CHAPTER 224

ANALYTICAL SOLUTION FOR THE WAVE-INDUCED EXCESS PORE-PRESSURE IN A FINITE-THICKNESS SEABED LAYER

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ABSTRACT

This paper presents the analytical solution to the wave-induced excess pore-pressure oscillations in permeable seabed sediments, derived for the case of limited thickness of the seabed layer. In order to show the utility of the analytical solution, a wide parameter study has been performed with a special emphasis to the relative compressibility of the two-phase (soil skeleton – pore-fluid) medium. The results of example computations are discussed with respect to the accuracy of the 'finite-thickness' analytical solution for the pore-pressure.

INTRODUCTION

The excess pore-pressure oscillations in permeable seabed sediments, induced by regular surface waves, have been the subject of several investigations and theoretical considerations over the last 40 years.

For many purposes in soil mechanics, it is permissible to uncouple the soil and fluid parts of any analysis in order to treat them separately. However, it may also be desirable on occasion to analyse the true coupled performance of a composite continuum, in which the two phases interact. Examples of practical importance would involve external loads which vary in time, and structure-foundation interaction analyses where the generation of foundation pore-pressure is completely dependent upon the relative stiffness of the components of the system. That is, a stiff or inhomogeneous structure causes different pore-pressures from a flexible or homogeneous one.

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In many practical problems appeared in the coastal engineering, there are many subsoil stratifications and hydro-engineering structures that can be treated as vertically two-dimensional. Assuming also that the seabed is loaded by harmonic waves characterized by long crests parallel to each other, then as a result the seabed is deformed under plain strain conditions. Under these conditions, the following two equations, describing elastic deformations, together with the 'storage' equation constitute the coupled problem and can be written as:

$$G\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+\frac{G}{1-2\mu}\frac{\partial}{\partial x}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right)=\frac{\partial p}{\partial x} \tag{1a}$$

$$G\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial z^2}\right)+\frac{G}{1-2\mu}\frac{\partial}{\partial z}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right)=\frac{\partial p}{\partial z} \tag{1b}$$

$$\frac{k_x}{k_z}\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial z^2}-\frac{\gamma n\beta'}{k_z}\frac{\partial p}{\partial t}=\frac{\gamma}{k_z}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right) \tag{1c}$$

where: p is the wave-induced excess pore-pressure, u_x and u_z are the horizontal and vertical displacements of soil skeleton, respectively, G is the shear modulus of the soil skeleton, μ is the Poisson's ratio, k_x and k_z are the coefficients of permeability of soil in horizontal and vertical direction, respectively, γ is the unit weight of the pore-fluid, β' is the compressibility of the pore-fluid, n is the porosity of soil, x and z are the horizontal and vertical coordinates of the Cartesian system, respectively.

The problem described in Eqs. (1a) to (1c) is very often called as a 'storage' problem because the third is based on the 'storage' equation given by Verruijt (1969), as the governing equation for flow of a compressible fluid in a homogeneous compressible porous medium.

The earlier simplified theoretical solutions give only approximated values for the pore-pressure response. Others, more advanced theories (*e.g.*: Madsen, 1978; Yamamoto *et al.*, 1978; Okusa, 1985) seem to be enough developed in order, at least, to describe the governing problem by introducing additional, meaningful parameters giving thereby a possibility of wide analyses of real soil-water conditions. All these three theories have the same physical background and they differ from each other in the applied method of deriving the particular solution. Okusa (1985), as a first one, discussed also the problem of phase-lag (time-shift) phenomenon existing in a gas-laden sediment for the wave-induced pore-pressure and stresses. The theoretical values calculated from the 'storage' theory lie far away from the 'potential' solution, originally presented by Putnam (1949) and based on the assumptions of incompressible pore-fluid and soil skeleton. These rigorous assumptions differ strongly from the realistic conditions of the soil and pore-fluid two-phase medium. The differences between the 'storage' solution and the 'consolidation' solution proposed by Moshagen & Tørum (1975) and assuming compressibility only for the pore-fluid, are not so drastic but they might become meaningful especially when a relatively compressible soil (*e.g.*, loose sand) is concerned.

'FINITE-THICKNESS' ANALYTICAL SOLUTION

The solutions obtained from the 'potential' problem and 'consolidation' problem approximate roughly the pore-presure response. Both of these solutions can be used in a feasibility study. More precise information on the pore-pressure attenuation and phase-lag distribution with depth can be obtained at the preliminary or detailed design level where the both components of two-phase system (*i.e.*, soil skeleton and pore-fluid) are considered to be compressible.

The appropriate analytical solutions for the pore-pressure response in case of a homogeneous, poro-elastic, semi-infinite seabed were derived by many authors (e.*g*.: Madsen, 1978; Yamamoto *et al.*, 1978; Okusa, 1985) and obtained in terms of the pore-pressure and effective stresses in seabed sediments. Analytical solutions of the 'potential' and 'consolidation' equation for the case of limited thickness of the seabed layer are also well known (*e.g.*, Moshagen & Tørum, 1975). Using the general solution presented by Madsen (1978), Magda (1989, 1992) derived a particular solution for the wave-induced excess pore-pressure under the assumption of a finite thickness of the seabed layer. A 'finite thickness solution' has a very important bearing on further analysis and verification of laboratory test results but would also have a wide application to several engineering problems where natural soil-water conditions exist.

