CHAPTER 38

New Optimization Method for Paddle Motion of Multi–Directional Wavemaker

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Abstract

Uniformity of monochromatic oblique waves in a wave basin is invcstigated. A new method, using non-linear least square formulation, to determine individual wave paddle motions of multi-directional wavemaker to improve uniformity, is proposed. The possibilities and limitations of the method are discusscd. Trial computations and comparisons with experiments demonstrate the validity and usefulness of the method.

Introduction

It is of prime importance to reproduce the desired oblique planar wave train in a wave basin. If an accurate and simple method were available for this purpose, directional random waves could also be reproduced through superposition of various oblique waves for constituent elements. Up to now, however, monochromatic oblique waves generated by a multi-directional wavemaker using the conventional snake principle (Biesel, 1954) have shown considerable spatial variations in wave height and wave propagating direction due to the finite length of the entire wavemaker and the finite width of each paddle (Takayama, 1982).

To reduce these spatial variations, Dalrymple (1989) presented a theory which utilizes reflection from the sidewall. Although his method is widely employed, if a reflective structure is to be tested using it, significant re-reflection will occur from the sidewall and the resultant wave field will become contaminated. In this context, a method which does not use sidewall reflection is quite promising. This kind of approach was first proposed by Ishida and Watanabe

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(1984) for a discrete-type multi-directional wavemaker under the condition of normal incidence. They showed that such spatial variations can be suppressed by controlling the amplitude of some of the wave paddles at both ends of the wavemaker without using the sidewall reflection. Mizuguchi (1993,1994) established a theory for continuous-type multi-directional wavemaker and proposed a general method to produce a uniform wave field for waves of oblique incidence. Toita *et al.* (1994) validated his method experimentally. The method employs linear decrease of paddle amplitude at both ends of the wavemaker while the conventional method uses a constant paddle amplitude. Although this method is very successful, the total length of the wave board to be controlled must be equal to one wavelength for the best result. The method is thought to be somewhat empirical. It cannot specify the target area in the basin, and the experimental work does not refer to uniformity of the wave propagating direction. Hanzawa *et al.* (1994) applied an electromagnetic wave theory for the generation of oblique waves. The applicability of this was investigated numerically using the boundary element method presented by Isaacson (1989). They suggested that the Dolph-Chebyshev distribution (Dolph, 1946), which was originally derived to determine the current distribution feeding to broadside antenna array and is optimum to minimize the beam width when a side lobe level is specified, showed possibility to improve the uniformity of an oblique water wave field.

In this paper, a new optimization method is proposed to determine each wave paddle motion to improve the uniformity of the monochromatic oblique wave field without using sidewall reflection. First, the amplitude of each wave paddle is determined using a non-linear least square method instead of the intuitive linear decrease proposed by Mizuguchi. Second, the wave heights and water particle motions are computed to investigate the generated wave fields. Finally, the method is validated by comparing computed and experimental wave fields. In comparison to Mizuguchi's method, the proposed method improves the uniformities of both the wave propagating direction and wave height, and the method can specify the location of the target area in the basin arbitrarily.

Formulation of the Problem

A wavemaker of continuous piston type is assumed to be installed in constant depth in a very wide wave basin with negligible sidewall reflection. In the following linear wave analysis, $2N$ is the total number of driving rods, a_i is the amplitude of the i-th rod, M is the total number of reference points distributed within the reference area, and A_j is the wave amplitude at the j-th reference point. Within the reference area, the uniformity of the wave field is improved. In this study, the magnitude of a_i is determined as the optimum value while the phase difference between adjacent paddles is simply decided in accordance with the conventional snake principle. Fig.1 gives a schematic view of the formulation.

The Hankel function gives a unit solution of the Helmholtz equation, expressing waves spreading from a point source over a horizontal bottom. Linearity of the equation permits an arbitrary superposition of such unit solutions for multiple sources distributed along the wave paddles. A particular solution is obtained by determining the alignment of the point sources so that the resultant oblique wave field has a uniform wave height distribution within a given area in the basin. Fig.2 shows the coordinate system and waves emitting from a point source on the wave board.

