distribution of wave energy by breaking waves and secondly, the criterion for computing $H_{\max}$ should be adapted to avoid the double counting of breaking on wave steepness in the presence of a white-capping dissipation source term, and thirdly to replace the peak frequency f_p with a more stable measure of a representative frequency, for instance by the mean frequency f_{m01}.

The most simple method of distributing the energy dissipation by breaking waves over the spectrum is to assume that this dissipation rate is in proportion to the energy density of each spectral component:

$$S_{brk} = -\frac{1}{4}\alpha f_{m01} Q_b H_{\max}^2 \frac{E(k,\theta)}{E_{tot}} \tag{10}$$

This method seems to be supported by laboratory experiments of Beji and Battjes (1993). The criterion for computing the maximum wave height is simplified to:

$$H_{max} = \gamma_2 h \tag{11}$$

An advantage of expression (11) is that it becomes negligible for deep water. The above formulation has also been included in a recently developed spectral wave model for the coastal zone, developed by Delft University of Technology (Holthuijsen et al., 1993).

The triad interactions are computed by the method proposed by Abreu et al. (1992) on the basis of their equation (34). Their model contains the parameter kd_{lim} which limits the range in which interactions between triads can take place. Based on theoretical arguments they set this limit at $kd_{lim} = \pi/10$, but on the basis of a comparison against field measurements they suggest that this limit should be close to 1. Triad interactions conserve energy and they do not directly affect the significant wave height.

Performance studies

A number of studies has been performed to compare the extended PHIDIAS wave model in shallow water conditions against two sets of laboratory and one set of field data. The laboratory data were collected in wave flumes, using mechanically-generated uni-directional random waves. Field data were obtained from the Egmond site along the coast of the Netherlands for the case of a double bar system. The primary objective of these studies was to investigate the performance of the wave model with respect to the prediction of both the significant wave height and mean wave period. For the field experiment also the relative importance of the modelled physical processes was examined.

The first set of laboratory data were collected by Battjes and Janssen (1978). For the present study results of their experiment numbers 13 and 15 were used, corresponding to wave propagation over an underwater bar. The incident wave conditions and parameter settings are summarized in Table 1. In the Battjes-Janssen experiments no attention was paid to changes in wave periods, and related results will not be shown here.

Experiment number	H_{rms0} (m)	bar-depth (m)	γ_1	γ_2	f_p (Hz)
13	0.113	0.267	0.88	0.75	0.497
15	0.154	0.120	0.88	0.75	0.530

The results obtained with the PHIDIAS model are shown in Fig. 1. The computed data for the significant wave height show good agreement with the measured data.

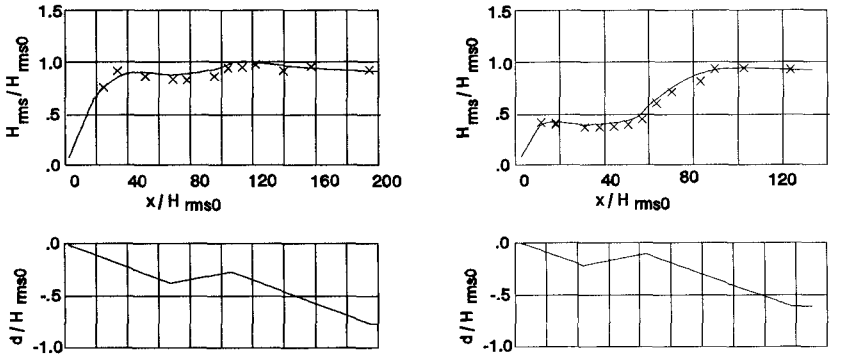

Figure 1: Observed (crosses) and computed (solid line) variation of the root mean square wave height H_{rms} and water depth d, normalized with the deep water rms wave height H_{rms0}, for the cases 13 (left panel) and 15 (right panel) of Battjes and Janssen (1978).

The second set of laboratory wave data were collected in the Schelde flume of DELFT HYDRAULICS in the framework of the MAST program of the European Community. One of these experiments consisted of random wave propagation over an underwater bar. Detailed spectral measurements were performed at 26 locations along the wave flume.

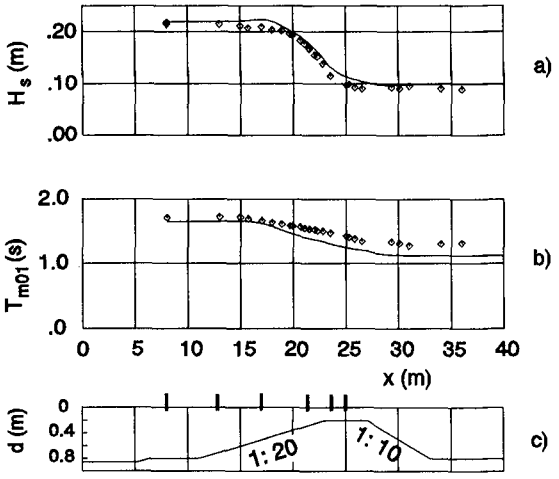

Figure 2: Observed (diamonds) and computed (solid line) variation of significant wave height (H_s) and mean wave period (T_{m01}) in Schelde flume experiment. The tick marks in panel c) refer to the locations for which frequency spectra are shown in Fig. 3.

