# CHAPTER 58

## WATER WAVE FLUCTUATIONS INDUCED BY IRREGULAR BATHYMETRY

Lulin Guo[1] and Robert A. Dalrymple[2]

**Abstract:** Small irregular water depth variations may cause relatively large variations of the wave field, which may affect the results of various water wave models. Statistical properties of depth variation are obtained from real bathymetry and idealized water depths are generated to study this influence on wave fields. An angular spectral model and a parabolic model are examined for their sensitivity to depth variations. It is shown that diffraction and nonlinear effects are dominant to the wave energy scattering. Correlation functions, especially correlation lengths, play an important role in the wave field variation.

## INTRODUCTION

In shallow water, waves are strongly affected by depth variations, yet the water depth near a site of interest is often not well known and may vary rapidly in time and space. Slight depth variations can cause relatively large variations of the wave properties particularly after significant propagation distance. Recent studies showed that some models are very sensitive to bathymetry variation; fluctuations of depth can lead to chaotic patterns in the wave field (Brown *et al.* 1991; ray tracing model).

An important input to numerical wave models is a bathymetric grid, containing digitized water depths. These digitized depths may contain errors caused by sounding errors, errors in digitizing the chart, unknown variation in depth between survey points and bottom variation since the sounding. These errors can be regarded as random depth variations. The influence of the random variations can be fairly large depending on the model, the initial wave field, the systematic bottom bathymetry, and the statistical properties of the random bottom variation. The statistical properties can be described by distribution function and correlation functions.

A variety of numerical models, which differ in their underlying theories and their numerical implementations, are used to simulate wave fields, . Results exist for a wave ray model, mild-slope equation, Boussinesq equations, an Eulerian

[1]Graduate student, Center for Applied Coastal Research, University of Delaware, Newark, DE 19716, USA.

[2]Professor and Director, Center for Applied Coastal Research, University of Delaware, Newark, DE 19716, USA.

model, which is a stationary model based on the action balance of short-crested waves ( Holthuijsen *et al.*, 1989 ), and some other models. The traditional ray tracing model does not consider diffraction and nonlinearity effects. It is not applicable for the study on wave fluctuations when these effects are important to be neglected. Other models contain effects of refraction, diffraction and nonlinearity, and their sensitivity to the random bottom variation are different. Numerical methods include finite element, spectral, pseudo-spectral and parabolic methods. The WANGLE model (Dalrymple *et al.* 1989) is an angular spectrum model to solve the modified mild-slope equation. It permits wide wave propagation angles which could appear in the situations when the refraction and diffraction of the waves are strong. The nonlinearity is included through the nonlinear dispersion relationship. The REF/DIF model (Kirby and Dalrymple, 1992) is a parabolic model originating from the mild-slope equation with the influence of current added. The sensitivity of these two models are examined.

Some studies of the influence of bottom variation have been done recently. Brown *et al.* (1991) showed that a ray tracing model is very sensitive to the bottom perturbations. When the bottom fluctuations are 20% of the mean water depth, the wave field can become chaotic. Holthuijsen and Booij studied the effects of the water depth variations using HISWA (Holthuijsen and Booij, 1989), a predictive model for stationary, short-crested waves in shallow water, in 1994. Their results showed that the bottom-induced fluctuation may be a serious problem and the effects of bottom variations on long-crested waves was more dominant than on short-crested waves. Reeve (1992) applied an analytical method and Monte Carlo simulations. Some quantitative results were obtained with the help of the angular spectrum of the bathymetry variation.

In this paper, we examine the influence of the random variability of the ocean bottom on waves in shallow water and the sensitivity of various numerical models to these depth variations. Some idealized bathymetries and the measured bathymetry near Duck, North Carolina is used for the numerical simulation. The REF/DIF and WANGLE models are examined. Statistical analyses of the bathymetry will be done. Bathymetries with the same statistical properties will be used to obtain the average wave properties and the fluctuations of the wave field. Results are compared with that of the traditional wave ray model. The relationship between the fluctuation in the wave field and the randomness of water depth is obtained.