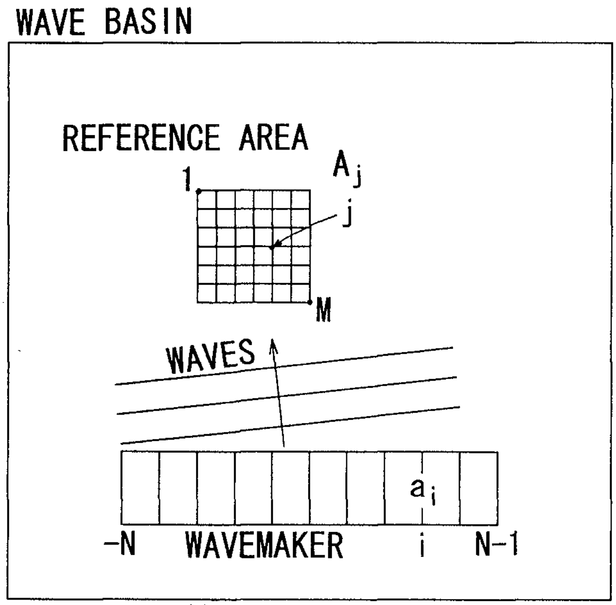

Figure 1: Schematic view of the formulation.

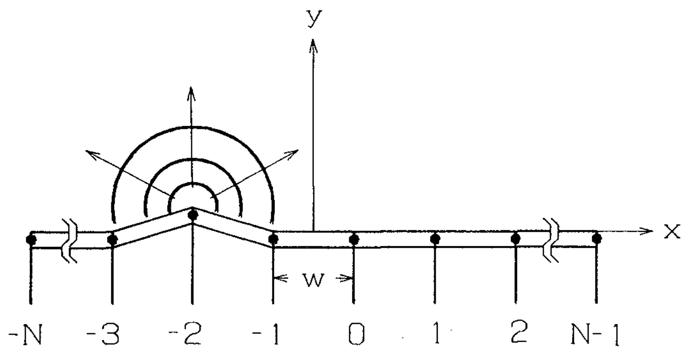

Figure 2: Coordinate system.

Neglecting the evanescent mode waves, the water surface profile at the j-th reference point (x_j, y_j) can be written as follows (Mizuguchi, 1993).

$$\eta_j = \alpha\sqrt{E_s^2 + E_c^2}\cos\left\{\arctan\left(\frac{E_s}{E_c}\right) - \sigma t\right\} \tag{1}$$

The wave amplitude at the j-th reference point normalized by the representative amplitude of the driving rod, b_0, is thus expressed as

$$A_j = \frac{\alpha}{b_0}\sqrt{E_s^2 + E_c^2} \tag{2}$$

where

$$\alpha = \frac{2\sinh^2 kd}{\sinh 2kd + 2kd} \tag{3}$$

and

$$E_s = \sum_{i=-N}^{N-1} a_i \cos\beta \left\{N_{ij}\cos\left(ikw\sin\beta\right) - J_{ij}\sin\left(ikw\sin\beta\right)\right\} \tag{4}$$

$$E_c = \sum_{i=-N}^{N-1} a_i \cos\beta \left\{N_{ij}\sin\left(ikw\sin\beta\right) + J_{ij}\cos\left(ikw\sin\beta\right)\right\} \tag{5}$$

$$N_{ij} = \int_{-kw}^{kw}\left(1 - \frac{|q|}{kw}\right) N_0\left(\sqrt{\left(k\left(x_j - iw - \frac{w}{2}\right) - q\right)^2 + (ky_j)^2}\right) dq \tag{6}$$

$$J_{ij} = \int_{-kw}^{kw}\left(1 - \frac{|q|}{kw}\right) J_0\left(\sqrt{\left(k\left(x_j - iw - \frac{w}{2}\right) - q\right)^2 + (ky_j)^2}\right) dq \tag{7}$$

in which, t is the time, σ is the angular frequency, k is the wave number, d is the water depth, β is the wave propagating direction measured counterclockwise from the y-axis indicated in Fig.2, w is the width of a wave paddle, and N_0 and J_0 denote the Bessel and Neumann functions of zeroth-order. In Eq.(6) and (7), when i=$-N$ or i=$N$$-$1, the integration is performed in the range of $[0, kw]$ or $[-kw, 0]$ respectively.