The input wave conditions comprised a JONSWAP spectrum with a peak period T_p=1.77 s, a significant wave height H_s=0.22 m and a peak enhancement factor γ=3.3. After a number of trial runs with different parameter settings for surf breaking and the triad interactions, good agreement was found for the settings kd_{lim} = 0.75 and γ_2 = 0.75. The cross-section of this flume and the results for the change in significant wave height and the mean wave period (T_{m01}) are shown in Fig. 2. The observed and computed frequency spectra for six locations are shown in Fig. 3.

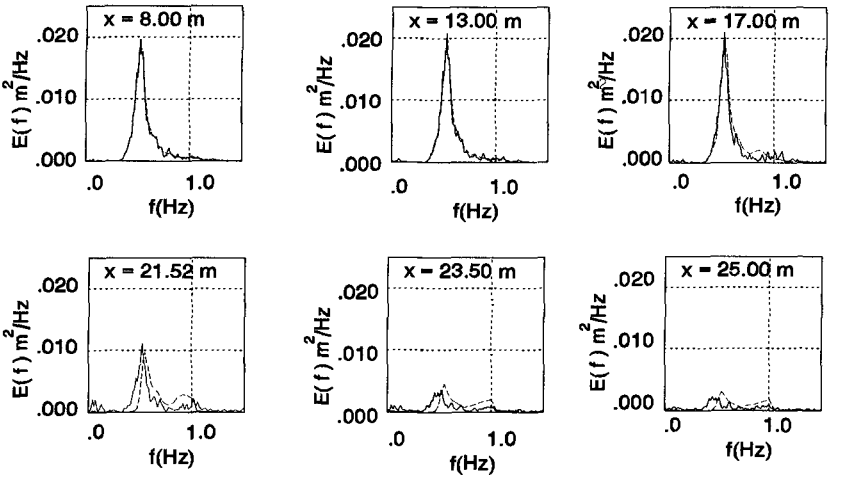

Figure 3: Observed (solid line) and computed (dashed line) frequency spectra in Schelde flume experiment at 6 locations.

The results of the computations show good agreement, not only for the significant wave height but also for the change in the mean wave period. As can be seen in Fig. 3, the change in the mean wave period is due to the generation of a second spectral peak in the vicinity of twice the peak frequency. The generation of this second peak is basically due to the effect of triad interactions. This was demonstrated by performing a computation in which the effect of triad interactions was omitted. Results of other computations (not shown here) with different parameters setting for the triad interactions indicate that the generation of the second spectral peak is controlled by the choice of the upper limit kd_{lim}. In the case of a high value, the second spectral peak showed excessive growth, such that this peak became larger than the first spectral peak.

Field data were collected for a site along the coast of the Netherlands near the town of Egmond. The bottom profile consists of almost parallel bottom contours with a double bar system. Wave data were collected at four locations along a ray protruding into the sea. The bottom profile of this ray and the locations of wave measurement

instruments are shown in Fig. 4. The outer measurement system consisted of a WAVEC, a directional wave buoy. Closer to the coast three wave poles were used which collected time series of the surface elevation. Directional information was not obtained with the wave poles. Unfortunately, no reliable wave data were obtained with wave pole 2.

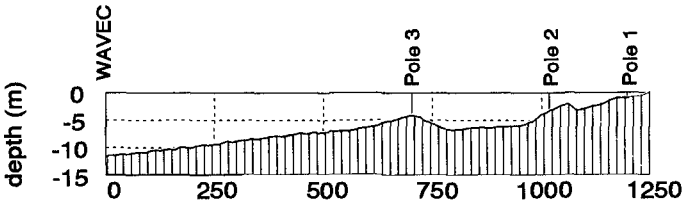

Figure 4: Bottom profile and location of measuring instruments along the Egmond ray.

The wave characteristics observed with the WAVEC buoy were transformed into two-dimensional wave spectra that were used as the boundary conditions for wave computations with the PHIDIAS model. Per frequency a directional distribution was reconstructed on the basis of the mean wave direction and directional spreading according to the $\cos^{2s}(\theta/2)$-model. Wind and water level information was obtained from nearby coastal stations.