## GENERATION of WATER DEPTH

As an exact complete measurement of real bathymetry is almost impossible, a statistical analysis of the properties of bathymetry, such as the distribution of the depths and the correlation length scales in different directions, are needed. For the numerical study, different water depth profiles with different statistical properties are needed to study the wave field response.

The depth variation can be described statistically by its distribution and its correlation function. First we generated depth variations with uniform distribution, whose correlation function is the Dirac delta function (white noise) on a rectangular horizontal grid. The standard deviation is controlled to be some ratio of the mean water depth. Such variations are not related to the horizontal distance as the depths are taken firstly to be uncorrelated, and they lead unfortunately to rapid changes of water depth between grid points. As the grid size is reduced, larger depth gradients occur.

We also generated depth variations with Gaussian distributions and smooth correlation functions. Figure 1.a shows the generated correlation function in a 2-dimensional case. Its form is roughly similar to that of the bathymetry in Duck, NC. Correlation length is defined by the length at which the correlation function has the value around 0.6. It describes the relationships among the randomly varied depths. We will compare the correlation length to wave length to determine if there is an effect (such as Bragg scattering) when they are similar. The following formulae are used to calculated the generated depth variations.

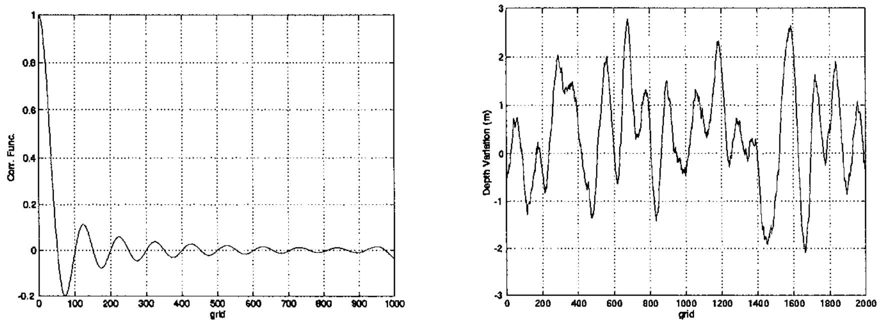



Figure 1: A sampled correlation function and a generated water depth variations for 2-D case. a). Correlation function; b). Water depth variation. The correlation length is about 30 grid sizes.

$$h(x) = \sum_{i=1}^{N} A_i \sin(i\Delta kx + \epsilon_i)$$

$$A_i = \sqrt{2F(c)_i}; \Delta k = 2\pi/L$$

where $N$ is the number of grids, the correlation function $c(x_1 - x_2) = < h'(x_1)h'(x_2) >$ is the ensemble average of $h'(x_1)h'(x_2)$ and it is assumed a function of $x_1 - x_2$, where $h'$ is the depth variation and $x_1$ and $x_2$ are two locations, $F(c)$ is the Fourier transform of $c$, $L$ is the length of the domain, and $\epsilon \in [0, 2\pi]$ is an uniformly distributed which serves as the random phase. Using different $\epsilon$, we can obtain different results of the same probability properties. Figure 1.b shows one of the results.

The 3D case is similar to that of 2D. A correlation function is used to generate the depth variation. Note that the correlation function has two arguments $x$ and $y$, and there are two correlation lengths, in $x$ and $y$ direction. We are going to compare these two lengths to the incident water wave length separately to see which one is more important. Figure 2 shows the results. The formulae used are