Denoting the target wave amplitude normalized by the representative amplitude of the driving rod, b_0, by A_c, the total residual squared is given by

$$r^2 = \sum_{j=1}^{M}\left[A_c - A_j\right]^2 \tag{8}$$

The problem can then be regarded as a non-linear least square problem. The optimum amplitude of each rod is obtained by finding the values of a_i which minimize r^2. In this study, as expressed in Eq.(9), A_c is taken to be 1/2 of the wave generating efficiency of a two dimensional piston type wavemaker.

$$A_c = \frac{2(\cosh 2kd - 1)}{\sinh 2kd + 2kd} \tag{9}$$

Marquardt's method is suitably employed to solve Eq.(8) because of its stability in computation. Since this type of approach allows arbitrary collocation of rcference points, it makes experiments more flexible.

Optimum Distribution of Driving Rod Amplitude

The general description of the least square problem is to find a set of N unknown parameters

$$(x_1, x_2, \ldots, x_N) \tag{10}$$

which minimizes the value of

$$r^2(\boldsymbol{x}) = \sum_{j=1}^{M} f_j^2(\boldsymbol{x}) \tag{11}$$

where $f_j(x_1, x_2, \ldots, x_N)$ $(j = 1, \ldots, M)$ are arbitrary functions of $\boldsymbol{x}$.

When $r^2(\boldsymbol{x})$ depends nonlinearly on a set of N unknown parameters x_i, $(i = 1, \ldots, N)$, the minimization must proceed iteratively. From the given initial trial values for the parameters, the calculation proceeds in a way that improves the trial solution. Such a procedure is then repeated until $r^2(\boldsymbol{x})$ reaches an equilibrium (Press *et al.*, 1992).

Table 1 and Fig.3 summarize the conditions and the setup for computation and experiments in this study. The wave basin was 50m long, 40m wide, 1.5m high. Wave absorbers placed along the side wall and the onshore slope were used to prevent waves from reflecting from the wall. The number of driving rods was 28 (the number of wave paddles was 27) and the width of a wave paddle was 0.9m.

Table 1: Experimental conditions.

Case	Water depth (m)	Wave period (s)	Wave direction (°)	Reference area
1	0.6	1.8	0.0	A
2	0.6	1.8	22.5	B
3	0.6	1.8	22.5	A

The reference area of the uniform wave field is the rectangular region of x=–4 to 4m, y=4 to 12m (Area A) and x=–9 to –1m, y=4 to 12m (Area B).

Within this area, 81 reference points are collocated at every 1m grid point. A uniform distribution of the driving rod amplitude is used for the initial values and the iteration count of the minimization procedure is one. Computation to calculate the wave amplitude at the j-th reference point in Eq.(8) is made in the manner presented by Mizuguchi under the assumption of small amplitude wave theory.

