CHAPTER 60

TWO-DIMENSIONAL ANALYSIS OF WAVE TRANSFORMATION BY RATIONAL-APPROXIMATION-BASED, TIME-DEPENDENT MILD-SLOPE EQUATION FOR RANDOM WAVES

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ABSTRACT

The method for determining coefficients in the rational approximation is improved and a numerical method is developed for application of time-dependent mild-slope equation for random waves to two-dimensional wave fields. The validity of the method is verified through comparisons with analytical solutions in typical situations and an experimental result of wave transformation around a man-made island on a uniform slope.

1. INTRODUCTION

Time-dependent mild-slope equations for random waves were derived from Berkhoff's mild-slope equation by approximating frequency-independent expressions to frequency-dependent coefficients (Kubo *et al.*, 1992; Kotake *et al.*, 1992; Isobe, 1994). It can be used to simulate directly the time evolution of irregular waves. The approximation by a rational function (Padé approximation) has a high accuracy to the coefficient over a wide frequency range (Isobe, 1994). However, the method for determining the coefficients in the rational function and the numerical calculation method for applying to two-dimensional problems were not established. In this paper, a method was developed to apply the time-dependent mild-slope equation for random waves to practical problems. The results of calculations are compared with analytical solutions in typical situations and an experimental result of wave transformation around a man-made island on a uniform slope.

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2. A TIME–DEPENDENT MILD–SLOPE EQUATION FOR RANDOM WAVES BASED ON RATIONAL APPROXIMATION

The governing equation is as follows:

$$\nabla^2\tilde{\phi} - ia_1\nabla^2\left(\frac{\partial\tilde{\phi}}{\partial t}\right) + (b_0 + ic_0)\tilde{\phi} + i(b_1 + ic_1)\frac{\partial\tilde{\phi}}{\partial t} - b_2\frac{\partial^2\tilde{\phi}}{\partial t^2} = 0 \tag{1}$$

where t is the time, ∇ the differential operator in the horizontal two directions, i the imaginary unit and $\tilde{\phi}$ the unknown variable which is related to complex amplitude $\hat{\phi}$ as follows.

$$\tilde{\phi} = \hat{\phi}e^{-i\omega' t}, \qquad \omega' = \omega - \overline{\omega} \tag{2}$$

where $\overline{\omega}$ is a representative angular frequency, ω and ω' the angular frequency and its deviation from $\overline{\omega}$, a_1, b_0, b_1 and b_2 the coeffcients in the rational function in (4) which is approximated to k^2 (k: wave number) in a Helmholtz equation (3) (Radder, 1979), and c_0 and c_1 the energy dissipation term to model the wave breaking.

$$\nabla^2\tilde{\phi} + k^2\tilde{\phi} = 0 \tag{3}$$

$$k^2 = \frac{b_0 + b_1\omega' + b_2\omega'^2}{1 - a_1\omega'} \tag{4}$$

3. IMPROVEMENT OF METHOD FOR DETERMINING COEFFICIENTS

3. 1. Condition of numerical stability

For monochromatic progressive waves, $\tilde{\phi}$ is expressed as

$$\tilde{\phi} = ae^{i(kx\cos\theta + ky\sin\theta - \omega' t)} \tag{5}$$

Then, equation(1) becomes

$$-k^2 + a_1k^2\omega' + (b_0 + ic_0) + (b_1 + ic_1)\omega' + b_2(\omega')^2 = 0 \tag{6}$$

Equation(6) can be solved for ω' as

$$\omega' = \left\{-(a_1k^2 + b_1 + ic_1) \pm \sqrt{(a_1k^2 + b_1 + ic_1)^2 - 4b_2(-k^2 + b_0 + ic_0)}\right\}/(2b_2) \tag{7}$$

To avoid numerical divergence, $Im\{\omega'\} \leq 0$. This requires that the magnitude of the imaginary part for $\sqrt{\quad}$ should not exceed c_1. Let the real and imaginary parts in the $\sqrt{\quad}$ be denoted by X and Y, respectively, then the condition is written as

$$X \geq 0 \quad (c_1 = 0) \tag{8}$$

$$X \geq (Y/2c_1)^2 - c_1^2 \quad (c_1 > 0) \tag{9}$$

The above condition should be satisfied for an arbitrary k, which yields

$$b_1^2 - 4b_0b_2 \leq 0, \quad c_0 = 0 \quad (c_1 = 0) \tag{10}$$

$$\left(\frac{c_0}{c_1}\right)^2 - \left(\frac{b_1}{b_2}\right)\left(\frac{c_0}{c_1}\right) + \left(\frac{b_0}{b_2}\right) \leq 0 \quad (c_1 > 0) \tag{11}$$