Boundary conditions

The boundary conditions at the surface of the seabed require that both the vertical wave-induced effective stress σ_z and the wave-induced shear stress τ are zero, and additionally, the pore-pressure p at the seabed surface is induced by the hydrodynamic bottom pressure. Therefore, one has to fulfil the following boundary conditions at the surface of the seabed ($z = 0$):

$$p = P_0 \exp[i(ax - \omega t)] \tag{2a}$$

$$\sigma_z = 0 \tag{2b}$$

$$\tau = 0 \tag{2c}$$

Field investigations frequently prove the existence of a limited thickness of permeable and isotropic seabed layer or layers with different properties in the upper boundary of seabed. In such cases, the soil profile may look like a sand layer, possibly with permeable sub-layers, a few metres thick overlaying an impermeable clay stratum.

If a finite, permeable layer overlaying a stiff and impermeable base is considered the boundary conditions at the bottom of permeable seabed layer ($z = -d$) have to be specified. And thus,

$$u_z = 0 \tag{2d}$$

$$\frac{\partial p}{\partial z} = 0 \tag{2e}$$

$$u_x = 0 \qquad \text{completely rough base} \tag{2f}$$

$$\tau = 0 \qquad \text{perfectly smooth base} \tag{2g}$$

where: p is the wave-induced excess pore-pressure, σ_z is the wave-induced effective vertical stress, τ is the wave-induced shear stress, u_x and u_z are the horizontal and vertical displacements of the soil skeleton, respectively, z is the vertical coordinate of the Cartesian system, and d is the thickness of a permeable seabed layer.

Finite thickness solution

Below, a solutions for the instantaneous wave-induced excess pore-pressure in the seabed sediments is presented whereas the problem is considered in terms of the wave response of a single layer resting on a stiff impermeable base. The solution derived in order to obtain the response of a single layer, can be successfully used for the case of a multi-layered seabed; however, the number of mathematical manipulations will increase substantially.

Taking the general solution for the wave response together with the boundary conditions for a finite layer overlaying a perfectly smooth, stiff and impermeable base, a set of six linear and complex-valued equations is obtained. This system of equations can be solved either in its complex form or in form of real values; this would induce however twelve equations involved in the solution procedure.

An implementation of the six boundary conditions leads to a system of six coupled linear equations. Because of rather lengthy form of each of 36 constant coefficients accompanied by unknowns $Y_{1,\ldots,6}$, the calculation procedure has been programmed. The matrix representation of the equation system is as follows:

$$[\mathbf{D}]\{\mathbf{Y}\} = [\mathbf{B}] \tag{3}$$

where each of these three matrices can be written as:

$$\mathbf{D} = \begin{bmatrix} D_1^{(1)} & D_2^{(1)} & D_3^{(1)} & D_4^{(1)} & D_5^{(1)} & D_6^{(1)} \\ D_1^{(2)} & D_2^{(2)} & D_3^{(2)} & D_4^{(2)} & D_5^{(2)} & D_6^{(2)} \\ D_1^{(3)} & D_2^{(3)} & D_3^{(3)} & D_4^{(3)} & D_5^{(3)} & D_6^{(3)} \\ D_1^{(4)} & D_2^{(4)} & D_3^{(4)} & D_4^{(4)} & D_5^{(4)} & D_6^{(4)} \\ D_1^{(5)} & D_2^{(5)} & D_3^{(5)} & D_4^{(5)} & D_5^{(5)} & D_6^{(5)} \\ D_1^{(6)} & D_2^{(6)} & D_3^{(6)} & D_4^{(6)} & D_5^{(6)} & D_6^{(6)} \end{bmatrix} \tag{4}$$

$$\mathbf{Y} = \begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \end{Bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -P_0 \\ 0 \end{bmatrix} \tag{5,6}$$

Matrix $\mathbf{D}$ contains constant coefficients accompanied by proper unknowns in equations of the boundary value problem. And thus, $D_j^{(i)}$ represents a set of constant coefficients related to j-th unknown in i-th equation. There are six equations where i-equation ($i = 1, ..., 6$) relates to the boundary condition (Eqs. 2f, 2d, 2b, 2c, 2a, and 2e, respectively). Matrix $\mathbf{Y}$ represents unknowns Y_j ($j = 1, ..., 6$), and matrix $\mathbf{B}$ is composed of free term in each expression for the boundary condition problem. The 36 coefficients of matrix $\mathbf{D}$ can be computed using the following relationships (Magda, 1992):