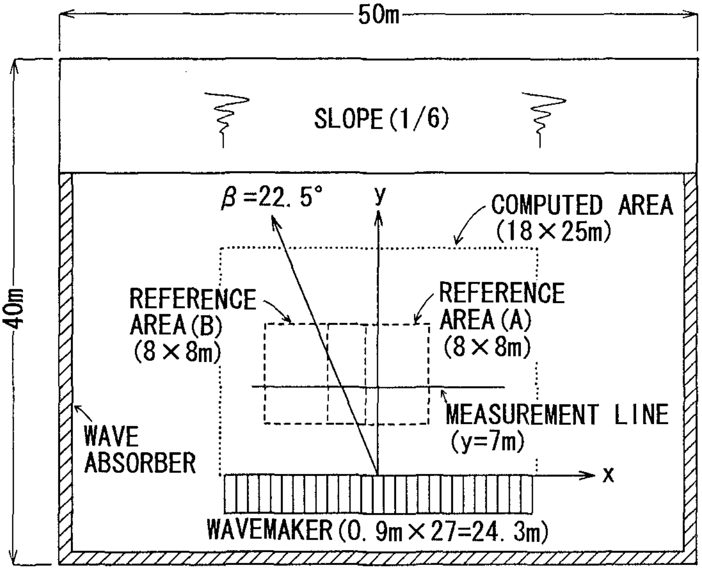

Figure 3: Wave basin.

Fig.4 gives the computed distributions of the rod amplitude for Case 1 and Case 2. This figure shows that the present method yields the distribution of the rod amplitude with high peaks at both ends of the wavemaker. The conventional method uses constant rod amplitude, while Mizuguchi proposed a linear decrease at both ends. These peaks, representing singularities at both ends, produce an improved uniformity of the wave field in the reference area. Such a non-uniform distribution of the point source intensities was correctly suggested by Nishimura *et al.* (1994) for the continuation condition at the artificial boundary of a wave field calculation.

The relationship between the iteration count of the minimization procedure and the values of residual squared for Case 1 is illustrated in Fig.5. This figure shows that only one time of iteration can reduce the value of residual squared up to approximately 1/100 of the original value. A preliminary investigation demonstrates that the uniformity of the wave field can be well improved by the distribution of the rod amplitude obtained by only one time of iteration. It is worth noting that not only the uniformity of the wave field in the reference area, but also that in its peripheral region can be widely improved to coincide with its target value because of the continuity of the fluid, and the fact that too much iteration count narrows the area of such a peripheral region. As this study stresses the development of the new method and the examination of properties of generated wave fields, hereafter the number of iteration counts will be fixed to one.

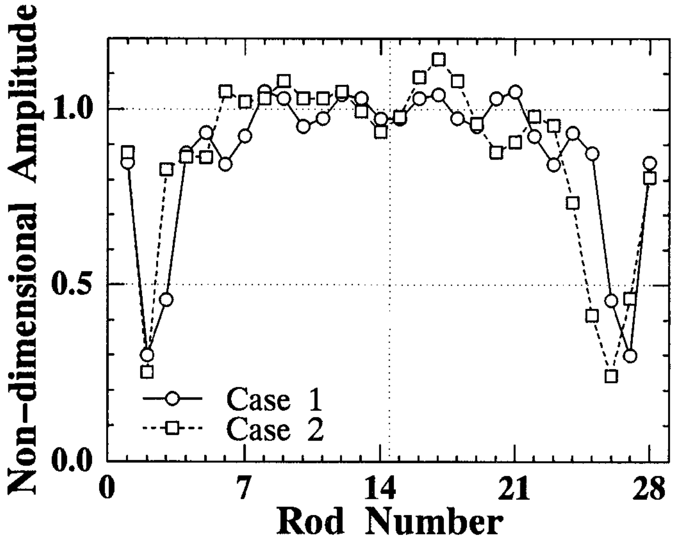

Figure 4: Computed distribution of the driving rod amplitude.

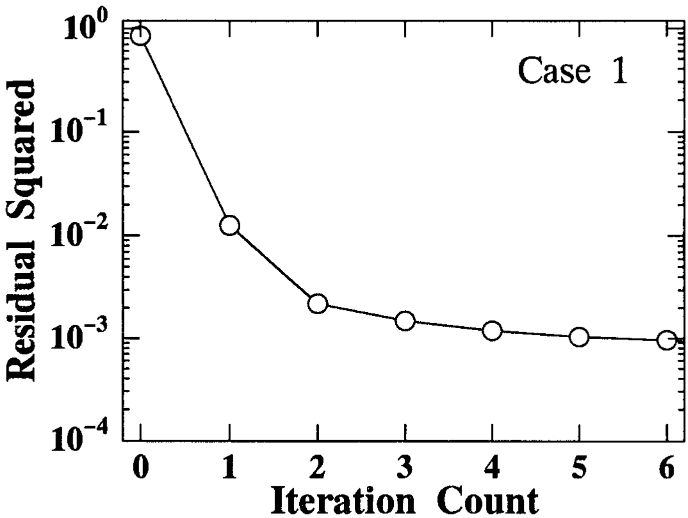

Figure 5: Changes of the residual squared.

