CHAPTER 63

Estimation of Persistence Statistics of the Waves Observed on Japanese Coast in the Light of Recent Studies

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Abstract

The NMI method proposed by Kuwashima and Hogben (1986) for estimating the wave persistence statistics and Mathiesen's Model (1994) which was expected to develop the NMI method were applied to the wave data observed at four sites at Japanese coast to investigate the validity of the models. Mathiesen's model was found not to be superior to NMI method. We show the modification of the NMI parameters based on the data improve the accuracy of the estimated persistence statistics.

1. Introduction

The estimation of wave height duration statistics is needed in various engineering fields such as planning and execution of maritime construction works as well as oil industry operations. For the case when continuous data sequences are not available Graham (1982) proposed an approximate method based on the probability distribution of significant wave height. Kuwashima and Hogben (1986) generalized the method using mainly North Sea wave data, and proposed an empirical method which they called the NMI method. Recently Mathiesen (1994) proposed a theoretically founded parametric model for the estimation of duration statistics for wave height. Although he suggested that his study established a theoretical basis for the development of NMI method, the fact was not confirmed.

In this study we estimate the wave persistence statistics from the wave data observed at the coasts of Japan using both NMI and Mathiesen's method. We pay attention to the local differences of persistence statistics examining the applicability of the two methods to the present wave data of Japanese area.

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2. Data and method of analysis

2.1 Wave data used

We used the wave data observed at the four sites in the Japanese coast. Fig.1 shows the locations of the four sites. Hitachinaka and Kashima are at the coast of Ibaraki facing to the Pacific Ocean, and Sakata and Wajima are in the Sea of Japan. They are hereafter abbreviated as HI, KA, SA and WA. The name of Ibaraki will also be used to denote both HI and KA. Those data are measured every two hours and the time spans are 13 years for HI and 8 or 9 for other sites.

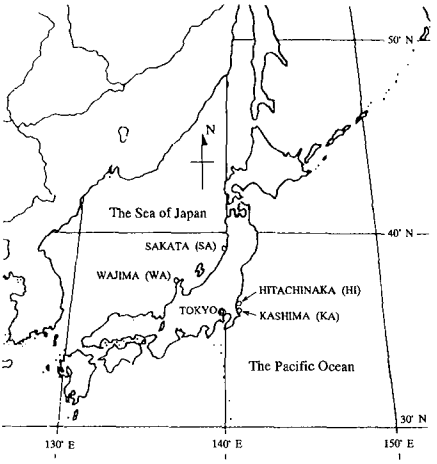

Fig.1 Map of the sites of the wave data used in this study.

2.2 Estimation of wave height persistence statistics by the NMI method

In the NMI method, the input data used are cumulative probability distributions of significant wave height H_s which is written for the case of height exceedance as $Q(\geqq H_s)$. The mean duration of exceedance waves $\overline{\tau}_g$ for a threshold height H_s' are expressed as:

$$\overline{\tau}_g = A[-\ln Q(\geqq H_s')]^{-\beta} \qquad (1)$$

where A and β are constants and Q is cumulative probability distribution for a threshold H_s'. The equation (1) was originally proposed by Graham (1982) with constants A=20.0 and $1/\beta$ =1.3 for the North Sea wave data.

Kuwashima and Hogben (1986) allowed variations in the parameters A and β by deriving empirical relations (based on exceedance data) between the parameters and a shape parameter γ for the input distribution defined in terms of a 2 parameter Weibull distribution. At first the distribution is written in terms of the normalized significant wave height defined by $H_n = \overline{H}_s / H_s$ as:

$$Q(\geqq H_n) = \exp(-B H_n^{\gamma}) \qquad (2)$$

where the parameter B and γ are related by

$$B = [\Gamma(1+1/\gamma)]^{\gamma} \qquad (3)$$

and Γ is a gamma function. Then the relevant relations of A and β with γ have been obtained from many data sets as follows:

$$A=35\,\gamma^{-0.5}, \quad \beta=0.6\,\gamma^{0.287} \qquad (4a,b)$$

Although Eqs.(4a,b) are empirical formulas, they are expected applicable to any world wide sites since the wave statistics is well represented by the parameter γ. When $\overline{\tau}_g$ is obtained from Eq.(1), then the mean duration of non-exceedance waves $\overline{\tau}_l$ is obtained as

$$\overline{\tau}_l = \overline{\tau}_g [1 - Q(\geqq H_s')] / Q(\geqq H_s') \qquad (5)$$

The cumulative distribution of duration for exceedance and non-exceedance waves are also expressed in terms of a 2-parameter Weibull distribution. Writing $x = \tau / \overline{\tau}$ where $\overline{\tau}$ is the mean duration, the distribution is written as:

$$Q_g(\tau / \overline{\tau}_g) = Q_g(\geqq x_g) = \exp[-C_g x_g^{\alpha_g}] \qquad (6a)$$

$$Q_l(\tau / \overline{\tau}_l) = Q_l(\geqq x_l) = \exp[-C_l x_l^{\alpha_l}]. \qquad (6b)$$