$$D_1^{(1)} = \exp(-ad) \tag{7.1a}$$
$$D_2^{(1)} = -d\exp(-ad) \tag{7.1b}$$
$$D_3^{(1)} = \exp(ad) \tag{7.1c}$$
$$D_4^{(1)} = -d\exp(ad) \tag{7.1d}$$
$$D_5^{(1)} = \exp(-\bar{k}ad) \tag{7.1e}$$
$$D_6^{(1)} = \exp(\bar{k}ad) \tag{7.1f}$$

$$D_1^{(2)} = \left(a^3 - X_2 a + X_3 \frac{1}{a}\right)\exp(-ad) \tag{7.2a}$$
$$D_2^{(2)} = \left(3a^2 - a^3 d - X_2 + X_2 ad - X_3 \frac{ad+1}{a^2}\right)\exp(-ad) \tag{7.2b}$$
$$D_3^{(2)} = -D_1^{(2)}\exp(2ad) \tag{7.2c}$$
$$D_4^{(2)} = \left(3a^2 + a^3 d - X_2 - X_2 ad + X_3 \frac{ad-1}{a^2}\right)\exp(ad) \tag{7.2d}$$
$$D_5^{(2)} = \left(\bar{k}^3 a^3 - X_2 \bar{k} a + X_3 \frac{1}{\bar{k}a}\right)\exp(-\bar{k}ad) \tag{7.2e}$$

$$D_6^{(2)} = -D_5^{(2)} \exp(2\bar{k}ad) \tag{7.2f}$$

$$D_1^{(3)} = X_1 a^4 - X_1 X_2 a^2 + X_1 X_3 + X_5 ia \tag{7.3a}$$
$$D_2^{(3)} = 4X_1 a^3 - 2X_1 X_2 a \tag{7.3b}$$
$$D_3^{(3)} = D_1^{(3)} \tag{7.3c}$$
$$D_4^{(3)} = -D_2^{(3)} \tag{7.3d}$$
$$D_5^{(3)} = X_1 \bar{k}^4 a^4 - X_1 X_2 \bar{k}^2 a^2 + X_1 X_3 + X_5 ia \tag{7.3e}$$
$$D_6^{(3)} = D_5^{(3)} \tag{7.3f}$$

$$D_1^{(4)} = a + X_1 ia^4 - X_1 X_2 ia^2 + X_1 X_3 i \tag{7.4a}$$
$$D_2^{(4)} = 1 + 3X_1 ia^3 - X_1 X_2 ia - X_1 X_3 i \frac{1}{a} \tag{7.4b}$$
$$D_3^{(4)} = -D_1^{(4)} \tag{7.4c}$$
$$D_4^{(4)} = D_2^{(4)} \tag{7.4d}$$
$$D_5^{(4)} = \bar{k}a + X_1 \bar{k}^3 ia^4 - X_1 X_2 \bar{k} ia^2 + X_1 X_3 \frac{i}{\bar{k}} \tag{7.4e}$$
$$D_6^{(4)} = -D_5^{(4)} \tag{7.4f}$$

$$D_1^{(5)} = X_7(a^4 - X_2 a^2 + X_3) + X_4 ia + X_6 \frac{a}{i} \tag{7.5a}$$
$$D_2^{(5)} = 2X_7 a(2a^2 - X_2) + 2X_6 \frac{1}{i} \tag{7.5b}$$
$$D_3^{(5)} = D_1^{(5)} \tag{7.5c}$$
$$D_4^{(5)} = -D_2^{(5)} \tag{7.5d}$$
$$D_5^{(5)} = X_7(\bar{k}^4 a^4 - X_2 \bar{k}^2 a^2 + X_3) + X_4 ia + X_6 \frac{\bar{k}^2 a}{i} \tag{7.5e}$$
$$D_6^{(5)} = D_5^{(5)} \tag{7.5f}$$

$$D_1^{(6)} = D_1^{(5)} a \exp(-ad) \tag{7.6a}$$
$$D_2^{(6)} = (D' + D'') \exp(-ad) \tag{7.6b}$$
$$D_3^{(6)} = -D_1^{(5)} a \exp(ad) \tag{7.6c}$$
$$D_4^{(6)} = (D' - D'') \exp(ad) \tag{7.6d}$$
$$D_5^{(6)} = D_5^{(5)} \bar{k}a \exp(-\bar{k}ad) \tag{7.6e}$$
$$D_6^{(6)} = -D_5^{(5)} \bar{k}a \exp(\bar{k}ad) \tag{7.6f}$$