In a previous study (Isobe, 1994), equal signs were taken for the sake of convenience within the above restrictions. Then,

$$b_2 = b_1^2/(4b_0) \tag{12}$$

$$c_1 = (2b_2/b_1)c_0 \tag{13}$$

However, numerical divergence occurs due to round-off error if the coefficients are determined by the above equations. An example is in the calculation of wave propagation over a submerged circular shoal. In the present study, the conditions were modified to avoid the numerical instability as follows:

$$b_1^2 - 6b_0b_2 \leq 0, \quad c_0 = 0 \quad (c_1 = 0) \tag{14}$$

$$\left(\frac{c_0}{c_1}\right)^2 - \left(\frac{b_1}{b_2}\right)\left(\frac{c_0}{c_1}\right) + \frac{3}{2}\left(\frac{b_0}{b_2}\right) \leq 0 \quad (c_1 > 0) \tag{15}$$

Within the above restrictions, we take equal signs. Then,

$$b_2 = b_1^2/(6b_0) \tag{16}$$

$$c_1 = (2b_2/b_1)c_0 \tag{17}$$

By considering the above two equations, independent parameters are a_1, b_0, b_1 and c_0.

3. 2. Determination of coefficients

The coefficients a_1, b_0, b_1 and b_2 can be determined from three sets of ω' and k which satisfy the dispersion relation exactly and equation(16) which is the condition to avoid numerical divergence. In the previous method (Isobe, 1994), equation(4) was applied piece by piece to three intervals of frequency range to approximate dispersion relation accurately, but it required extensive computational time for calculation of two-dimensional wave transformation of random waves. In the present study, equation(4) is applied for the whole range of frequency and the coefficients are determined by using three sets of ω' and k which satisfy the dispersion relation exactly on f/f_p =0.76, 1.22, 1.92, where f is frequency, f_p the peak frequency. Figure 1 shows the dispersion relation which was obtained by applying equation (4). The relative error due to the method is at most 10 % in the range of $f/f_p = 0.68 \sim 2.07$ which occupies the major portion of wave energy in frequency spectrum.

When we determine the values of coefficients a_1, b_0, b_1 and b_2, we compensate for the error included in the finite difference form of the equation in the ADI method of which the detail is given later. The finite difference expressions for each term in equation(1) are related with the corresponding derivatives as

$$\left.\frac{\partial^2\tilde{\phi}}{\partial x^2}\right|_{\text{F.D.}} = \left(\frac{\sin(\frac{1}{2}k\Delta x\cos\theta)}{\frac{1}{2}k\Delta x\cos\theta}\right)^2 \left(\cos(\frac{1}{2}\omega'\Delta t)\right)^2 \frac{\partial^2\tilde{\phi}}{\partial x^2} \tag{18}$$

$$\left.\frac{\partial^2\tilde{\phi}}{\partial y^2}\right|_{\text{F.D.}} = \left(\frac{\sin(\frac{1}{2}k\Delta y\sin\theta)}{\frac{1}{2}k\Delta y\sin\theta}\right)^2 \left(\cos(\frac{1}{2}\omega'\Delta t)\right)^2 \frac{\partial^2\tilde{\phi}}{\partial y^2} \tag{19}$$

$$\left.\frac{\partial^3\tilde{\phi}}{\partial x^2\partial t}\right|_{\text{F.D.}} = \left(\frac{\sin(\frac{1}{2}k\Delta x\cos\theta)}{\frac{1}{2}k\Delta x\cos\theta}\right)^2 \frac{\sin(\frac{1}{2}\omega'\Delta t)}{\frac{1}{2}\omega'\Delta t}\frac{\partial^3\tilde{\phi}}{\partial x^2\partial t} \tag{20}$$

$$\left.\frac{\partial^3\tilde{\phi}}{\partial y^2\partial t}\right|_{\text{F.D.}} = \left(\frac{\sin(\frac{1}{2}k\Delta y\sin\theta)}{\frac{1}{2}k\Delta y\sin\theta}\right)^2 \frac{\sin(\frac{1}{2}\omega'\Delta t)}{\frac{1}{2}\omega'\Delta t}\frac{\partial^3\tilde{\phi}}{\partial y^2\partial t} \tag{21}$$

$$\left.\tilde{\phi}\right|_{\text{F.D.}} = (\frac{2}{3}\cos\omega'\Delta t + \frac{1}{3})\tilde{\phi} = \beta_0\tilde{\phi} \tag{22}$$