Properties of Generated Waves

A series of computations on wave height, wave propagating direction and flatness of the hodograph of two component horizontal composed velocities were conducted to investigate the properties of oblique waves generated by the proposed method. Computations were carried out at every 50cm grid point over the area indicated in Fig.3

Fig.6 shows definitions of the wave propagating direction and the flatness of the hodograph. Because of the phase difference between one velocity component and the other, the path of the water particle does not follow a straight line but it forms an ellipse (Takayama, 1982). The wave direction is defined by the direction of longer radius. It should be noted that this definition gives a mean value of the wave direction because the wave direction at a fixed location varies from time to time. The flatness of the hodograph χ, is defined by using a longer and shorter radius as follows:

$$\chi = r_2/r_1 \tag{12}$$

where, r_1 is the longer radius and r_2 is the shorter one. The flatness of the hodograph indicates the magnitude of fluctuation of the wave propagating direction and is equal to zero if the wave field has complete uniformity.

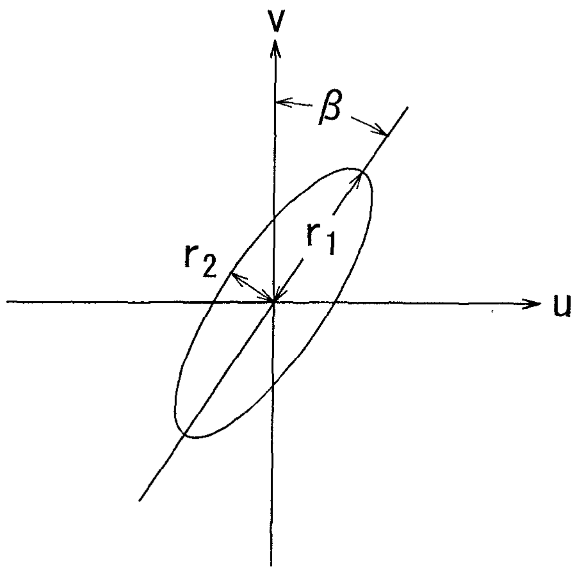

Figure 6: Trajectory of a water particle.

Fig.7 shows the computed distributions of the normalized wave height, the deviation of the wave propagating direction, and the flatness of the hodograph for Case 2. These results were obtained by the conventional method, Mizuguchi's method, and the proposed method. The normalized wave height was defined as the ratio of generated wave height to twice the amplitude of the driving rod. The target value is 0.97 according to the water depth and wave period used. The dark areas indicate that the normalized wave height is between 0.9 and 1.1, deviations of the wave propagating direction is between $-2.5°$ and $2.5°$, and the flatness of the hodograph is less than 0.05 respectively.

The wave fields computed by the conventional method display wavy features of considerable variations around 30% in the normalized wave height and the wave propagating direction differs by nearly $10°$ from the target values over a large area. Furthermore, the flatness of the hodograph shows the maximum value to be over 0.15. These wavy features are considered to be largely related to the phenomenon that the wave energy of the oblique uni-directional random waves have directional spreading and the multi-directional random waves show a spatial variation of the directional spectrum.

As shown in the figure, the proposed method reduced these wavy features to produce a wave field in which the error in the normalized wave height was less than 10%, the error in the wave propagating direction less than $2.5°$, and the error in flatness of the hodograph less than 0.05. It is interesting to note that although the design procedure for paddle amplitude was, as was described above, derived from the viewpoint of wave height, not only the uniformity of the wave height but also the uniformities of both the wave propagating direction and flatness of the hodograph were significantly improved simultaneously. This may be explained by using the transfer function between the time series of the water particle velocities and the water surface elevation. The uniformity of the wave field for Case 1 produced by the proposed method is also improved with favorable results. See Matsumoto *et al.* (1995) for the reference.