The above presented set of coefficients is derived to solve the pore-pressure problem with the assumption of rough basement under the permeable seabed layer. Principally, the same procedure is applied for a finite layer overlaying a completely smooth base. The only difference is that the boundary condition (2f) has to be replaced by the relation (2g) which is foreseen for the smooth base condition. It means that the sub-set of constant coefficients $D_j^{(6)}$ is built up using constant coefficients of Eq. (2g). All the elements in the other two matrices (*i.e.*, **X** and **B**) stay unchanged. Equations (7.6a) to (7.6f) were derived for the case of perfectly rough basement. If a perfectly smooth basement is assumed Eqs. (7.6a) to (7.6f) have to be replaced by the following set of equations in the computational procedure, respectively:

$$D_1^{(7)} = X_6(a + X_1 ia^4 - X_1X_2ia^2 + X_1X_3i)\exp(-ad) \tag{7.7a}$$

$$D_2^{(7)} = X_6\left[1 - d + 3X_1ia^3 - X_1dia^4 - X_1X_2ia + X_1X_2dia^2 - X_1X_3\frac{i(ad+1)}{a}\right]\exp(-ad) \tag{7.7b}$$

$$D_3^{(7)} = -D_1^{(7)}\exp(2ad) \tag{7.7c}$$

$$D_4^{(7)} = X_6\left[1 + d + 3X_1ia^3 + X_1dia^4 - X_1X_2ia - X_1X_2dia^2 + X_1X_3\frac{i(ad-1)}{a}\right]\exp(ad) \tag{7.7d}$$

$$D_5^{(7)} = X_6(\bar{k}a + X_1i\bar{k}^3a^4 - X_1X_2i\bar{k}a^2 + X_1X_3\frac{i}{a})\exp(-\bar{k}ad) \tag{7.7e}$$

$$D_6^{(7)} = -D_5^{(7)}\exp(2\bar{k}ah) \tag{7.7f}$$

where:

$$D' = X_7(5a^4 - 3X_2a^2 + X_3) + X_1X_3X_6 + X_4ia + 3X_6\frac{a}{i} \tag{8a}$$

$$D'' = -X_7ad(a^4 - X_2a^2 + X_3) + X_1X_3X_6 - X_4ia^2d - X_6\frac{a^2d}{i} \tag{8b}$$

$$X_1 = \frac{2(1-\mu)k_z}{a\omega\gamma\left(n\beta + \dfrac{1-2\mu}{G}\right) + i(k_x - k_z)a^3} \tag{8c}$$

$$X_2 = a^2 + \frac{[k_x(1-2\mu) + k_z]a^2 - i\omega\gamma n\beta(1-2\mu)}{2k_z(1-\mu)} \tag{8d}$$

$$X_3 = a^2\left(a^2\frac{k_x}{k_z} - \kappa^2\right) \tag{8e}$$

$$X_4 = -\frac{(1-\mu)E}{(1+\mu)(1-2\mu)} \tag{8f}$$

$$X_5 = \frac{\mu}{1-\mu} \tag{8g}$$

$$X_6 = -G \tag{8h}$$

$$X_7 = X_1(X_4X_5 + X_6) \tag{8i}$$

Solution of the coupled linear equations' system [Eq. (3)], obtained in terms of coefficients $Y_{1,\ldots,6}$, constitutes simultaneously an explicite solution to the pore-pressure (and also to the wave-induced effective stresses and strains) within the seabed layer of finite thickness. The pore-pressure solution can be written as:

$$p = -\left(\sigma_x + \frac{1}{ia}\frac{\partial \tau}{\partial z}\right) \tag{9}$$

where:

$$\sigma_x = X_4(T_1 + X_5T_2)\exp[i(ax - \omega t)] \tag{10}$$

$$\begin{aligned}\frac{\partial \tau}{\partial z} = X_6[&2Y_2a\exp(az) + (Y_1 + Y_2z)a^2\exp(az)\\ &- 2Y_4a\exp(-az) + (Y_3 + Y_4z)a^2\exp(-az)\\ &+ Y_5\bar{k}^2a^2\exp(\bar{k}az) + Y_6\bar{k}^2a^2\exp(-\bar{k}az)\\ &+ T_2ia]\exp[i(ax - \omega t)]\end{aligned} \tag{11}$$

$$\begin{aligned}T_1 = ia[&(Y_1 + Y_2z)a^2\exp(az) + (Y_3 + Y_4z)a^2\exp(-az)\\ &+ Y_5\bar{k}^2a^2\exp(\bar{k}az) + Y_6\bar{k}^2a^2\exp(-\bar{k}az)]\end{aligned} \tag{12}$$

$$\begin{aligned}T_2 = X_1\{&4Y_2a^3\exp(az) + (Y_1 + Y_2z)a^4\exp(az)\\ &- 4Y_4a^3\exp(-az) + (Y_3 + Y_4z)a^4\exp(-az)\\ &+ Y_5\bar{k}^4a^4\exp(\bar{k}az) + Y_6\bar{k}^4a^4\exp(-\bar{k}az)\\ &- X_2[2Y_2a\exp(az) + (Y_1 + Y_2z)a^2\exp(az)\\ &\qquad - 2Y_4a\exp(-az) + (Y_3 + Y_4z)a^2\exp(-az)\\ &\qquad + Y_5\bar{k}^2a^2\exp(\bar{k}az) + Y_6\bar{k}^2a^2\exp(-\bar{k}az)]\\ &+ X_3[(Y_1 + Y_2z)\exp(az) + (Y_3 + Y_4z)\exp(-az)\\ &\qquad + Y_5\exp(\bar{k}az) + Y_6\exp(-\bar{k}az)]\}\end{aligned} \tag{13}$$

in which:

$$\bar{k} = \sqrt{\frac{k_x}{k_z} - \frac{\kappa^2}{a^2}} \tag{14}$$

and

$$\kappa^2 = i\frac{\omega\gamma\{n\beta' + (1-2\mu)/(2(1-\mu)G)\}}{k_z} \tag{15}$$

where, additionally to the formerly described parameters: a is the wave number, ω is the wave angular frequency, and i is the imaginery unit.