$$h(x, y) = \sum_m \sum_n A_{mn} \cos(k_{mx}x + k_{ny}y + \epsilon_{mn})$$

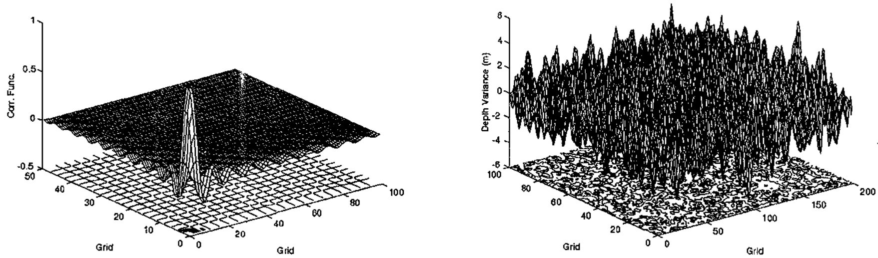



Figure 2: A sampled correlation function and a generated water depth variations for 3-D case. a). Correlation function; b). Water depth variation. The correlation lengths in the two directions are about 3 and 4.5 grid sizes, respectively.

$$A_{mn} = \sqrt{F(\mathrm{c})_{mn}}; kmx = \Delta k_x \cdot m; kny = \Delta k_y \cdot n; \epsilon_{mn} \in [0, 2\pi]$$

where $\Delta k_x = 2\pi/L_x$, $\Delta k_y = 2\pi/L_y$ and $L_x$ and $L_y$ are the size of the domain in $x$ and $y$ directions, respectively. The correlation function is $c(x_1 - x_2, y_1 - y_2) = < h'(x_1, y_1)h'(x_2, y_2) >$.

## NUMERICAL EXPERIMENTS AND RESULTS

Both REF/DIF and WANGLE models involve the effects of refraction, diffraction and shoaling. Weak nonlinearity can also be included. We examined the sensitivity of the two models, and then we studied the fluctuations of wave field response to different bottom changes. To demonstrate the importance of refraction and diffraction, we did two cases, one of which included only shoaling effects and the other included refraction, diffraction and shoaling effects. To study the model sensitivity, we compared Holthuijsen and Booij's results with our results obtained by REF/DIF and WANGLE models. Their results showed quantitatively that long-crested waves were affected more than short-crested waves by the bottom variation. In our cases, we only used plane waves for the comparison.

To study water wave fluctuations induced by the bottom randomness, we did some numerical experiments using the generated water depth and the measured bathymetry in Duck, NC.

Figure 3 shows the comparison of shoaling effect and the combination of refraction, diffraction and shoaling effects. We used the bathymetry in Duck, NC measured in August, 1994. (Thanks to Casey Church who supplied the data to us) In Fig. 3.a, the normalized amplitude variation calculated from the alongshore-averaged bathymetry by the REF/DIF model is shown. The incident wave is normal to the shoreline, so only shoaling effect is involved. The wave amplitude increases from 1 to about 1.09, a 9% increase. In Fig 3.b, we used the real bathymetry, with its alongshore variation. Now the fluctuation of the wave amplitude is much larger than that with shoaling effect only. The largest amplitude increase is about 55%, over 6 times larger larger than the shoaling-only case.

When water waves propagate in shallow water toward the shoreline, the water depth variation in the propagation direction causes the waves to shoal, while the variation in the transverse direction causes refraction and diffraction. It is the transverse variations that cause the variation of wave direction scattering and wave focusing. The transverse variation, rather than the onshore variation, is more important to the wave scattering.

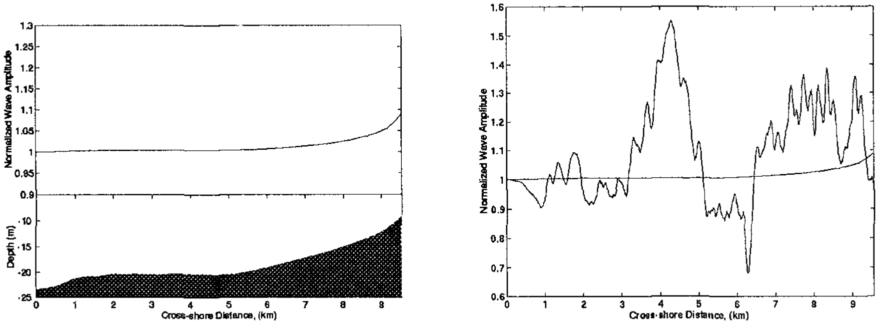



Figure 3: REF/DIF model, nonlinear version. Wave period = 10 sec. a). Upper: Normalized wave amplitude; Lower: Alongshore-averaged water depth (Duck, NC ); b). Comparison of shoaling effect and refraction, diffraction and effects. Real bathymetry

