CHAPTER 3

THE EFFECTS OF CURRENTS ON ESTIMATIONS OF DIRECTIONAL WAVE SPECTRA

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Abstract

To investigate the effects of currents on estimations of directional wave spectra, the problems with the existing methods for obtaining directional wave spectra are discussed when they are applied to wave fields in the existence of currents. The characteristics of distortion of the estimated spectra by current effects are examined through numerical simulations and analyses of experimental data. Both the numerical and experimental tests with wave gages, show that the directional spread of estimated spectra becomes narrower than that of the actual wave field and the value of the spectral peak is overestimated for the case of adverse currents. The extent of these distortions depends on the relative speed and direction of the current and waves. The relation of the error and these factors is summarized in the present study.

Introduction

Several methods have been developed for estimating directional wave spectra. These techniques, however, are generally restricted for the analysis of directional random sea waves in the absence of currents. When conventional methods are applied to wave fields with currents, misinterpretations of the actual phenomena may result. Methods are, therefore, needed to evaluate how currents will modify conventional directional spectral analyses so that corrective measures can be taken. This paper discusses the limitations associated with the existing methods for computing directional spectra of waves on currents. The effects of uniform currents on spectra are then estimated using numerical simulation techniques and these results are compared with the estimated directional spectra of wind waves propagating on uniform currents in an experimental channel.

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Estimation of Directional Wave Spectra on Currents

(1) *Existing methods*

The basic equation for estimating directional wave spectra is described as the simultaneous integral equation,

$$\Phi_{mn}(f) = \int_0^{2\pi} H_m(f,\theta)\cdot H_n^*(f,\theta) \times\left[\cos\{k(x_{mn}\cos\theta + y_{mn}\sin\theta)\} - i\sin\{k(x_{mn}\cos\theta + y_{mn}\sin\theta)\}\right]\times S(f,\theta)d\theta, \tag{1}$$

where $\Phi mn(f)$ is the cross power spectrum between the m-th and n-th wave motion parameters, $H(f,\theta)$ is the transfer function from surface elevation to other wave motion parameters and * denotes their complex conjugates. $S(f,\theta)$ is the directional spectrum, $xmn = xm - xn$, and $ymn = ym - yn$.

Based on some assumptions and approximations to directional distribution functions in Equation (1), the existing methods try to determine the unique solution of a directional spectrum with a limited number of wave motion parameters (See e.g. Hashimoto *et al.* 1994). In the process of getting a solution with these methods, the wave numbers and the transfer functions of wave components with arbitrary wave frequency are evaluated by using linear wave theory, and do not consider the presence of currents. Therefore, simply applying the existing methods to the analysis of the combined wave-current field may not produce accurate results. The following points have to be noted when analyzing such cases.

(2) *Dispersion Relation in Currents*

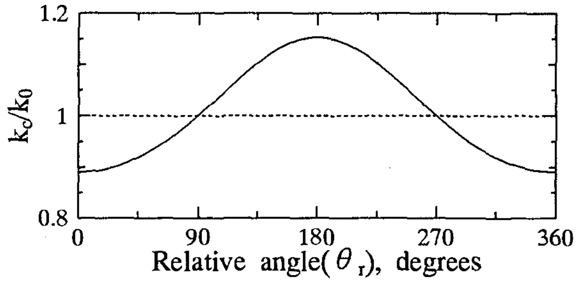
When the analysis is carried out with wave records measured at separate locations, e.g. a measurement with an array of wave gages, the wave number has to be put into Equation (1) corresponding to arbitrary frequency. In order to do this, the existing methods use the following equation as a dispersion relation,