Mizuguchi's method also reduced the wavy features. However, it produced some spots of unacceptable errors. These may be ascribed to the fact that his method modifies only a few wave paddles at both the generator ends empirically while the proposed method controls all the paddle amplitudes as an optimum distribution.

The normalized wave height computed by the proposed method for Case 3 is indicated in Fig.8. In this case, the wave ray from the center of the whole wave paddle did not go through the center of the reference area. The figure demonstrates that, even in such a case, the uniformity of the wave field can be improved by the proposed method. This supports the idea that the proposed method has a high possibility to reproduce a desired directional random wave field, which does not show any spatial variation of directional spectrum, by the superposition of oblique waves.

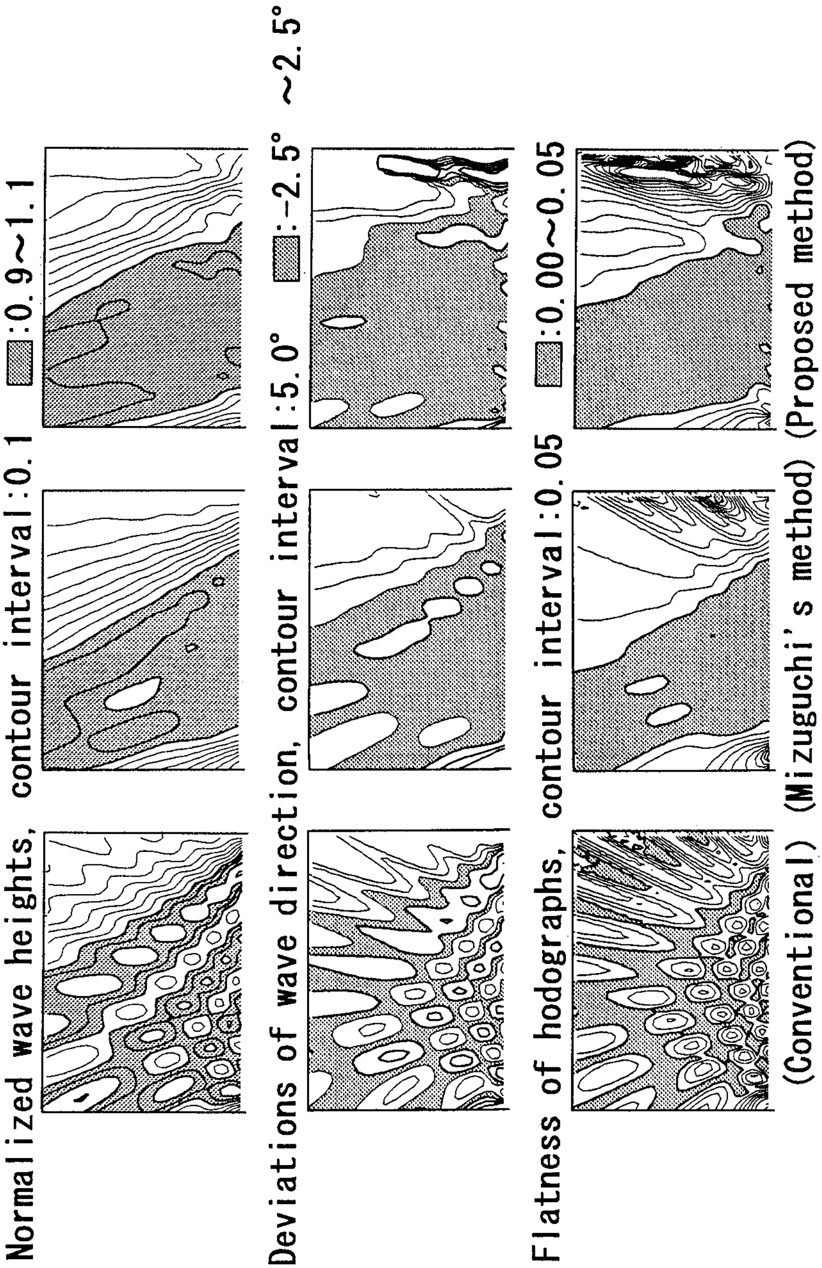

Figure 7: Computed wave fields (Case2).