Having the formula for the pore-pressure p distribution with depth z, which is written in a complex-number form, the formulae describing the amplitude, $|p|$, and the phase-lag, δ, of the pore-pressure oscillated in seabed sediments, with respect to the phase of bottom pressure oscillations, can be easily obtained.

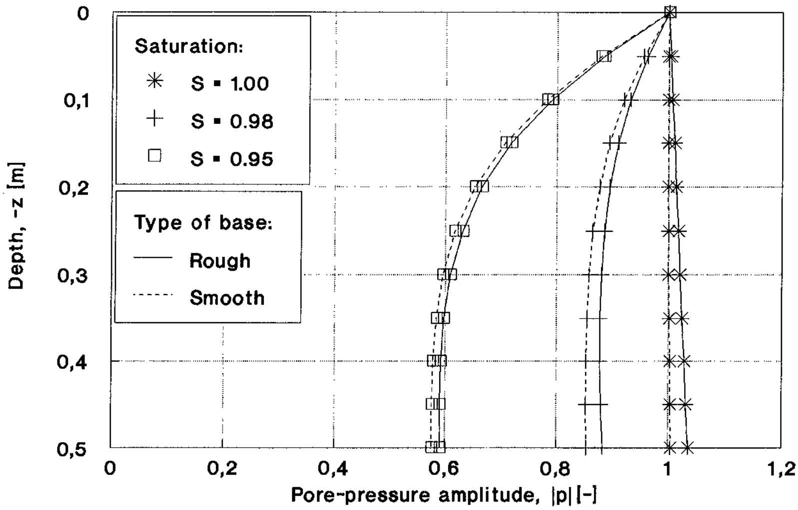
RESULTS OF EXAMPLE CALCULATIONS

In order to perform illustrative calculations the following input data were used:

- coefficient of isotropic permeability: $k_x = k_z = 0.0001\,\mathrm{m/s}$
- porosity of soil: $n = 0.4$ (loose sand), $n = 0.36$ (dense sand)
- Poisson's ratio: $\mu = 0.3$
- Young's modulus of soil: $E = 10^4\,\mathrm{kN/m^2}$ (loose sand), and $E = 10^5\,\mathrm{kN/m^2}$ (dense sand)
- degree of saturation: from $S = 1.00$ (fully saturated soil conditions) to $S = 0.95$ (unsaturated soil conditions)
- seabed layer of finite thickness: $d = 0.5\,\mathrm{m}$ (smooth or rough basement)
  - wave period: $T = 6\,\mathrm{s}$
  - water depth: $h = 4.5\,\mathrm{m}$

Using the analytically derived finite-thickness layer solution (Magda, 1989, 1992), two different boundary conditions assumed at the bottom of the permeable seabed layer [Eqs. (2f) and (2g)] were introduced in order to simulate either rough ($u_x = 0$, *i.e.* no movement of the soil skeleton) or smooth ($\tau = 0$, *i.e.* no friction and free movement) surface of a rigid and impermeable basement underneath the permeable seabed layer.

The results of pore-pressure response in a permeable seabed layer of finite thickness, overlaying either a smooth or rough impermeable stiff base, are given in Fig. 1 (the solid-line denotes the rough base condition and the dashed-line denotes the smooth base condition) in terms of the pore-pressure amplitude and phase-lag as functions of depth in the seabed. Different soil saturation conditions ($S = 0.95 - 1.00$) as well as different compressibilities of soil skeleton, *i.e.*: $E = 10^5\,\mathrm{kN/m^2}$ for dense or semi-dense sandy sediments (see Fig. 1), and



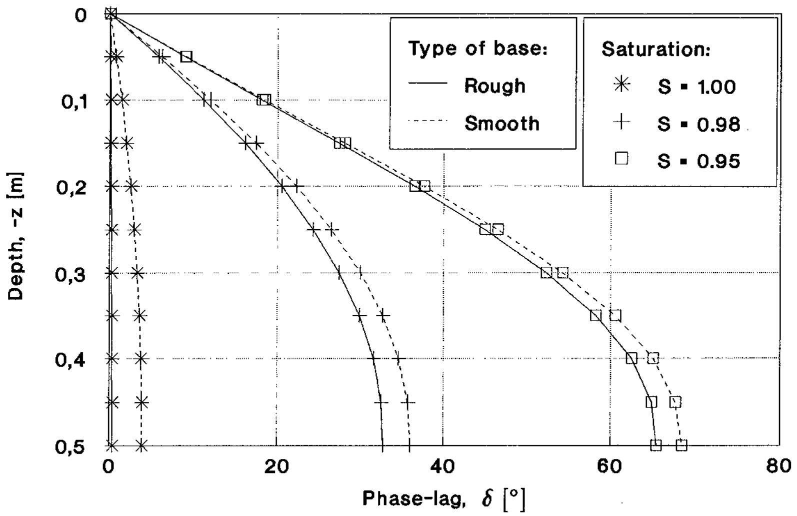


Figure 1. Pore-pressure amplitude and phase-lag distribution with depth for different saturation conditions and type of impermeable basement (finite-thickness-layer solution, $E = 10^5\ \mathrm{kN/m^2}$ – dense sandy sediments)