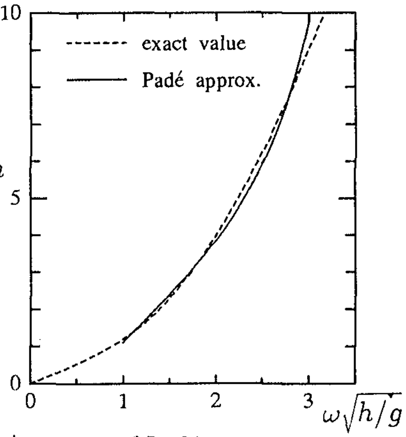

Figure 1: Accuracy of Padé approximation to k

$$\left.\frac{\partial\tilde{\phi}}{\partial t}\right|_{\text{F.D.}} = \frac{\sin\omega'\Delta t}{\omega'\Delta t}\frac{\partial\tilde{\phi}}{\partial t} = \beta_1\frac{\partial\tilde{\phi}}{\partial t} \tag{23}$$

$$\left.\frac{\partial^2\tilde{\phi}}{\partial t^2}\right|_{\text{F.D.}} = \left(\frac{\sin(\frac{1}{2}\omega'\Delta t)}{\frac{1}{2}\omega'\Delta t}\right)^2\frac{\partial^2\tilde{\phi}}{\partial t^2} = \beta_2\frac{\partial^2\tilde{\phi}}{\partial t^2} \tag{24}$$

where $|_{\text{F.D.}}$ denotes the finite difference expressions. The wave directions θ at calculation grid points are assumed as 45°, which makes the values of correction factors the same for x- and y-directions and closer to unity. Under the condition that $\Delta x = \Delta y = \Delta l$, the finite difference expressions and correction factors are as follows:

$$\left.\begin{array}{ll} \frac{\partial^2\tilde{\phi}}{\partial x^2}|_{\text{F.D.}} = \alpha_1\frac{\partial^2\tilde{\phi}}{\partial x^2}, & \frac{\partial^2\tilde{\phi}}{\partial y^2}|_{\text{F.D.}} = \alpha_1\frac{\partial^2\tilde{\phi}}{\partial y^2} \\ \frac{\partial^3\tilde{\phi}}{\partial x^2\partial t}|_{\text{F.D.}} = \alpha_2\frac{\partial^3\tilde{\phi}}{\partial x^2\partial t}, & \frac{\partial^3\tilde{\phi}}{\partial y^2\partial t}|_{\text{F.D.}} = \alpha_2\frac{\partial^3\tilde{\phi}}{\partial y^2\partial t} \end{array}\right\} \tag{25}$$

$$\left.\alpha_1 = \left(\frac{\sin(\frac{1}{2\sqrt{2}}k\Delta l)}{\frac{1}{2\sqrt{2}}k\Delta l}\right)^2\left(\cos(\tfrac{1}{2}\omega'\Delta t)\right)^2, \quad \alpha_2 = \left(\frac{\sin(\frac{1}{2\sqrt{2}}k\Delta l)}{\frac{1}{2\sqrt{2}}k\Delta l}\right)^2\frac{\sin(\frac{1}{2}\omega'\Delta t)}{\frac{1}{2}\omega'\Delta t}\right\} \tag{26}$$

Then, instead of equation (6), the finite difference equation for equation(1) implies

$$-\alpha_1 k^2 + a_1\alpha_2 k^2\omega' + b_0\beta_0 + b_1\beta_1\omega' + b_2\beta_2\omega'^2 = 0 \tag{27}$$

for $c_0 = c_1 = 0$. By rearranging the equation with $b_2 = b_1^2/6b_0$, the equation is written

$$-b^*\alpha_1 k^2 + a^*\alpha_2 k^2\omega' + \beta_0 + \xi\beta_1\omega' + \frac{1}{6}\beta_2\xi^2\omega'^2 = 0 \tag{28}$$

where

$$b^* = 1/b_0, \quad \xi = b_1/b_0, \quad a^* = a_1/b_0 \tag{29}$$

Three independent parameters can be determined from three set of exact values.

Since equation (29) is linear in a^* and b^*, these parameters can be eliminated to yield a parabolic equation in terms of ξ. After solving for ξ, we can determine a_1, b_0, b_1 and b_2 by equations (16) and (29).