$$\omega^2 = (2\pi f)^2 = gk_0 \tanh k_0 h , \tag{2}$$

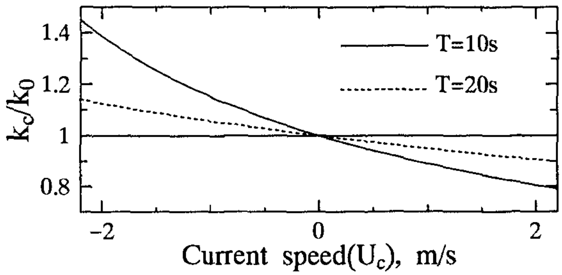
where $k0$ is the wave number, and h is the water depth. The subscript 0 denotes in the absence of currents. This relation is, however, not appropriate in the wave field on currents. For the simple case, assuming the propagation of waves in a uniform current in space and time, the relation becomes,

$$(\omega - U_c k_c \cos\theta_r)^2 = (2\pi f - U_c k_c \cos\theta_r)^2 = gk_c \tanh k_c h , \tag{3}$$

where Uc is the speed of the uniform current, and θr is the relative angle between the directions of current and wave propagation. The subscript c denotes in the presence of currents. This equation shows that, the wave number for a wave of given frequency depends on the direction and the speed of the current. **Figure 1(a)** shows this dependence of the change in wave number with the relative direction of the current. Here the wave period is assumed to be 10 s, the water depth 100 m and the speed of the current 1.0 m/s. In the region where adverse currents are dominant, around θ_r = 180°, the wave numbers become larger than that of the wave of the same period and depth condition in the absence of a current. The computation of wave numbers with Equation (2), therefore, can not describe the distribution of the wave number correctly in the current field. This is one source of inaccuracies in present methods. These deviations of wave numbers are also governed by the speed of currents in relation to the wave celerity as shown in **Figure 1(b)**. In the figure, where a negative current speed means adverse current, the deviation of the wave number increases with the magnitude of the current speed. The effect of the currents is significant for waves of higher frequency.



(a) Variation of wave numbers in relation to the relative angle



(b) Variation of wave numbers in relation to current speed

Figure 1. Deviations of wave numbers by the effect of currents

(3) *The Transfer Functions*

As a set of data for the analysis, several wave motion parameters measured at the same location are also used for the computation of directional wave spectra. In this case, the appropriate transfer functions from free surface elevation to other wave properties must be evaluated for the estimation procedure with the existing methods. For example, the transfer function to the horizontal particle velocities induced by the wave motion is expressed as,

$$H_0(f,\theta) = \frac{gk_0}{\omega} \frac{\cosh k_0(h+z)}{\cosh k_0 h} , \tag{4}$$

where z is the vertical coordinate with positive value taken upwards from the mean water level and k_0 is determined by the relation of Equation(2). Assuming again that the current is uniform, the expression becomes,

$$H_c(f,\theta) = \frac{gk_c}{(\omega - U_c k_c \cos\theta_r)} \frac{\cosh k_c(h+z)}{\cosh k_c h} , \tag{5}$$

where the relation between frequencies and wave numbers, k_c, is based on Equation (3). Since the wave numbers are modified by the currents, as mentioned in the previous