The shape parameters α_g and α_l of the above equations are also given in the following empirical forms:

$$\alpha_g = 0.267\,\gamma\,(H_s'/\bar{H}_s)^{0.4} \qquad (7a)$$

$$\alpha_l = 0.267\,\gamma\,(H_s'/\bar{H}_s)^{-0.4} \qquad (7b)$$

C_g and C_l of Eq.(6) are related to α_g and α_l as :

$$C_g = [\,\Gamma(1+1/\alpha_g)]^{\alpha_g} \qquad (8a)$$

$$C_l = [\,\Gamma(1+1/\alpha_l)]^{\alpha_l} \qquad (8b)$$

If we know the probability distribution of wave height for a certain area, then from the cumulative probability distribution $Q(\geqq H_s')$ and the shape parameter γ of a 2-parameter Weibull distribution, we can estimate the mean duration $\overline{\tau}_g$ (or $\overline{\tau}_l$) and the cumulative probability of duration Q_g (or Q_l) of exceedance (or non-exceedance) wave for a threshold wave height by using the equations stated above.

2.3 Mathiesen's model

Mathiesen (1994) assumed that the time variation of significant wave height is a continuous stationary process and applied a level crossing theory for water waves to the time series of wave heights. In this section we write a significant wave height H_s simply as H and a threshold wave height H_s' as H_c.

The average number of exceedances or level-upcrossings per unit time $\nu(H)$ is given by:

$$\nu(H) = (1/2) \int_{-\infty}^{\infty} | H' | \, p(H, H')\, dH' \qquad (9)$$

where $H' = dH/dt$ and $p(H,H')$ is the joint probability of H and H'. Then Mathiesen obtained the relation:

$$\nu(H) = (1/2) f(H) S(H) \tag{10}$$

where $f(H)$ is the probability density function for H, and $S(H)$ the average absolute rate of change of significant wave height H. That is, he showed that the duration statistics depended not only on $f(H)$ but also $S(H)$, the average absolute rate of change of H. Considering the equation of mean duration of exceedance $\bar{\tau}_g$:

$$\bar{\tau}_g = T\, Q(\geqq H_c) / N \tag{11}$$

where N is the number of up-crossing for the total time length T, then $\bar{\tau}_g$ is given as :

$$\bar{\tau}_g = 2\, Q(\geqq H_c) / [\, f(H)\, S(H)\,] \tag{12}$$

$f(H)$ and $Q(H)$ are evaluated in the 3-parameter Weibull form as follows:

$$f(H) = (\gamma' / X_0)\, ((H-a)/X_0)^{\gamma'-1} \exp[-((H-a)/X_0)^{\gamma'}] \tag{13}$$

$$Q(H) = \exp[-((H-a)/X_0)^{\gamma'}] \tag{14}$$

where γ' is the shape parameter, a the local parameter which was set as zero in practice and X_0 the scale parameter.

Mathiesen (1994) found that $S(H)$ could be expressed as

$$S = q\, (H/H_0)^R, \quad (H_0 = 1.0\text{m}) \tag{15}$$

where q and R are constants different between the locations.

From Eqs.(11) to (15) and Eq.(3), the NMI parameters A and β are found to become to A_M and β_M in the following equations:

$$A_M = 2 / [\, \gamma' (X_0 / H_0)^{R-1} (q / H_0)] \tag{16}$$

$$\beta_M = (\gamma' + R - 1) / \gamma' \tag{17}$$

It is seen that the parameters A_M and β_M depend on both the shape of the distribution $Q(H)$ and the average absolute rate of change $S(H)$ in Mathiesen's model.

3. Applicability of the NMI method to the present wave data

In this study we used the measured wave data for calculating the wave height persistence statistics in three ways; (a) data sets of each year, (b) seasonally divided data sets for four years, and (c) monthly divided for 9 (KA and HI) or 8 (SA and WA) years. For testing the reliability of the NMI method Kuwashima and Hogben (1986) showed comparisons of the estimated persistence statistics with those directly derived from measured data in cumulative distribution form. We made the same comparisons for the test of

applicability of the NMI method to the present wave data. Figure 2 is one of the results which are calculated for a 4-year data set of HI (Hitachinaka). The upper one is for exceedance waves and the lower one non-exceedance. As seen from these figures the NMI method yielded moderately good but slightly deviated results. More or less, this was the case for most of other data sets.