4. METHOD OF NUMERICAL CALCULATION

The ADI method is employed in the numerical calculation to achieve high accuracy in a reasonable computational time. The calculation is carried out alternately in the x- and y-direction. In the discretization by the ADI method, the term $\nabla^2\tilde{\phi}$ cannot be ensured a second-order accuracy in time. In the present study, the term is averaged over time to ensure the accuracy. The finite difference equations of the equation (1) in the x- and y-directions are written

(t+1 time step : x-direction)

$$\begin{aligned}
&\frac{1}{2}\left(\frac{\tilde{\phi}^{t+1}_{i-1,j}-2\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t+1}_{i+1,j}}{(\Delta x)^2}+\frac{\tilde{\phi}^{t}_{i-1,j}-2\tilde{\phi}^{t}_{i,j}+\tilde{\phi}^{t}_{i+1,j}}{(\Delta x)^2}\right)\\
&\quad -ia_1\frac{(\tilde{\phi}^{t+1}_{i-1,j}-2\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t+1}_{i+1,j})-(\tilde{\phi}^{t}_{i-1,j}-2\tilde{\phi}^{t}_{i,j}+\tilde{\phi}^{t}_{i+1,j})}{(\Delta x)^2\Delta t}\\
&\quad +\frac{1}{2}\left(\frac{\tilde{\phi}^{t}_{i,j-1}-2\tilde{\phi}^{t}_{i,j}+\tilde{\phi}^{t}_{i,j+1}}{(\Delta y)^2}+\frac{\tilde{\phi}^{t-1}_{i,j-1}-2\tilde{\phi}^{t-1}_{i,j}+\tilde{\phi}^{t-1}_{i,j+1}}{(\Delta y)^2}\right)\\
&\quad -ia_1\frac{(\tilde{\phi}^{t}_{i,j-1}-2\tilde{\phi}^{t}_{i,j}+\tilde{\phi}^{t}_{i,j+1})-(\tilde{\phi}^{t-1}_{i,j-1}-2\tilde{\phi}^{t-1}_{i,j}+\tilde{\phi}^{t-1}_{i,j+1})}{(\Delta y)^2\Delta t}\\
&\quad +(b_0+ic_0)\frac{\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t}_{i,j}+\tilde{\phi}^{t-1}_{i,j}}{3}+i(b_1+ic_1)\frac{\tilde{\phi}^{t+1}_{i,j}-\tilde{\phi}^{t-1}_{i,j}}{2\Delta t}\\
&\quad -b_2\frac{\tilde{\phi}^{t+1}_{i,j}-2\tilde{\phi}^{t}_{i,j}+\tilde{\phi}^{t-1}_{i,j}}{(\Delta t)^2}=0
\end{aligned}\tag{30}$$

(t+2 time step : y-direction)

$$\begin{aligned}
&\frac{1}{2}\left(\frac{\tilde{\phi}^{t+1}_{i-1,j}-2\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t+1}_{i+1,j}}{(\Delta x)^2}+\frac{\tilde{\phi}^{t}_{i-1,j}-2\tilde{\phi}^{t}_{i,j}+\tilde{\phi}^{t}_{i+1,j}}{(\Delta x)^2}\right)\\
&\quad -ia_1\frac{(\tilde{\phi}^{t+1}_{i-1,j}-2\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t+1}_{i+1,j})-(\tilde{\phi}^{t}_{i-1,j}-2\tilde{\phi}^{t}_{i,j}+\tilde{\phi}^{t}_{i+1,j})}{(\Delta x)^2\Delta t}\\
&\quad +\frac{1}{2}\left(\frac{\tilde{\phi}^{t+2}_{i,j-1}-2\tilde{\phi}^{t+2}_{i,j}+\tilde{\phi}^{t+2}_{i,j+1}}{(\Delta y)^2}+\frac{\tilde{\phi}^{t+1}_{i,j-1}-2\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t+1}_{i,j+1}}{(\Delta y)^2}\right)\\
&\quad -ia_1\frac{(\tilde{\phi}^{t+2}_{i,j-1}-2\tilde{\phi}^{t+2}_{i,j}+\tilde{\phi}^{t+2}_{i,j+1})-(\tilde{\phi}^{t+1}_{i,j-1}-2\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t+1}_{i,j+1})}{(\Delta y)^2\Delta t}\\
&\quad +(b_0+ic_0)\frac{\tilde{\phi}^{t+2}_{i,j}+\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t}_{i,j}}{3}+i(b_1+ic_1)\frac{\tilde{\phi}^{t+2}_{i,j}-\tilde{\phi}^{t}_{i,j}}{2\Delta t}\\
&\quad -b_2\frac{\tilde{\phi}^{t+2}_{i,j}-2\tilde{\phi}^{t+1}_{i,j}+\tilde{\phi}^{t}_{i,j}}{(\Delta t)^2}=0
\end{aligned}\tag{31}$$

where $(i,\ j)$ is the grid number in the $(x,\ y)$ coordinate system, t the time, Δx and Δy the grid size in x- and y-direction, respectively.