For a number of situations wave model computations have been performed with the PHIDIAS model with the objectives of predicting the changes in the spectral shape and of studying the relative importance of the various physical processes in the coastal zone.

Results are presented of one computation for the situation on October 16, 1992, 3 hours, the wind speed was 5 m/s, blowing to the shore. The variation of the significant wave height, mean wave period, incident wave direction and directional spreading are shown in Fig. 5. This figure clearly shows the effect of the underwater bars on the above-mentioned wave parameters. As expected, energy dissipation takes place on the bars and in the area closer to the coast. The spatial variation of the incident wave direction and directional spreading resemble the bottom profile. The waves turn towards the coast and the directional spreading becomes smaller as the water becomes shallower, two effects which are both in agreement with the theoretical expectation.

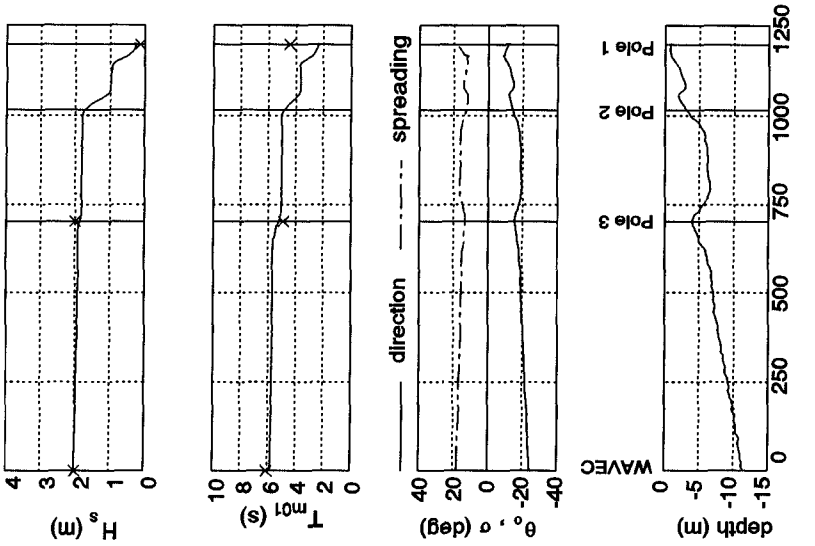

Figure 5: Observed (crosses) and computed (lines) variation of significant wave height (H_s), mean wave period (T_{m01}), mean wave direction (θ_0) and directional spreading (σ) along the Egmond ray on October 16, 1992, 3 hours.

A comparison between the computational results with the measurements shows good agreement for the significant wave height, but not for the mean wave period. As can be seen in Fig. 6, the change of the mean wave period is due to the growth of a strong second spectral peak which reduces the mean wave period. Different parameter settings for the triad interactions were tried to obtain better agreement with the observations. However, this was not possible. The origin of the observed second spectral peak at wave pole 3 could not be predicted with the present implementation of the triad interactions. A closer look at the computational results revealed that this second peak could not be generated by the effect of wind. Clearly, the Abreu model is not capable of handling this situation.

The results of the computations for the Egmond site were also analyzed with respect to the strength of the various source terms, representing the various physical processes. The strength of a source term is defined as the integral over all spectral bins. For the nonlinear interaction source terms (quadruplets and triads) the integral over the absolute values was taken because these processes conserve energy. For the same case as above, the spatial variation of the strength of each source term is shown in Fig. 7. For the present case, the source terms for surf breaking and triads interactions are dominant over all other source terms. It can be seen that the strength

of all source terms is influenced by the local water depth. The wind input source function increases in strength because the waves are slowed down on the bar systems such that the relative wind speed increases. All dissipation source terms become stronger (more negative) over the bars, which is an effect that can also be seen in the strength of the nonlinear interaction source terms.

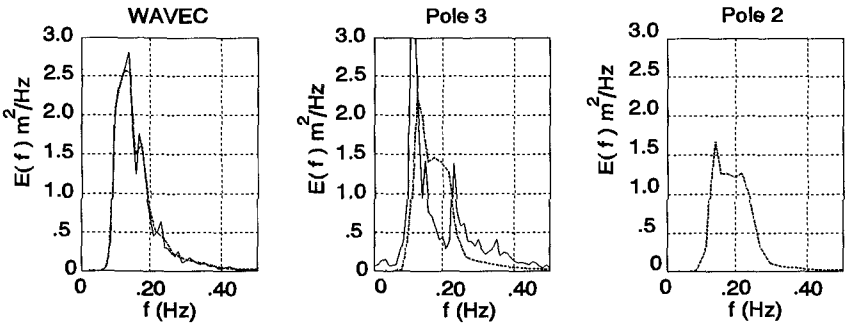

Figure 6: Observed (solid line) and computed (dashed line) frequency spectra along Egmond ray on October 16, 1992, 3 hours.