This may be due to the fact that the parameters A, β, α, *etc.* given by Eqs.(4) and (7) does not sufficiently fit to the present wave data. So following Kuwashima and Hogben (1986) we tried to obtain more suitable expressions from the measured data in stead of Eqs.(4) and (7). To complete this, we calculated A, β, α_M (value of α for $H_s'=\bar{H}_s$) from the measured wave data and plotted them against the parameter γ as Fig.3 in which the original values of NMI method are shown with dotted lines. Although there are considerable

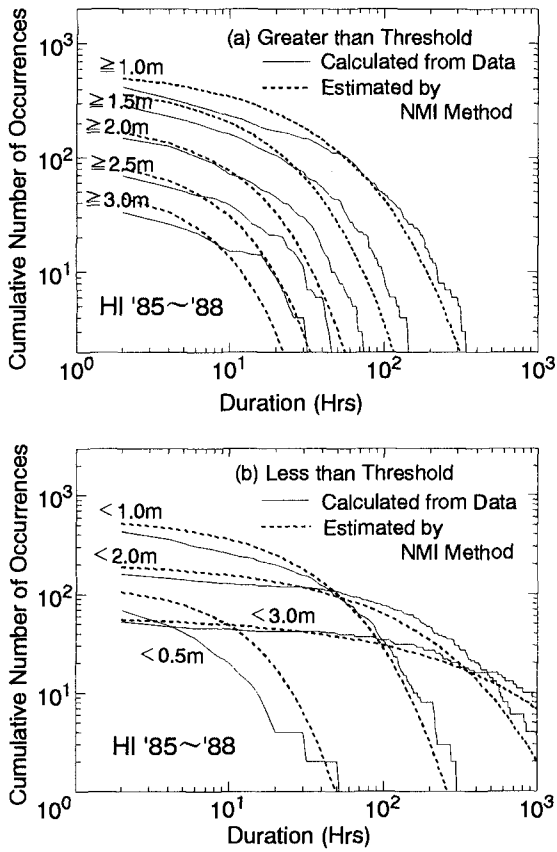

Fig. 2 Comparison of the estimated persistence statistics by NMI method with those directly derived from the measured wave data.

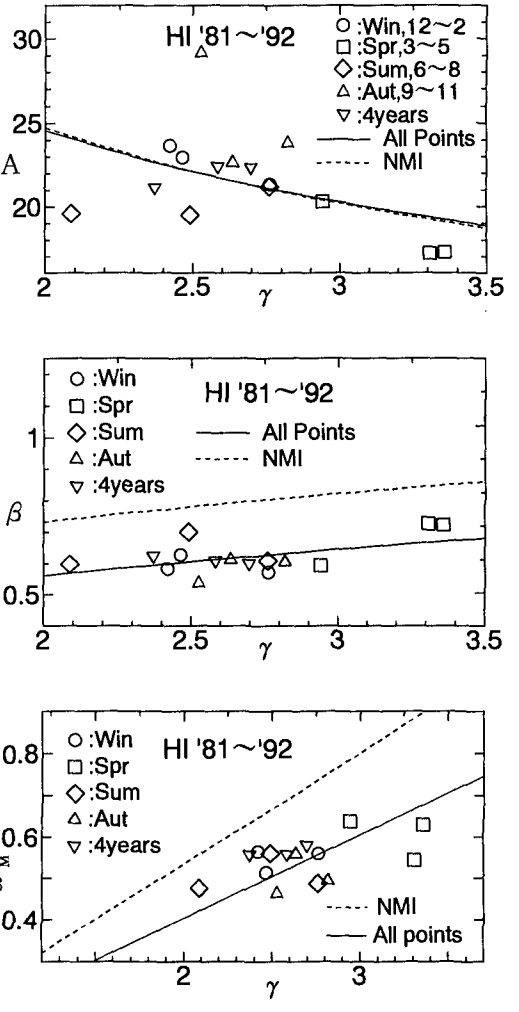

Fig. 3 Plots of NMI parameters A, β, α_M against γ
Seasonal dataset of HI for 4 years were used.

scatter, we obtained the empirical relations instead of Eqs.(4). For example the relations for HI are as follows:

$$A=32.3\,\gamma^{-0.464}, \qquad \beta=0.552\,\gamma^{0.100} \tag{18a,b}$$

$$\alpha_M = 0.202\,\gamma \tag{19}$$

The relation of A is almost the same as that of NMI. However the values of β are smaller than those of NMI about by 2.0 and α_M is as small as nearly 3/4

of NMI values.

Figure 4 shows the variations of α_g and α_l of HI normalized by α_M against various threshold height H_s' in the form of Eqs.(7a,b). From the results of these figures we can obtain the following relations:

$$\alpha_g = 0.202\,\gamma\,(H_s'/\bar{H}_s)^{0.318}, \tag{20a}$$

$$\alpha_l = 0.202\,\gamma\,(H_s'/\bar{H}_s)^{-0.044} \tag{20b}$$

When we used Eqs.(18) and (19) in estimating persistence statistics for HI we found that the agreement of the estimated values with those calculated from the measured data increased in most cases. Figure 5 shows the results corresponding to the previous figure 2 and it is seen that the modification of coefficients such as Eqs.(18) to (19) yields better agreements.