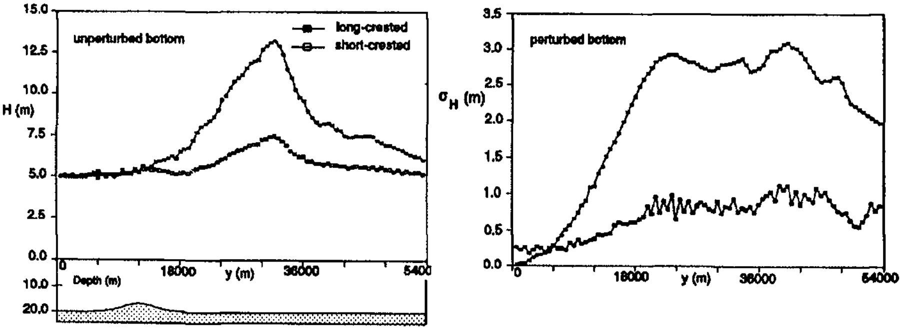



Figure 4: Results obtained by the HISWA model (Holthuijsen and Booij, 1994.) for the circular shoal. a). Wave heights along centerline profile for circular shoal with unperturbed bottom; b). The standard deviations of the wave height for the perturbed bottom.

Holthuijsen and Booij studied an idealized case numerically using the HISWA model. The bathymetry is a isotropic Gaussian-shaped shoal superimposed on an otherwise flat horizontal bottom. The ambient depth is 20 m and the minimum depth over the shoal is 17 m. The width of the shoal is 6 km. The presence of the shoal causes the waves to focus behind it. For the cases when the bottom is perturbed, the superimposed variation on the bathymetric grid points is Gaussian uncorrelated noise of 0.5 m standard deviation, 2.9% of the minimum depth. The wave height of the incident wave is 5 m and the wave period is 10 sec. Holthuijsen and Booij considered both long- and short-crested incident waves. Only long-crested waves were simulated in our experiments. Their results are shown in Figure 4. Fig. 4.a shows the results of unperturbed bottom, the bathymetry profile and the wave amplitude profiles of long- and short-crested incident waves along the centerline which is parallel to the initial wave propagation direction and crosses the top of the shoal. Fig. 4.b shows the depth perturbation induced wave height standard deviations on the centerline, averaged by the results of 25 cases.

We can see both long- and short- waves focus at the same position, about 20,000 m behind the shoal. The wave height at the focusing point is 12.5 m for long-crested wave, 2.5 times larger than the initial wave height of 5 m. The standard deviation of the wave height is 2.5 m, half as the initial waves. But for short-crested waves, the focusing wave height and the wave fluctuation are much smaller.

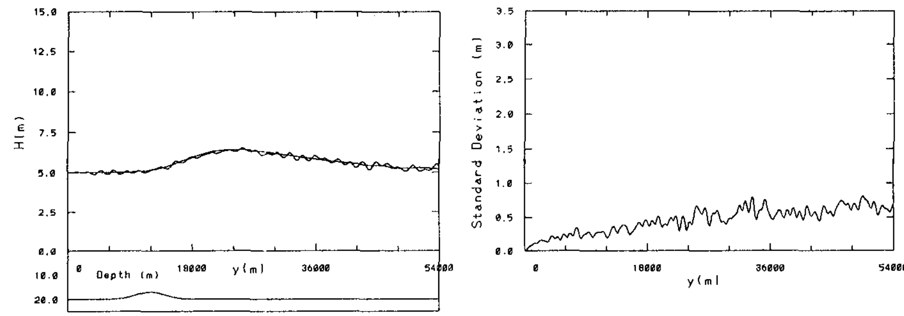



Figure 5: Results obtained by the nonlinear version of the REF/DIF model for the circular shoal. a). Wave height along the centerline with unperturbed bottom and the average of the results of 25 perturbed bottoms; b). The standard deviation of the wave height for the perturbed bottom.