$E = 10^4\,\mathrm{kN/m^2}$ for loose sandy sediments (the results are not presented here in graphical form) were introduced into computations.

In case of dense sandy sediments (Fig. 1), the difference between the rough and smooth base is relatively small. The influence of introduction of the smooth base is magnified when loose sandy sediments are considered. In both cases (*i.e.*, loose and dense sediments) the smooth base condition makes the pore-pressure attenuation as well as the phase-lag larger with respect to the rough base condition. Althoug the pore-pressure gradient is more inconvenient (more dangerous for the stability of seabed sediments) when the smooth base is assumed, although the rough base condition seems to be more natural.

Performing comparative calculations and wide parameter studies, it was found that fully saturated soil conditions ($S = 1.00$) and a finite thickness of the seabed layer cause some unexpected disturbances in the pore-pressure distribution with depth, namely, the pore-pressure value at the impermeable base exceeds the value of inducing hydrodynamic pressure at the seabed surface (see Fig. 1) which normally – from the physical point of view – should not appear. This phenomenon can be explained if the value of relative compressibility of both components in the two-phase system (*i.e.*, soil skeleton and pore-fluid) as well as the boundary condition choice are investigated carefully.

It is believed that the discrepancy between the calculated and logically expected (*i.e.*, a vertical-profile distribution with depth) values of the pore-pressure amplitude might be caused by:

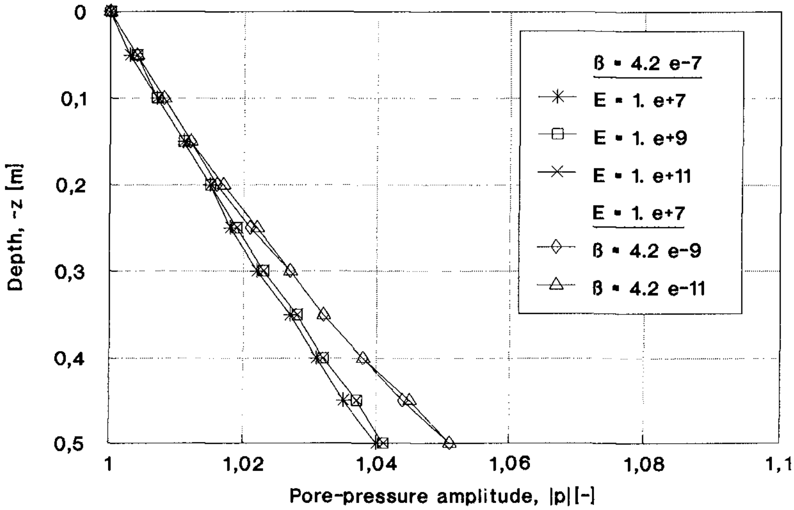
- ill-conditioned coefficient matrix $\mathbf{D}$ (numerical problem), or
- unrealistic boundary conditions assumed at the surface of seabed sediments (boundary-condition problem), or
- interaction of both of them (mixed problem).

In order to get more detailed insight into the problem, some additional computations were performed where the influence of different extreme values of the relative compressibilities of the two-phase medium was studied (Figs. 2 and 3).

The ratio between the absolute smallest and largest values in the coefficient matrix $\mathbf{D}$ differs in order between 10^{14} and 10^{22}, in the case analysed and presented in Fig. 3. On the other hand, it is commonly known that the matrices like this one are ill-conditioned and can very often cause some serious numerical problems.

In direct methods of solution, roundoff errors accumulate, and they are magnified to the extend that the governing matrix is close to singular (Press *et al.*, 1989). In spite of the Gaussian elimination method, the following three methods, dealing with sets of equations that are either singular or else numerically very close to singular, were applied:

- LU decomposition,
- iterative improvement of a solution,
- singular value decomposition (SVD).