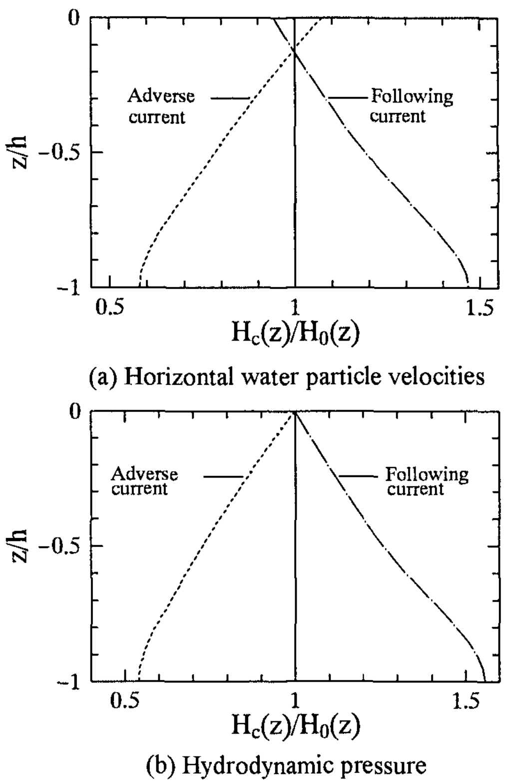


Figure 2. Deviations of transfer functions by the effect of currents

section, the transfer function also changes depending on the relative speed and direction of the current to the direction of wave propagation. The deviation of the transfer function of horizontal water particle velocities with frequency in the adverse and following current cases are shown in **Figure 2(a)** as a ratio of the function in currents, $Hc(z)$, to that in no current, $H_0(z)$. In the figure, it is seen that the deviation of the functions are maximum at the bottom, and the trend decreases towards the surface, and reverses just below the free surface. This means that estimations based on data of horizontal water particle velocities may be different depending on the depth of the measuring instruments such as a current meter.

In the case of the transfer function of the hydrodynamic pressure it is expressed, in the absence of currents, as,

$$H_0(f,\theta) = \rho g \frac{\cosh k_0(h+z)}{\cosh k_0 h} \ . \tag{6}$$

It can be shown that the hydrodynamic pressure transfer function in a uniform current is identical to Equation (6), with wave number and frequency governed by Equation (3). Their ratio is compared in **Figure 2(b)** for the same condition as **Figure 2(a)**. The maximum deviations by currents are at the bottom in the same manner as the horizontal water particle velocities. The transfer function ratio in a following current is greater than one and less than one in an adverse current, at all depths. Therefore the estimation, by existing methods which do not consider these differences, may also lead to inaccurate results of the directional wave spectrum.

Numerical Simulations

(1) Procedure of Simulation

To investigate the effects of currents on the directional spectrum estimation, a series of numerical simulations was performed by using an existing estimation method, known as Extended Maximum Entropy Principle method, EMEP by Hashimoto *et al.* (1994). The numerical tests were conducted in the same manner as the examination of another estimation method, Extended Maximum Likelihood Method, EMLM by Isobe *et al.* (1984). Prior to the estimation of the directional spectrum in the simulation, the cross power spectra of wave motions with target frequency are computed by numerical integration of the basic equations expressed by Equation (1). The directional spreading function is given arbitrarily as a model function. In the present study, since the objective wave field is supposed to be in uniform currents, the dispersion relation expressed by Equation (2) is used to obtain the cross power spectra. Based on the computed cross power spectra for the assumed wave field with currents, the analyses with the existing method are used to estimate directional spectra. Since the presence of currents is ignored at this stage of the estimating procedure, the estimation results may be distorted from the model function due to the incompatibilities of the existing methods, as mentioned in the previous section. In the following sections, some estimation results and their comparisons with the model function are demonstrated.

(2) Results of Simulation

A 10 s wave propagating in a uniform current of 1.0 m/s, in a water depth of 100 m was used as the target wave field for the simulation. The model function is expressed here as a conventional cosine-powered function,

$$G(\theta) = \cos^{2S}\left(\frac{\theta - \theta w}{2}\right) , \tag{7}$$

where S is the spreading parameter and θw the principal wave direction. The estimated directional spreading functions by the EMEP for the above condition are shown in **Figure 3**. A star array consisting of four wave gages is assumed as the simulated observation condition. In the case of adverse currents to the principal wave propagation, the estimated directional spectrum shows a narrower distribution and

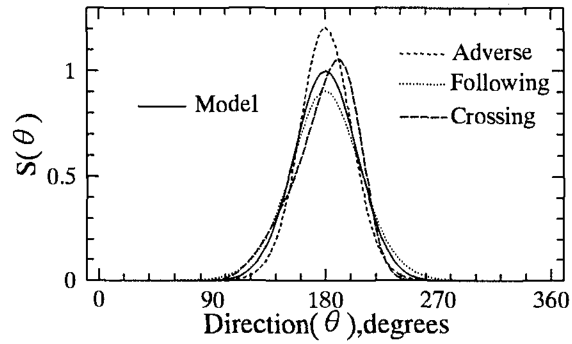


Figure 3. Estimated directional wave spectra by EMEP for various current directions using star array