Using the REF/DIF and WANGLE models, both the linear and nonlinear versions, we replicated the same case. We obtained the wave field with an unperturbed bottom for each model and version. Then 25 cases were run with different perturbed bottoms to get the ensemble-averaged water wave height and standard deviation. The ensemble-averaged wave height was calculated by averaging the 25 cases. The ensemble-averaged standard deviation was calculated by computing the standard deviation for each of the 25 cases, as compared to the unperturbed case, and then averaging the results.

Results obtained by the nonlinear version of the REF/DIF model are shown in Fig. 5. The wave height at the focusing point is 6.3 m, 26% larger than that

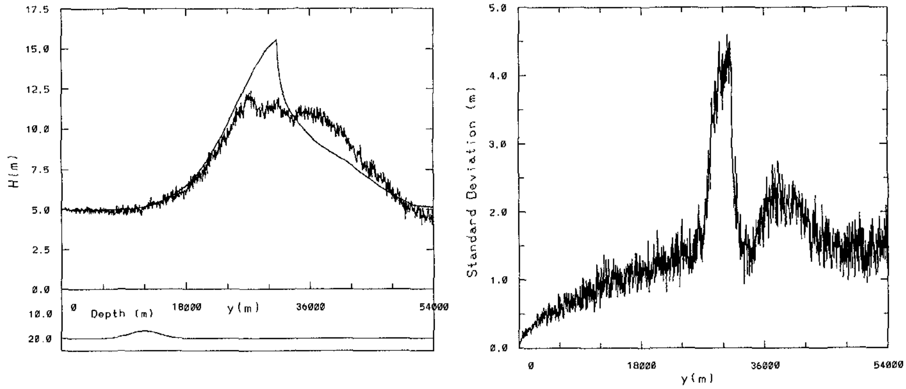



Figure 6: Results obtained by the nonlinear version of the REF/DIF model for the circular shoal. Same conditions as that for Fig. 5.

of the incident wave. The maximum standard deviation is about 0.8 m, 16% of the initial wave height. It is even smaller than the result of HISWA model of the short-crested waves. The focusing happens earlier due to diffraction and nonlinear effects. These effects help to transfer wave energy to different locations and smooth down the roughness of the wave field.

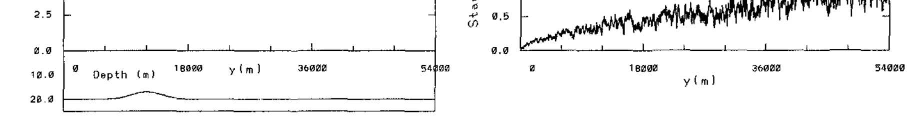



Figure 7: Results obtained by the nonlinear version of the WANGLE model for the circular shoal. Same conditions as that for Fig. 5.

Fig. 6 shows the results obtained by the linear version of the REF/DIF model. For the unperturbed bottom, there seems to be wave breaking when waves are focusing behind the shoal. But there is no wave breaking with the perturbed bottom. The reason is that the random depth variations make wave refract and diffract before focusing behind the shoal. Wave energy is scattered to other directions and eventually the wave height at the focusing position is reduced, avoiding breaking. The breaking position is further behind the shoal than the focusing position obtained by the nonlinear version model. The only difference between the two experiments is that nonlinear effect is not involved here. Nonlinearity can smooth down the amplitude variation, transfer wave energy to the area right behind the shoal, and make results more reasonable. The standard deviation shown in Fig. 6.b is very large around the breaking area due to the big difference between breaking and non-breaking waves.