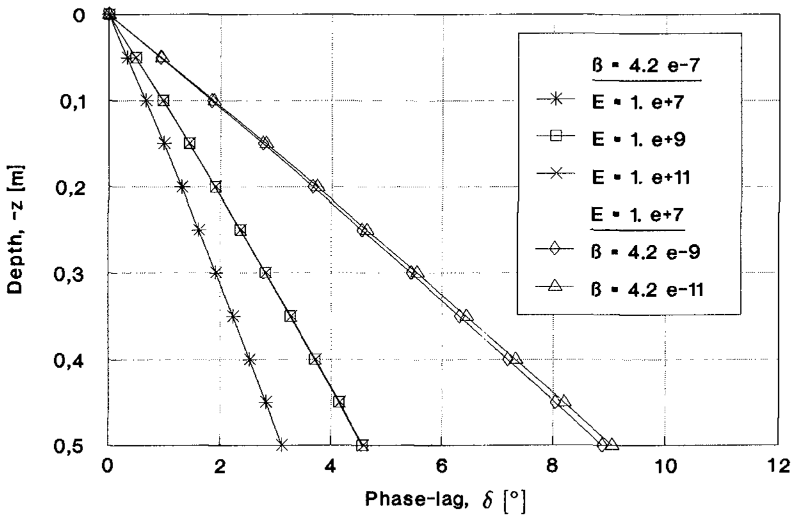


Figure 2. Test on numerical accuracy – pore-pressure amplitude and phase-lag distribution with depth for fully saturated soil conditions ($S = 1.00$) (finite-thickness-layer solution, rough basement, $\beta' E = 4.2 \times 10^{0}$ (2)(4) and $\beta' = 4.2 \times 10^{-7}$ m^2/kN or $E = 10^7$ kN/m^2)

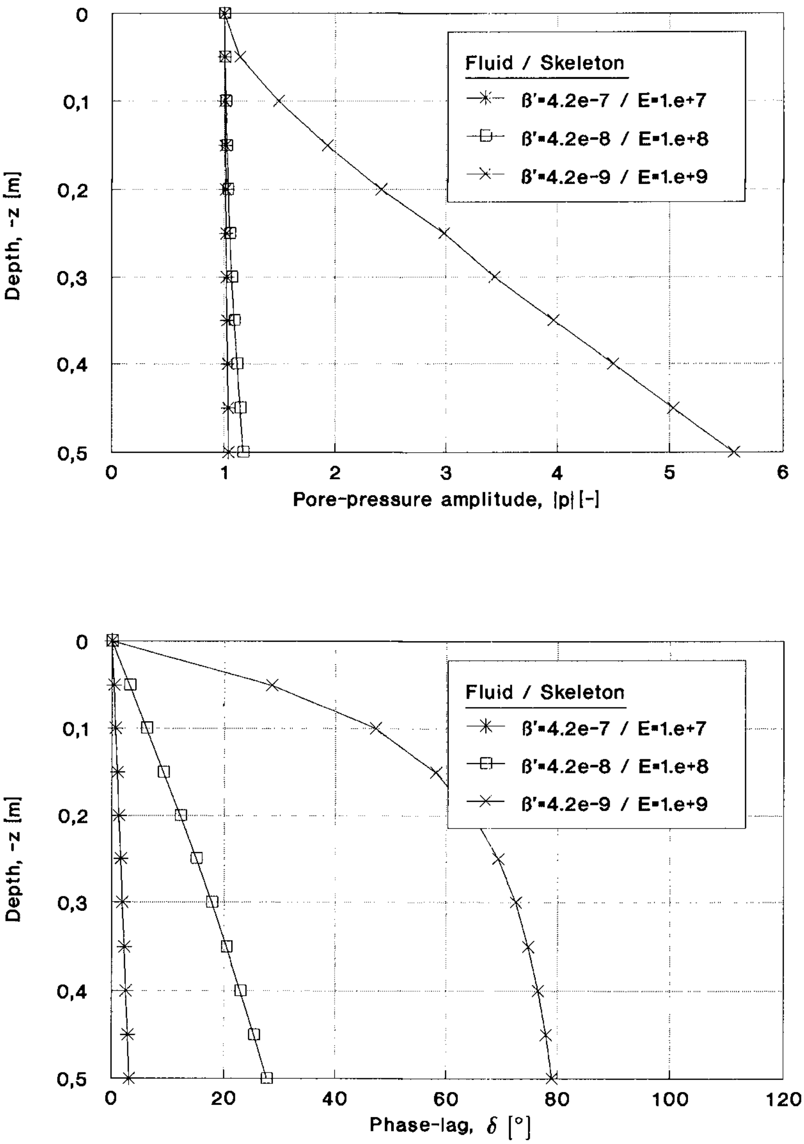


Figure 3. Test on numerical accuracy – pore-pressure amplitude and phase-lag distribution with depth for fully saturated soil conditions ($S = 1.00$) (finite-thickness-layer solution, rough basement, $\beta' E = 4.2$ (const) and $\beta' = 4.2 \times 10^{-7(-9)(-11)}\,\mathrm{m}^2/\mathrm{kN}$)

Although performing all computations in a double-precision mode, neither of these methods could avoid the numerical problems discovered in Fig. 1, confirmed in Fig. 2 and strong emphased in Fig. 3.