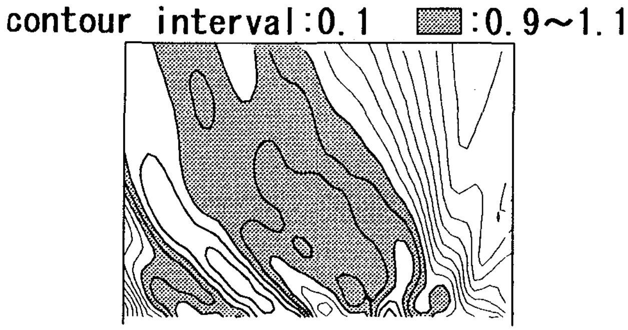

Figure 8: Computed normalized wave height (Case3).

Comparison with Experiments

Experiments were carried out to provide the wave height distribution and the water particle velocity to examine the performance of the proposed method. Equipment and experimental conditions are summarized in Fig.3 and Table 1. The representative amplitude of the driving rod was 3.7cm. Both the water surface elevation and two component horizontal water particle velocities were measured using capacitance type wave gauges and electromagnetic velocimeters along the measurement line (y=7.0m) as indicated in Fig.3. In the analysis, time serieses of the water surface elevation and water particle velocity before reaching the reflected waves from the sidewall were used.

Figs.9 and 10 compare the measured and computed distributions of the relative wave heights normalized with the target wave height for Case 1 and Case 2. In these figures, the loss of wave generating efficiency due to leakage of energy through the spacing between the wave board and the bottom of the wave basin, which was measured in the preliminary experiment, are taken into account. The agreement is good for both the proposed and conventional methods. A comparison of the measured and computed wave heights for Case 3 was also conducted. The measured wave heights agree well with those computed. This confirms the validity of the proposed method and the linear analysis of the wave field.

Figs.11 and 12 present the measured and computed hodographs of normalized horizontal velocities for Case 2 obtained at the location x=2.25m, y=7.00m, z=−0.15m. The diagonal dotted line shows the specified wave direction. The measured values follow computed ones quite well for both the conventional and proposed method. The water particle motions are also found to be reproduced well. Fig.12 indicates that the wave propagating direction coincides with its target direction. Moreover, the orbital curve becomes thinner, i.e., the magnitude of fluctuation of the wave direction decreases. These results confirm the validity of the proposed method from the viewpoint of the uniformity of wave direction.

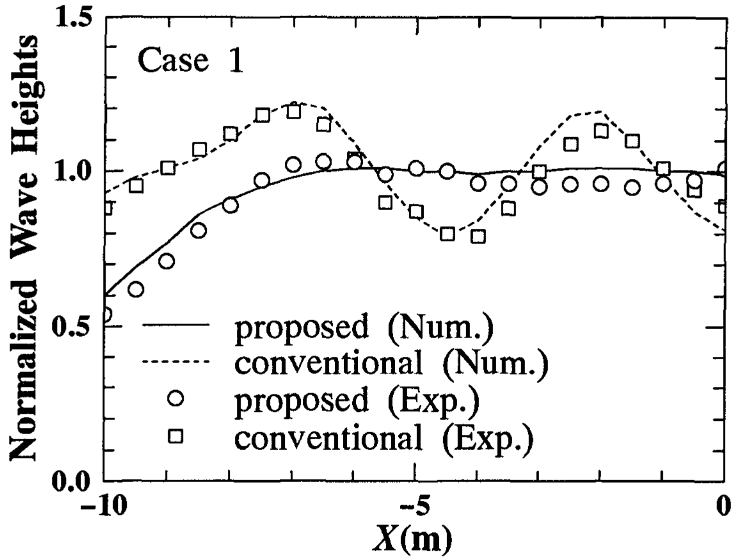

Figure 9: Measured and computed normalized wave heights (Case 1).