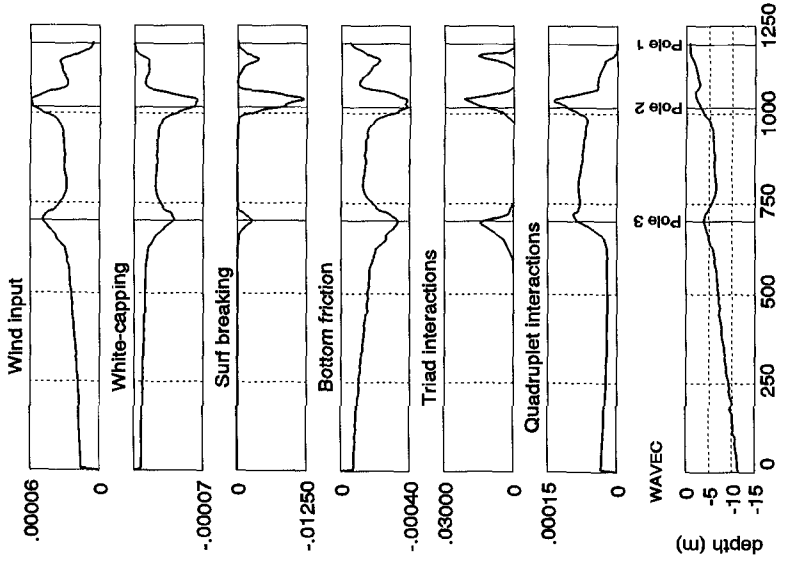

Figure 7: Computed spatial variation of the magnitude of the various physical processes for the Egmond computation of October 16, 1992, 3 hours.

Discussion

The computational results obtained with the extended PHIDIAS wave model show good agreement with measurements regarding the prediction of the significant wave height in situations were surf breaking plays a dominant role. This is not surprising since the source term for surf breaking is based on a well-tested dissipation model (Battjes and Janssen, 1978; Battjes and Stive, 1985). The spectral distribution of dissipation by surf breaking in proportion to the existing energy density was straightforward and seems to be supported by measurements, although a theoretical basis is still missing.

In the coastal zone the change in the mean wave period is mainly due to the effect of triad interactions, especially if waves propagate over an underwater bar. In such a case relatively large changes occur over short distances. The results of the computations for the Schelde flume indicate that the inclusion of a source term for triad interactions is essential for computing a change in the mean wave period. The results for the Egmond site show that it was not possible to find a proper parameter setting for the triad interactions such that the change of the mean wave period would be predicted correctly. This implies that the spectral method of Abreu et al. (1992) for computing the triad interactions is incomplete.

The problem with the Abreu method is that it is based on inconsistent assumptions (Elgar et al., 1993). It is based on the non-dispersive, shallow water equations and a natural asymptotic closure for directionally spread, non-dispersive waves. A consequence is that only exact resonance is taken into account. Moreover, since the waves are assumed to be frequency non-dispersive, only triads containing waves travelling in the same direction are considered resonant. One of the results is that too much wave energy is transferred to higher frequencies resulting in an excessive growth of secondary spectral peaks. This was found to occur on a sloping beach. In the Schelde flume, however, just enough high frequency energy was produced on the bar to obtain a good prediction of the mean wave period. In the deeper water behind the bar the effect of the triad interactions became negligible.

The nonlinear interactions between quadruplets and triads conserve energy. They do not affect the total amount of wave energy, but only the spectral shape. In shallow water triad interactions are much stronger than quadruplet interactions, whereas in deep water the latter process is more dominant.

The present results provide quantitative information about the relative importance of the various physical processes in the nearshore zone, see also Battjes (1994). Such knowledge can be useful in the preparation of a computational run, e.g. by omitting the relative costly computation of the nonlinear quadruplet interactions.

Conclusions and future work

The present study was aimed at obtaining a better understanding of modelling waves in shallow water. To that end a full spectral third-generation wave model was extended with formulations for surf breaking and triad interactions, and compared against shallow water wave data. The results of this study led to the following conclusions:

1 In the coastal zone surf breaking is dominant over the physical effects of wave growth, bottom friction and white-capping dissipation, and triad interactions are dominant over quadruplet interactions.
2 The spectral distribution of wave energy dissipation by surf breaking does not affect the spectral shape.
3 The inclusion of a source term for triad interactions is necessary for the prediction of changes in the mean wave period, especially if waves propagate over an underwater bar.
4 The method of Abreu et al. (1992) for the computation of triad interactions in a wave spectrum is based on inconsistent assumptions. An improved formulation for these interaction is needed, possibly by the inclusion of the proper frequency dispersion characteristics.

Acknowledgements

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