5. NUMERICAL RESULTS

As for the frequency spectrum and the directional spreading function, the Bretschneider-Mitsuyasu-type and the Mitsuyasu-type were employed. The incident waves consisted of 1000 components and were given by the single summation method.

5. 1. Refraction

The present method was applied to the calculation of refraction of multi-directional irregular waves on 1/50 slopes. The significant deepwater wave height and period are 1.0m and 8.0s. The principal wave direction is normal to the shoreline (0°). The maximum directional concentration parameter, S_{max}, is 10 in deepwater. In this case, the analytical solution can be obtained by means of linear superposition of the spectral components by Snell's law. Figure 2 and 3 compare the two-dimensional and cross-shore distributions of the significant wave height, respectively. Dashed lines show analytical solution and solid lines represent the results of numerical calculation. Good agreement is observed.

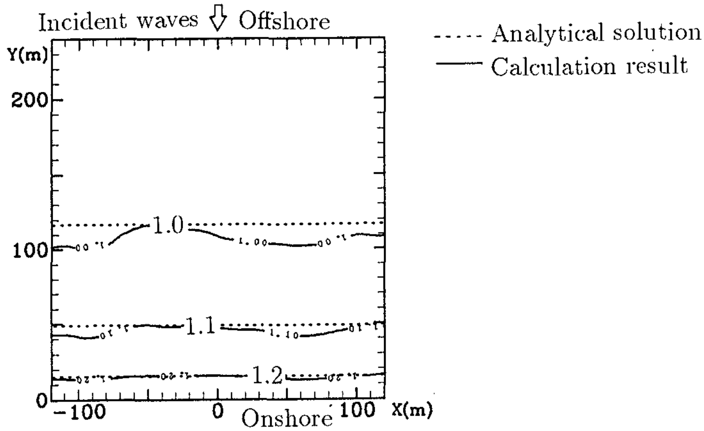

Figure 2: Significant wave height distribution due to refraction

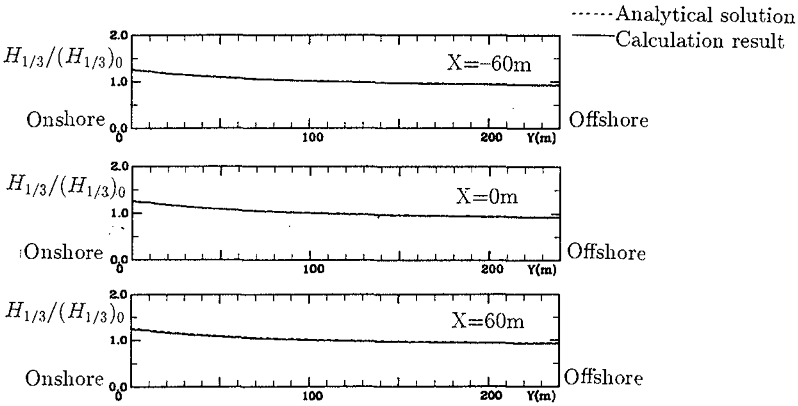

Figure 3: Significant wave height cross-shore distribution due to refraction

5. 2. Diffraction

Diffraction coefficients of multi-directional irregular waves for a semi-infinite breakwaters and through a breakwater gap have been calculated by Goda *et al.* (1978) by superposing analytical solutions. In the calculation, Bretschneider-Mitsuyasu-type frequency spectrum and Mitsuyasu-type directional distribution function are used. Dashed lines in Figures 4 and 5 show the results with the maximum directional concentration parameter, $S_{max} = 10$. In Figure 4, L denotes the wavelength corresponding to the significant wave period. In Figure 5, B denotes the gap width and this figure shows the case of B/L=8. Solid lines represent the results of the present method. These lines agree well with the dashed lines.