We also studied the same case with WANGLE model. Fig.7 and Fig. 8 show the results of WANGLE model of nonlinear and linear versions, respectively.

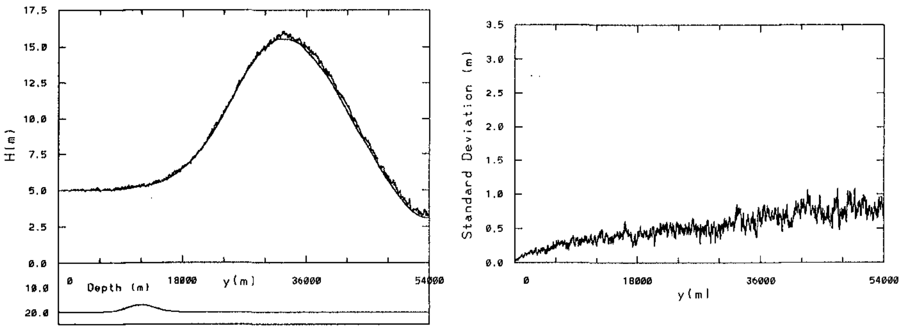



Figure 8: Results obtained by the nonlinear version of the WANGLE model for the circular shoal. Same conditions as that for Fig. 5.

We can see the focusing is still very large, about 15m here, larger than than of HISWA model. However, the standard deviation is smaller even than that obtained by REF/DIF model. For these cases, WANGLE is not very sensitive to bottom perturbation, so it does not suffer as much from bathymetry errors. Results of the linear and the nonlinear versions are not quite different. WANGLE model is an angular spectrum model that is supposed to be applicable to those cases where the wave direction varies widely. It emphasizes diffraction and refraction effects.

As mentioned above, we generated two kinds of water depth variations to superimpose on flat bottoms: uniformly distributed uncorrelated variations and Gaussian distributed correlated variations. Using the generated water depth and the nonlinear REF/DIF model, we did some numerical studies on depth variation influence. Fig. 9 and Fig. 10 are the results of the first kind of variations. They are the normalized standard deviations of wave height for bottom perturbations of different orders and different gradients, respectively. We choose bottom perturbations as 1%, 5% and 10% related to the mean depth. The corresponding normalized amplitude standard deviations are 10%, 30% and 50% of the incident wave, respectively. It is not surprising to see the larger the perturbation is, the larger the amplitude fluctuation is. But the ratio between amplitude variation and bottom perturbation becomes smaller as the perturbation increases. The reason is that as the bottom variation increases, the diffraction and nonlinearity effects becomes larger and they scatter and transfer wave energy, hence reduce the relative wave height fluctuation. To get the results in Fig. 10, we used the same water depth but different grid sizes, 5m, 2.5m and 1.25m, decreasing by half. The grid sizes are much smaller than the wave length. After waves propa-

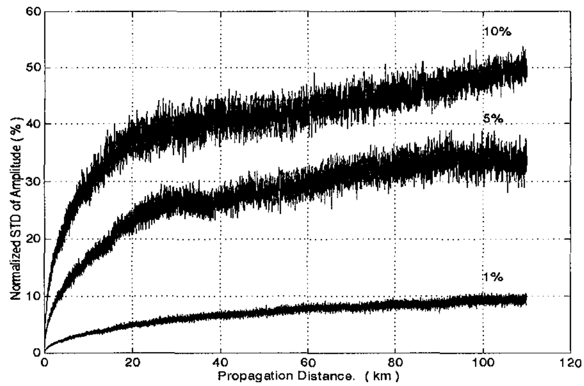



Figure 9: Amplitude standard deviations for different depth perturbations on a flat bottom, 1%, 5% and 10% of the mean water depth, respectively. Wave period = 5 sec. Mean water depth = 5 m. Grid size = 5 m. Incident wave amplitude = 0.5 m. REF/DIF, nonlinear version.