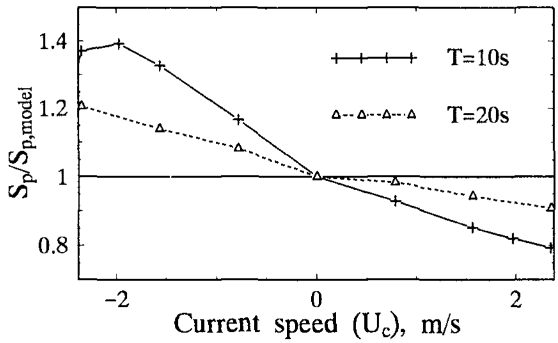


Figure 4. Relative errors of estimated peak value in relation to current speed

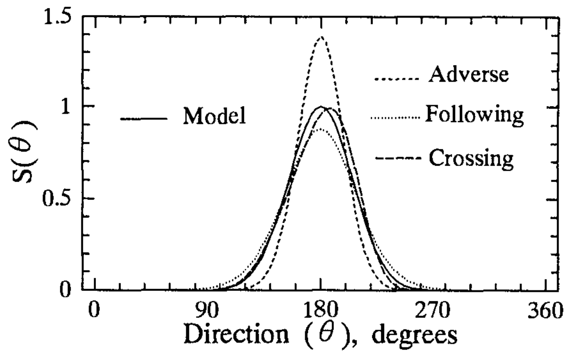
higher peak value than the model function which is represented by the solid line. The opposite trend appears in the following current case. When the direction of wave propagation and the current direction are at right angles, the peak of the estimated function is shifted downstream of the current but the effect of this current is considerably less than the adverse and following current cases. The over- and under-estimation of the peak of the directional distribution in the adverse and following current cases, respectively, are shown in **Figure 4**. In relation to the change in the wave numbers by the effect of currents shown in **Figure 1(b)**, the error in estimation results increases with the speed of currents and is greater for higher frequency waves.

Similar results have been obtained for the case of the three measured quantities; sea surface elevation and two components of horizontal water particle velocity at the same location. These results are shown in **Figure 5(a)**, with the water particle velocities assumed to be measured at the free surface. **Figure 5(b)** shows the estimated result for the same condition as in **Figure 5(a)** except now the depth at which the particle velocities were measured is 20 m. It should be noted that the trend of the error in the estimated results (**Figure 5(b)**) are opposite to the results obtained in **Figure 5(a)**. For example when the spectrum is estimated for the following current case using water particle velocities measured at the surface, the estimated peak is lower and the shape is wider than the model. For the same current scenario using water particle velocities measured 20 m below the surface, the estimated directional spectrum has a higher peak and is narrower than the model. This is ascribed to the deviation of the transfer functions and their vertical distribution in the presence of currents as shown in **Figure 2(a)**. The ratios of the transfer functions for adverse and following currents cross unity at a depth beneath the surface. This introduces a distortion of the estimated directional wave spectra depending on the depth at which the wave motion parameters were measured.

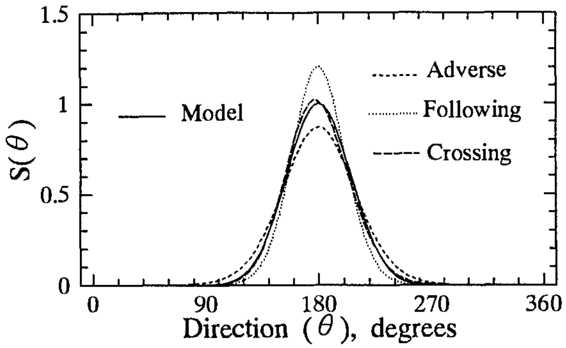
In shallow water regions near coasts, it is possible to use a measurement system at the sea bottom with sensors that measure hydrodynamic pressure and horizontal water particle velocities for directional spectra analysis. Assuming such observation conditions, directional wave spectra were estimated in cases for a wave period of 10 s, water depth 30 m and a current speed of 1.0 m/s. The results are shown in **Figure 6**. For reference, another estimated result is shown in **Figure 7** where the free surface elevation and horizontal water particle velocities at the surface were used as a measurement system for the same conditions as for the previous case in **Figure 6**. By the comparison of these figures, it is clear that the errors in **Figure 6** are extremely large. The reason of the large errors is that the deviation of the transfer functions for the wave properties are maximum at the bottom as explained in the previous section.