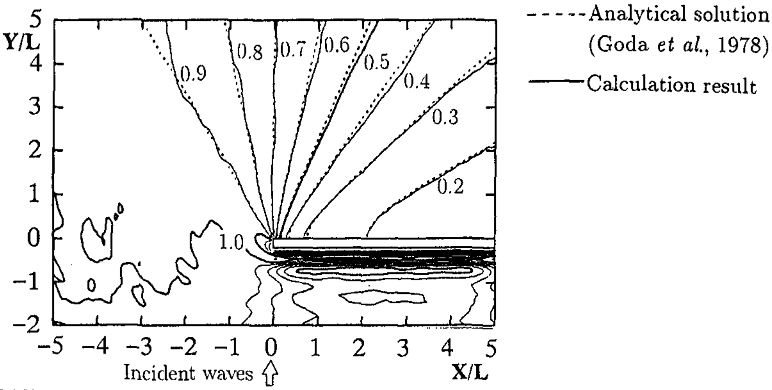

Figure 4: Diffraction coefficient around a semi-infinite breakwater for irregular waves

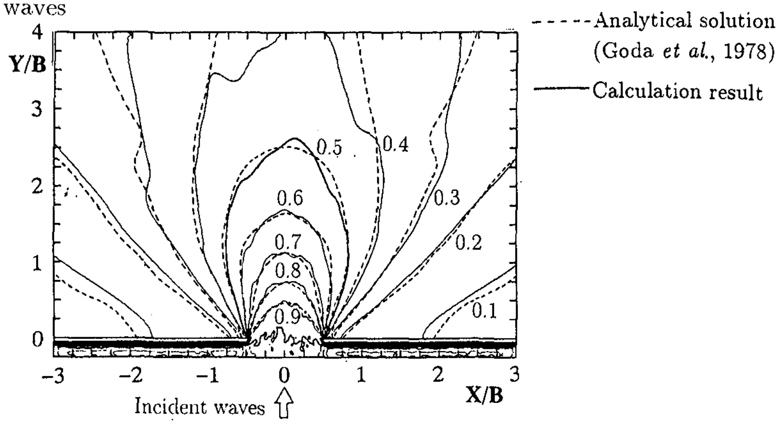

Figure 5: Diffraction coefficient around a breakwater gap for irregular waves (A gap width of 8 significant wavelength)

5. 3. Submerged circular shoal on a constant bottom

When waves propagate over a submerged circular shoal, the phenomenon of combined refraction-diffraction of waves occurs. To illustrate the capability of the present numerical model for the phenomenon, we compared the present results with that obtained by using the parabolic equation for a submerged circular shoal (Serizawa *et al.*, 1990). Figure 6 shows a submerged circular shoal on a constant depth. The constant water depth is 15m, the water depth at the peak of the circular shoal with radius 160m is 5m. Serizawa *et al.* (1990) calculated for this case. The significant wave height and period are 1.0m and 5.1s. The principal wave direction is in the positive y-axis (0°). The maximum value S_{max} of the parameter S in the Mitsuyasu-type directional spreading function was 10. Significant wave height distributions are presented in Figeres 7 and 8, to compare with the previous numerical result. Good agreement is observed.

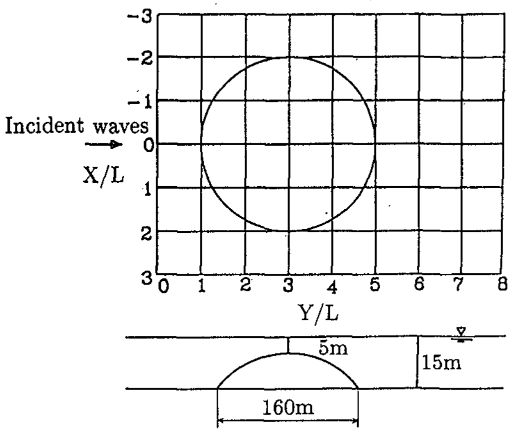

Figure 6: Sketch of submerged circular shoal on a constant bottom

5. 4. Wave breaking

Breaking wave model used is the same as Isobe (1987). The calculation of wave breaking was carried out in the cross-shore wave transformation problem for uni-directional irregular waves. The slope is 1/50. The significant wave period is 8.0s. The significant deepwater wave heights are 2.0m and 4.0m. In Figure 9, dashed lines show the results of calculation and solid lines show the results by Goda (1975). In the figure,H_0 is the significant wave height in deep water and h the water depth. The effect of wave nonlinearity and wave setup in shallow water is neglected in the present method. By considering this, the results seems to be in reasonable agreement with the results by Goda.