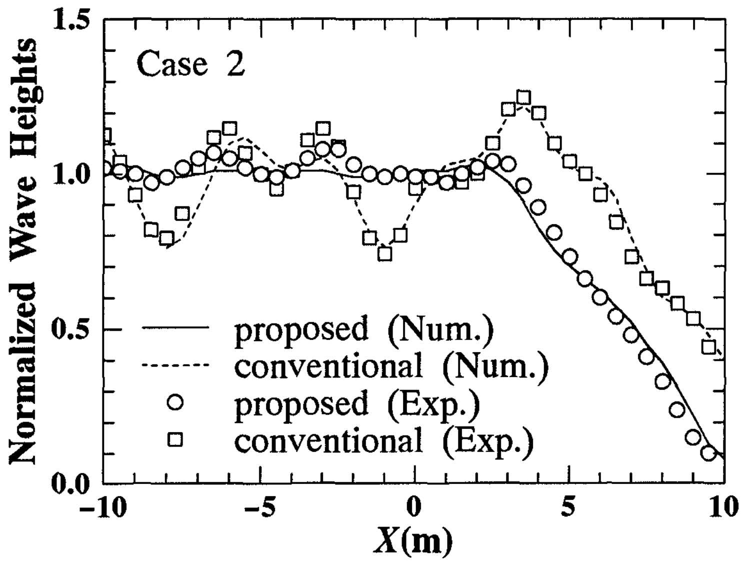

Figure 10: Measured and computed normalized wave heights (Case 2).

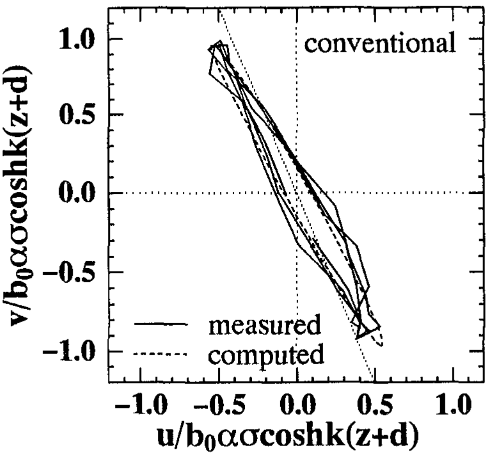

Figure 11: Hodographs of composed velocities (Case 2, Conventional method).

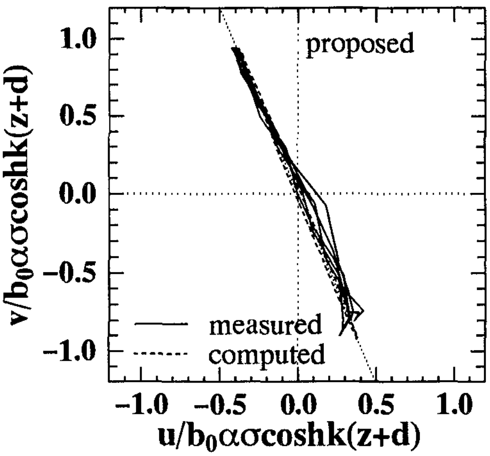

Figure 12: Hodographs of composed velocities (Case 2, Proposed method).

Concluding Remarks

Throughout this study, it was clarified that the method as presented actually improves the uniformity of both wave height and wave propagating direction. Since the method can specify the target area in the basin arbitrarily, it should prove very useful for applying to multi-directional random wave fields. As the method is formulated by using an inverse problem, it is applicable independent of the type of wavemaker and boundary conditions in the wave basin.

Acknowledgment

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