(3) Simulations with Consideration of Currents

As shown in the previous section, when the existing methods are applied to the wave field with currents, the estimated directional spectra are distorted by the effects of the current. Under the assumption of a uniform current, if the current speed is known, the wave numbers and the transfer functions can be modified appropriately by using Equations (3) and (5). The results of simulations using these modifications for the same following current conditions as **Figure 3** are shown in **Figure 8** by the dots.



(a) Using horizontal water particle velocities at z = 0 m



(b) Using horizontal water particle velocities at z = -20 m

Figure 5. Estimated directional wave spectra by EMEP for various current directions

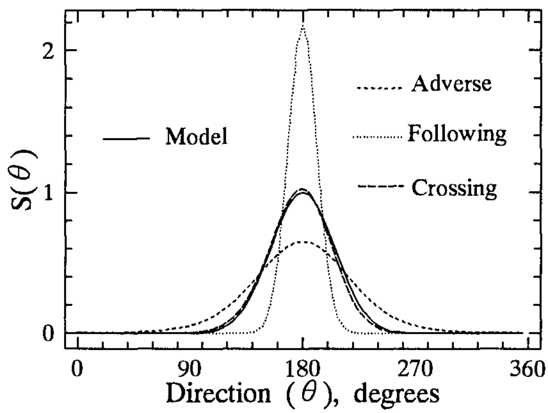


Figure 6. Estimated directional wave spectra by EMEP for various current directions using u, v, and p at the bottom

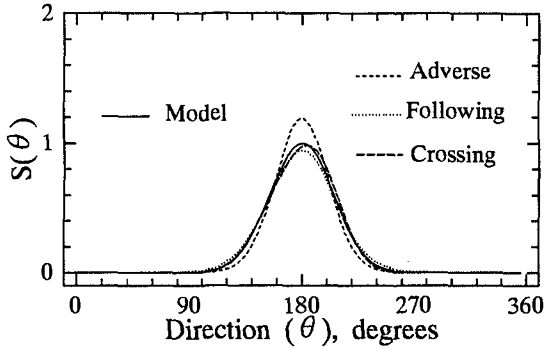


Figure 7. Estimated directional wave spectra by EMEP for various current directions using u, v, at the mean water level and η

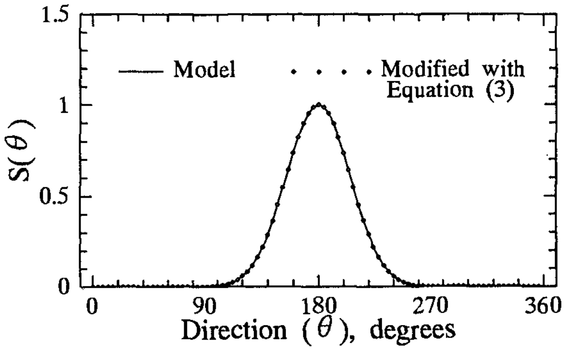


Figure 8. Estimated directional wave spectrum by EMEP with the modification of wave numbers by Equation (3)

It can be seen that the simulated results show almost complete agreement with the model. Simulations for adverse and crossing current scenarios were also conducted using Equation (3) with equally good agreements to the model.