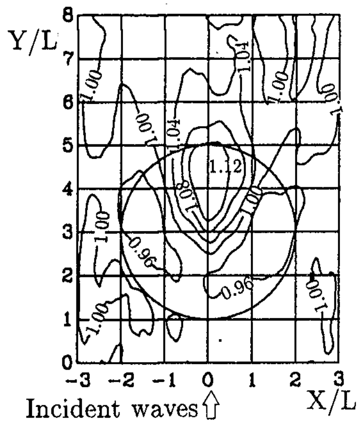

Figure 7: Present result for a submerged circular shoal on a constant bottom (the ratio of significant wave height to incident wave height)

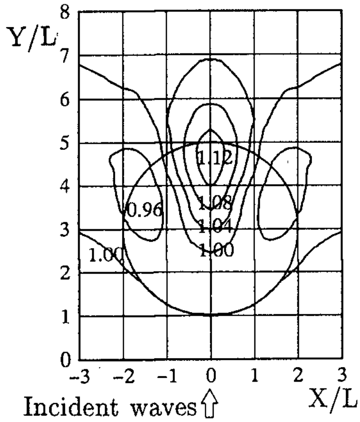

Figure 8: Previous result for a submerged circular shoal on a constant bottom (The ratio of significant wave height to incident wave height)

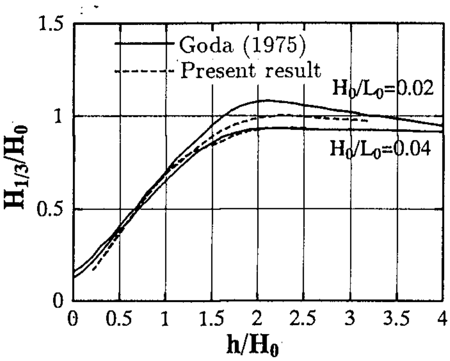

Figure 9: Significant wave height change of uni-directional irregular waves due to shoaling and breaking

6. WAVE HEIGHT DISTRIBUTION AROUND A MAN–MADE ISLAND

Numerical calculation of wave transformation around a rectangular man-made island on 1:50 slope is performed. It is an island of 1km long in the alongshore direction and 0.5km wide, and located about 1km from the coastal line. Hydraulic model tests with a multi-directional wave maker were carried out by Central Research Institute of the Electric Power Industry, Japan (Ikeno *et al.*, 1995) to investigate effects of the irregularity and directional spreading of waves behind the man-made island. The model is 1/150 in scale and the island is 6.6m long and 3.3m wide. The water depth at the man-made island is 20m at prototype scale which corresponds to 15cm at model scale. Side walls in the laboratory basin are installed on the slope along both edges of the wave maker, preventing waves from being diffracted.

Numerical calculation is performed under the same condition as that of the experiment. The offshore significant wave height and period are 9.2m and 14.3s. As for the frequency spectrum and the directional spreading function, the Bretschneider-Mitsuyasu-type and the Mitsuyasu-type were employed. The maximum values S_{max} of the parameter S in the Mitsuyasu-type directional spreading function were 25. The incident waves consisted of 512 components and were given by the single summation method, whereas input signal to each segment of wave maker was computed by superposing component waves with 512 frequencies and 90 directions by the double summation method.

Figure 10 and 11, respectively, depict the numerical result and the experimental result (Ikeno *et al.*, 1995) for significant wave height distribution. Figure 12 compares the longshore distributions of the significant wave height. The validity of the numerical model is verified through comparison with the experimental result.

7. CONCLUSION

We have proposed a method for determining coefficients in the rational approximation and an efficient numerical calculation method for applying the time-dependent mild-slope equation for random waves to two-dimensional problems. The results of numerical calculations were compared with analytical and experimental results, which confirmed the validity of the present method.

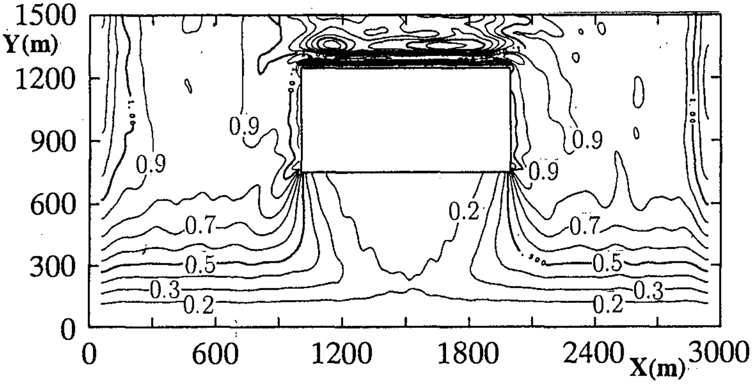

Figure 10: Calculation result (The ratio of significant wave height to incident wave height, $(H_{1/3})_0 = 9.2\text{m}$, $T_{1/3} = 14.3\text{s}$, $S_{\max} = 25$, $\theta_p = 0°$)