Figure 2 illustrates that by assuming a constant value of one of the compressibility parameters (*i.e.*, the compressibility of pore-fluid $\beta' = 4.2 \times 10^{-7}$ m^2/kN or Young's modulus of soil skeleton $E = 10^7$ kN/m^2) and enlarging a value of the second one, the numerical problems do not leed to a dramatical increase of both the pore-pressure amplitude and the phase-lag, which show to be coherent and vary: amplitude – from 1.04 to 1.05, phase-lag – from 3° to 9°, computed at the bottom of seabed layer. A simultaneous increase of both the compressibility parameters (see Fig. 3), however, brigs both the pore-pressure amplitude and the phase-lag onto an enormous and unrealistic level (amplitude – from 1.04 to 5.5, phase-lag – from 3° to 80°, computed at the bottom of seabed layer).

CONCLUSIONS

Taking into account the results of the above presented analysis, it is a rather lucky coincidence that the realistic values of compressibility of the pore-pressure and soil skeleton are: $\beta' \geq 4.2 \times 10^{-7}$ m^2/kN and $E \leq 10^5$ kN/m^2, respectively. These conditions guarantee that the order of maximum ratio between the absolute smallest and largest value in the coefficient matrix **D** will be always less than *ca* 10^{14}. Only by a near-incompressible system (*i.e.*, $\beta' = 4.2 \times 10^{-7}$ m^2/kN for fully saturated soil conditions $S = 1.00$, and $E = 10^5$ kN/m^2) some numerical problems are expected as indicated in Fig. 1. It was shown, however, that even in this case, the difference between the computed pore-pressure amplitude and the logically expected values (*i.e.*, a vertical-profile distribution with depth) have no practical meaning and can be neglected.

There is, however, another method that helps to solve the governing problem, correctly, omitting simultaneously the above mentioned numerical problems when solving the system of coupled linear equations. This method, based on a finite-element approximation of one-dimensional model for wave-induced excess pore-pressures and displacements in the soil skeleton, allows to omitt a bit artificial boundary conditions (*i.e.*, $\sigma_z = 0$ and $\tau = 0$) assumed at the seabed surface, and gives more realistic picture of the pore-pressure field in permeable seabed sediments. The results of such a numerical analysis was published by Magda (1991, 1992).

The preparation of all elements [Eqs. (7.1a) to (7.6f)] from the coefficient matrix **D** in the analytical solution requires much more mathematical operations with relatively small and large values before the equations system is solved. This complicated and superfluous procedure is omitted in the finite-element solution where the elements of coefficient matrix are computed using less mathematical operations.

ACKNOWLEDGEMENTS

This study has been financially supported by grants from the German Research Foundation (DFG) directed to the Special Collaborative Programme – 205 "Coastal Engineering" (Sonderforschungsbereich – 205 "Küsteningenieurwesen") at the University of Hannover as well as Grant No. 772629102 "Wave-induced excess pore-pressure oscillations in seabed sediments – computational methods for evaluating the hydrodynamic loadings acting on a submarine buried pipeline" at the Technical University of Gdańsk (Marine Civil Engineering Department).

REFERENCES

Madsen, O.S. (1978). Wave-induced pore pressures and effective stresses in a porous bed. *Géotechnique*, 28, No. 4, pp. 377-393.

Magda, W. (1989). Wave-induced pore water pressure generation in a sand bed. *Internal Report No. 1, SFB-205 'Küsteningenieurwesen', TP A13*, University of Hannover, pp. 1-94.

Magda, W. (1991). A finite-element approximation of the one-dimensional model for wave-induced pore-water pressure and displacements within the seabed. *Internal Report No. 8, SFB-205 'Küsteningenieurwesen', TP A13*, University of Hannover, pp. 1-89.

Magda, W. (1992). *Wave-Induced Pore-Pressure Oscillations in Sandy Seabed Sediments.* Ph. D. Thesis, Technical University of Gdańsk, pp. 1-160.

Magda, W. (1992). Finite-element approximation of one-dimensional model for wave-induced pore pressure and displacements within seabed sediments. *Archives of Hydroengineering*, Vol. XXXIX, No. 3, pp. 58-77.

Moshagen, H., Tørum, A. (1975). Wave-induced pressures in permeable seabeds. *Proc. ASCE. Journal of Waterways, Harbors and Coastal Engineering Division*, Vol. 101, No. WW1, Proc. Paper 11099, pp. 49-57.

Okusa, S. (1985). Wave-induced stresses in unsaturated submarine sediments. *Géotechnique*, 35, No. 4, pp. 517-532.

Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.T. (1989). *Numerical Recipes. The Art of Scientific Computing.* Cambridge University Press.

Putnam, J.A. (1949). Loss of wave energy due to percolation in a permeable sea bottom. *Transactions American Geophysical Union*, Vol. 30, No. 3, pp. 349-356.

Verruijt, A. (1969). *Elastic storage of aquifers.* Chap. 8 in: *Flow through porous media.* De Wiest (ed.), Accademic Press, London, pp. 331-376.

Yamamoto, T., Koning, H.L., Sellmeijer, H., Hijum, E. (1978). On the response of a poro-elastic bed to water waves. *Journal of Fluid Mechanics*, Vol. 87, Part 1, pp. 193-206.