In order to reduce the calibration error of field data measured at the same location, it was proposed by Isobe *et al.* (1984) to use the ratio between the power spectra of different wave properties instead of using theoretical expressions like Equation (7). The following expression was used for wave data to include surface elevation and the two components of horizontal water particle velocities,

$$H(\omega) = \sqrt{\frac{S_{uu}(\omega) + S_{vv}(\omega)}{S_{\eta\eta}(\omega)}} \,. \tag{8}$$

Here $S_{uu}(\omega)$, $S_{vv}(\omega)$ and $S_{\eta\eta}(\omega)$ represent the power spectra of horizontal water particle velocities, u and v and the surface elevation, η, respectively. The estimation result of the directional spectra for the same conditions as in **Figure 3** are shown in **Figure 9**, where Equation (8) is used for the determination of the transfer function. In each case, the degree of distortion is substantially diminished. The reason for this good agreement with the model function may be explained with **Figure 10**. In this figure, the transfer functions approximated with Equation (8) in the following and adverse current cases are indicated by the horizontal lines. Although they may not approximate the model transfer function in all directions, they come very close to the true transfer functions for the direction where the wave energy is highly concentrated. That is, for the estimation of directional spectra, the complete agreement with the theoretical function is not required for the directions where the wave energy does not exist. Equation (8) makes it possible to get reasonable transfer functions near the peak of the directional wave spectrum. Therefore, the analysis with Equation (8) is very effective in estimating directional spectra for wave fields with currents when the wave directionality is narrow.

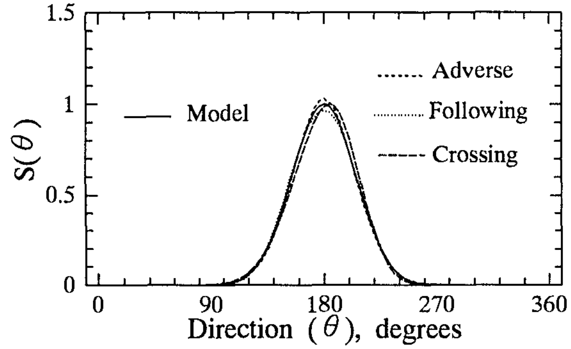


Figure 9. Estimated directional wave spectra by EMEP for various current directions with the transfer function approximated by Equation (8)

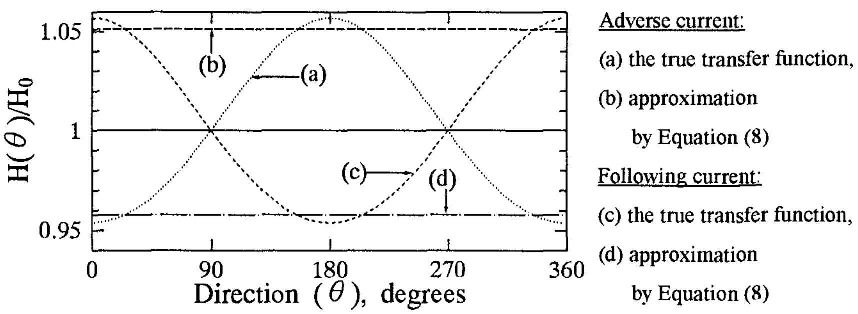


Figure 10. Approximation of transfer functions by Equation (8) for various currents

Analysis of Experimental Data

Estimations of directional spectra have also been carried out using the experimental data of directional waves in a wind-wave channel at the Port and Habour Research Institute (PHRI). The channel is 30 m in length, 1.5 m in width, and has a water depth of 0.5 m as shown in **Figure 11**. The waves were generated by wind blowing into the channel at a speed of 10.4 m/s at 40 cm above the still water level. Uniform currents were superimposed on the waves in following and adverse directions to the waves. For this analysis, surface elevations were measured with four wave gages installed at the location as shown in **Figure 11**. The sampling frequency was 50 Hz and the sampling duration was 2.5 minutes. The frequency power spectra of the wind waves on the currents are shown in **Figure 12**. The difference in peak frequencies of the spectra are