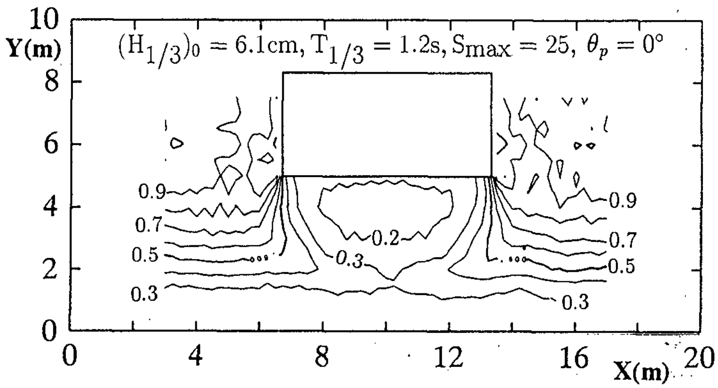

Figure 11: Experimental result (The ratio of significant wave height to incident wave height, 1/150 model)

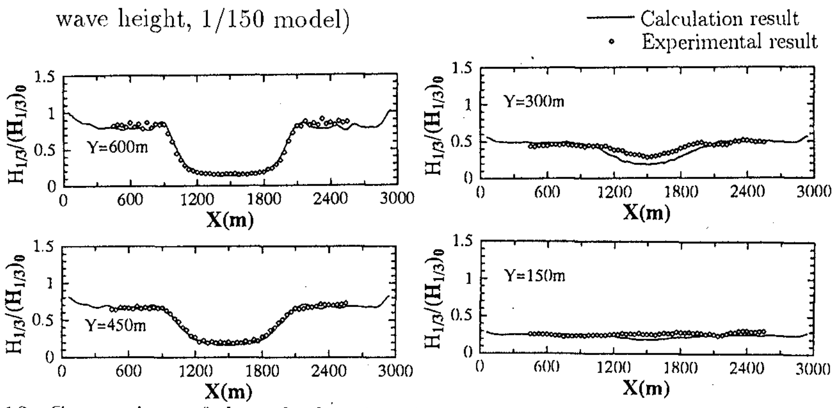

Figure 12: Comparison of the calculated and measured alongshore distribution of the wave height

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REFERENCES

Berkhoff, J. C. W. (1972): Computation of combined refraction-diffraction, Proc. 13th Int. Conf. on Coastal Eng., ASCE, pp. 471-490.

Goda, Y. (1975): Irregular wave deformation in the surf zone, Coastal Eng. in Japan, Vol. 18, pp.13-26.

Goda, Y., T. Takayama and Y. Suzuki (1978): Diffraction diagrams for directional random waves, Proc. 16th Coastal Eng. Conf., ASCE, 1978, pp. 628-690.

Ikeno, M., R. Kajima, M. Matsuyama and T. Sakakiyama (1995): Characteristics of Short-Crested Waves and Currents Behind Offshore Man-Made Island Type Power Plant, Proc. 5th Int. Offshore and Polar Eng. Conf., pp. 168-175.

Ishii, T., M. Isobe and A. Watanabe (1994): Improved boundary conditions to a time-dependent mild-slope equation for random waves, Proc. 24th Int. Conf. on Coastal Eng., ASCE, pp. 272-284.

Isobe, M. (1987): A parabolic equation model for transformation of irregular waves due to refraction, diffraction and breaking, Coastal Eng. in Japan, Vol. 30, pp. 33-47.

Isobe, M. (1994): Time-dependent mild slope equations for random waves, Proc. 24th Int. Conf. on Coastal Eng., ASCE, pp. 285-299.

Kubo, Y., Y.Kotake, M. Isobe and A. Watanabe (1992): Time-dependent mild slope equation for random waves, Proc. 23rd Int. Conf. on Coastal Eng., ASCE, pp. 419-431.

Kotake, Y., M. Isobe and A. Watanabe (1992): On the high-order time-dependent mild slope equation for irregular waves, Proc. 39th Japanese Conf. on Coastal Eng., pp. 91-95 (in Japanese).

Radder, A. C. (1979): On the parabolic equation method for water-wave propagation, J. Fluid Mech., Vol. 95, pp. 159-176.

Serizawa, M., K. Ozawa, S. Mifune and M. Isobe (1990): Application of the parabolic equation for irregular waves to practical problems, Proc. 37th Japanese Conf. on Coastal Eng., pp. 26-30 (in Japanese).