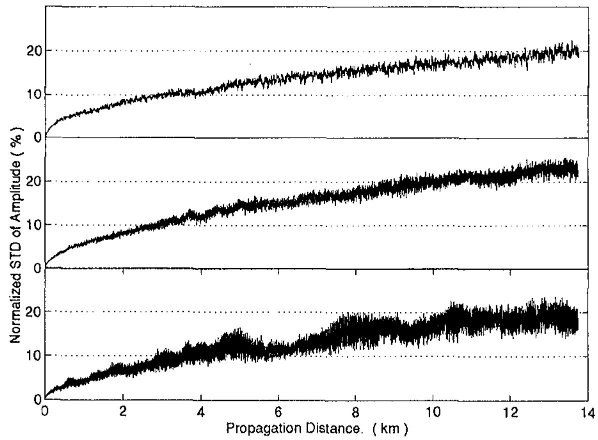



Figure 10: Amplitude standard deviations for different grid sizes, 5m, 2.5m and 1.25m, respectively. Wave period = 5 sec. Mean water depth = 5 m. Incident wave amplitude = 0.5 m. Depth perturbation = 5% of the mean depth. REF/DIF, nonlinear version.

gate over a very long distance, about 300 wave lengths, the amplitude variations approach roughly the same value. For large gradients, or rougher bottom surface, there are more grid points in one wave length and the wave field is more sensitive.

Results of the normal distributed and correlated water depth variations are shown in Fig. 11. The figures show the amplitude profiles in sections perpendicular to the initial wave direction. A normally distributed water depth can be described by its correlation function. We used water depths with different correlation lengths in different directions and examined how the wave field responds when the wave length ($L$) is similar to the correlation lengths in the wave propagation and the transverse directions ($L_x$ and $L_y$). In Fig. 11.a, we fixed $L_y$ and varied $L_x$; in Fig. 11b, $L_x$ was fixed and $L_y$ was varied. The results of wave focusing of the three cases shown in Fig. 11a are not very different and bottom variation correlation length in wave direction does not affect the wave field much. Fig. 11.b shows that when $L_y$ is much longer than wave length, the focusing is relatively small. When the two lengths are the same, focusing is large. So correlation length in the direction perpendicular to wave propagation is an important factor to the wave field. When the correlation length is much shorter than the wave length, the focusing is also fairly dominant. This is because for this case, the bottom is rougher, as there are more grid points in one wave length, and the wave fields are sensitive to this in short wave propagation distance.

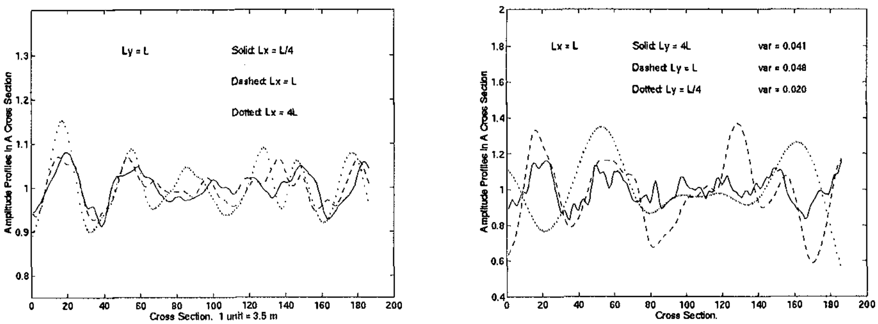
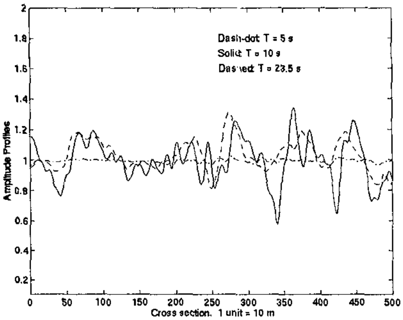



Figure 11: Cross-section amplitude profiles obtained by the REF/DIF model, nonlinear version. $L$: water wave length, $L_x$ and $L_y$: water depth correlation lengths in wave propagation and transverse directions, respectively. Wave period = 5 sec, Mean water depth = 5 m. Standard deviation of water depth = 0.5 m. Distance from where waves start: 700 m.