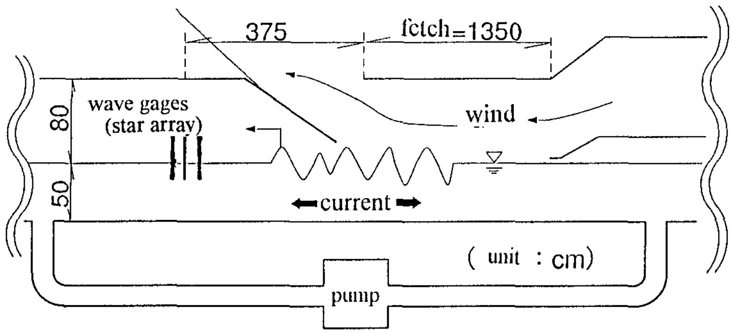


Figure 11. Sketch of wind-wave and current channel

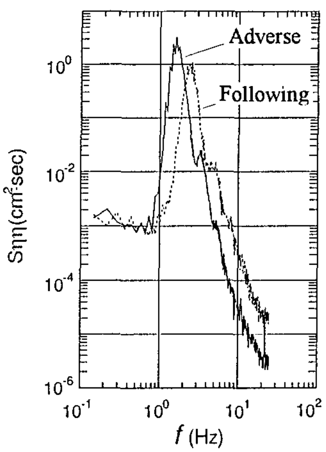


Figure 12. Power spectra of wind waves for various current directions

caused by the change in the effective fetch length (Kato and Tsuruya, 1978). By using these data, the analysis of directional spectra was estimated by EMEP. **Figures 13** and **14** show the results of estimated directional spectra near the peak frequency, for following and adverse currents, respectively. The dashed lines represent the estimated spreading function and the solid lines represent the results with the modification of wave numbers per Equation (3). In the following current case, **Figure 13**, the function estimated by EMEP predicts a wider distribution than the modified estimation. Under adverse current conditions, **Figure 14**, the estimated distribution has a narrower shape and a higher peak than the modified case. The trend of the results are the same as those obtained by the numerical simulations.

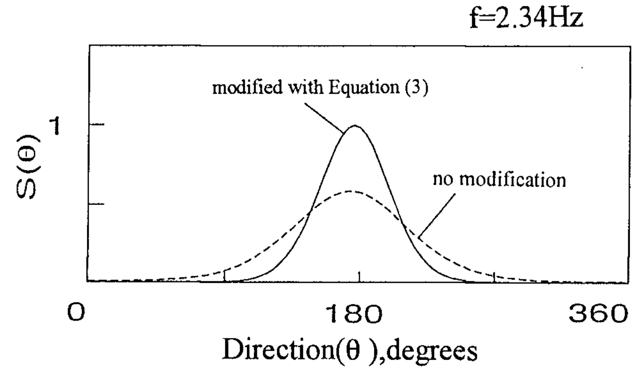


Figure 13. Estimated directional wave spectrum (following current)

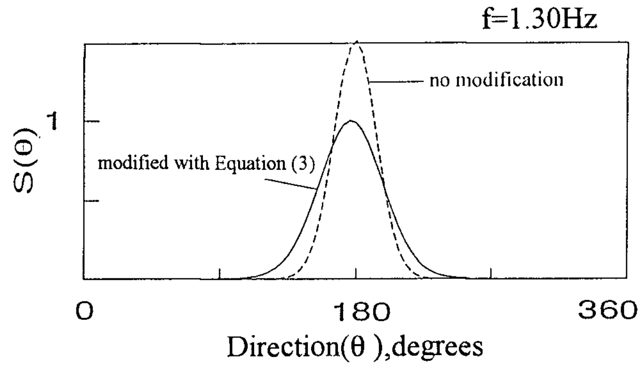


Figure 14. Estimated directional wave spectrum (adverse current)

Conclusion

Numerical simulations of directional wave spectra estimations for wave fields with currents have shown that the degree and trend of the distortion of wave spectra by the currents depends not only on the relative speed and direction of the waves and current but also on the measuring system of the wave properties. To reduce the errors of these estimation results, modifications of wave numbers and transfer functions by the theoretical relations for currents have been confirmed to be effective in uniform current cases. The transfer function defined as the ratio of each power spectra is particularly useful in reducing the error of estimated directional spectra in the presence of currents when the directionality of the waves is narrow. From analyses of experimental data with a wind wave channel, similar distortion trends to those estimated by the numerical simulations have been observed.

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