We also used the bathymetry in Duck, NC to study the influence of bottom randomness of different statistical properties. The correlation length is about 150 m in both alongshore and cross-shore directions. We used waves of different wave length for numerical studies. The results are similar to what we got from the generated water depth. When correlation length and wave length are similar, the effects on wave field is dominant. This can be observed quantitatively by the amplitude standard deviations. After waves propagate for certain distances ( 1



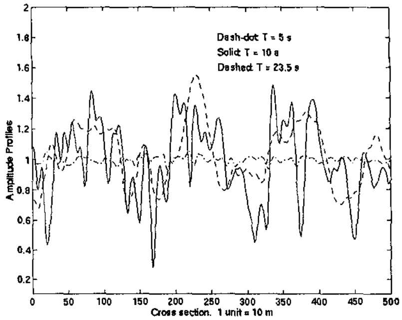




Figure 12: Cross-section amplitude profiles obtained by the REF/DIF model, nonlinear version, for incident waves of different wave periods $T$. Bathymetry in Duck, NC is applied. Distance from the incident waves, left figure: 1 km; right figure: 2 km.

km and 2 km ), the amplitude standard deviations are 0.0323, 4.4, 2.5 and 0.0982, 13.2, 8.6 for waves with time periods of 5sec, 10sec and 23.5sec, respectively. When the two lengths are the the most similar, the standard deviation is the largest.

## DISCUSSION AND CONCLUSIONS

We have shown that small water depth perturbation can cause large wave fluctuations based on the wave model used. Different numerical models are sensitive to depth variation in different ways, and models involving nonlinearity, refraction and diffraction effects can be applied to study the effects of random bottom variations, such as the REF/DIF and the WANGLE models. Large and rapid water depth variations cause large wave field fluctuations, and with the same variance, different statistical water depths have different effects on wave field. The effects of a Gaussian distributed bottom variation on wave field depends on its variance and correlation function. The numerical results show that correlation length in the direction perpendicular to water waves direction is important to the wave fluctuations. When it is similar to the incident wave length, the wave fluctuation becomes larger.

## ACKNOWLEDGMENTS

This work was sponsored by the U.S. Army Research Office, University Research Initiative under Contract No. DAAL03-92-G-0116.

## REFERENCE

Brown,M. G., Tappert, F. D. and Sundaram, S. E., Chaos in Small-Amplitude Surface Gravity Waves Over Slowly Varying Bathymetry, *J. Fluid Mech.* ,vol. 227, *pp*.35-46, 1991.

Dalrymple, R. A., Suh, K. D., Kirby, J. T., and Chae, J. W., Models for Very Wind-Angle Water Waves and Wave Diffraction. Part 2. Irregular Bathymetry, *J. Fluid Mech.*, 201, 299-322, 1989.

Holthuijsen, L. H. and Booij, N., Bottom Induced Scintillation of Long- and Short-crest Waves, *Proc. the Intl. Symp.: Waves-Physical and Numerical Modeling*, Vancouver, Canada, 1994.

Kirby, J.T. and Dalrymple, R.A., Combined Refraction/Diffraction Model–REF/DIF 1, Version 2.4, Documentation and User's Manual, Center for Applied Coastal Research, Res. Rpt. CACR-92-04, 1992.

Reeve, D. E., Bathymetric Generation of an Angular Wave Spectrum, *Wave Motion*, 16, 217-228, 1992.