We conducted the similar procedures for the monthly divided data sets. Figure 6 shows the calculated values of A, β, α_M for KA and HI, the location of which are the Pacific Ocean side. Since no different tendency was found between the values of HI and KA, both quantities are plotted together and the

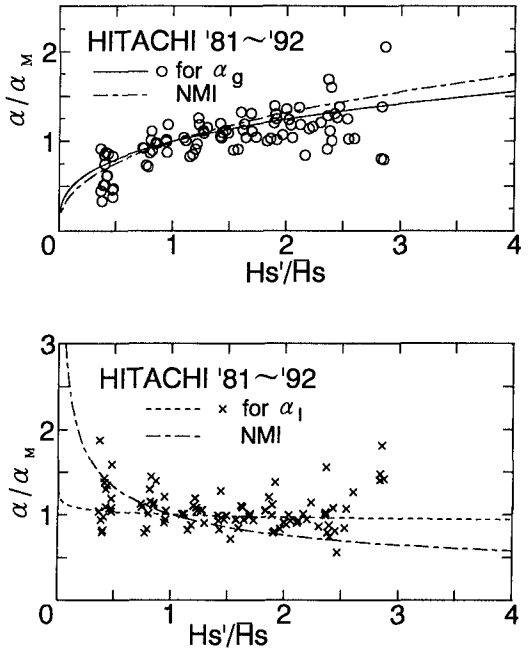

Fig. 4 Plots of α_g and α_l against threshold wave heights.

relations common to the two sites were obtained. Figure 7 shows the variation of α normalized by α_M for HI and KA. From these monthly data we obtained the following relations:

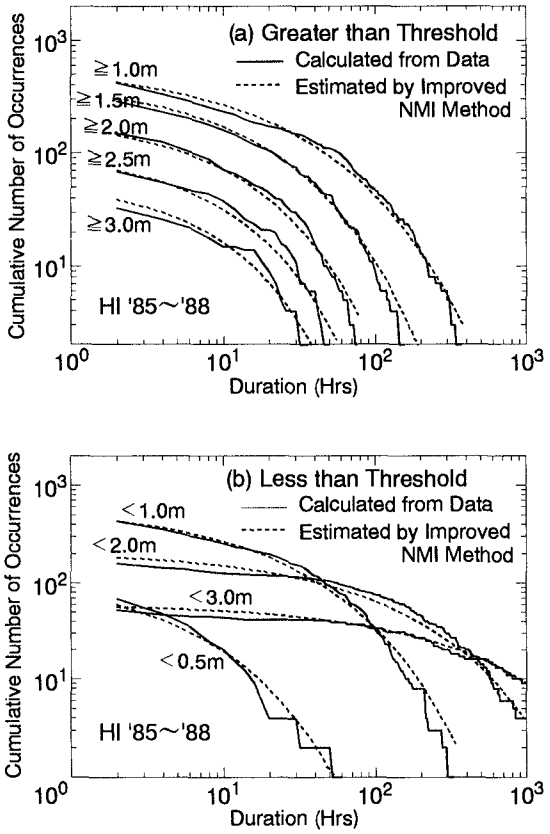

Fig. 5 Comparison of the estimates by the improved NMI formula with those directly derived from the measured wave data.

$$A=32.5\,\gamma^{-0.439}, \qquad \beta=0.618\,\gamma^{0.0052} \tag{21a,b}$$

$$\alpha_M = 0.195\,\gamma \tag{22}$$

$$\alpha_g=0.195\,\gamma\,(H_s'/\bar{H}_s)^{0.163} \tag{23a}$$

$$\alpha_l=0.195\,\gamma\,(H_s'/\bar{H}_s)^{-0.150} \tag{23b}$$

Optimum parameter formula for SA and WA

When we applied the NMI method for the wave data of SA and WA in the coast of the Sea of Japan, good agreements were not found similarly to HI and

KA in the coast of the Pacific Ocean. Therefore in the similar way as HI and KA, we tried to obtain the optimum relations for SA and WA. The results of A, β, α_M calculated from the measured data are plotted in Fig.7. Concerning A and β we may take the common formula, that is

$$A=31.1\,\gamma^{-0.451}, \qquad \beta=1.05\,\gamma^{-0.453} \qquad (24a,b)$$

Since, however, the magnitude of α_M are slightly but distinctly different between SA and WA, the different relations may be obtained as:

$$\alpha_M = 0.398\,\gamma \qquad \text{(SA)} \qquad (25)$$

$$\alpha_M = 0.355\,\gamma \qquad \text{(WA)} \qquad (26)$$

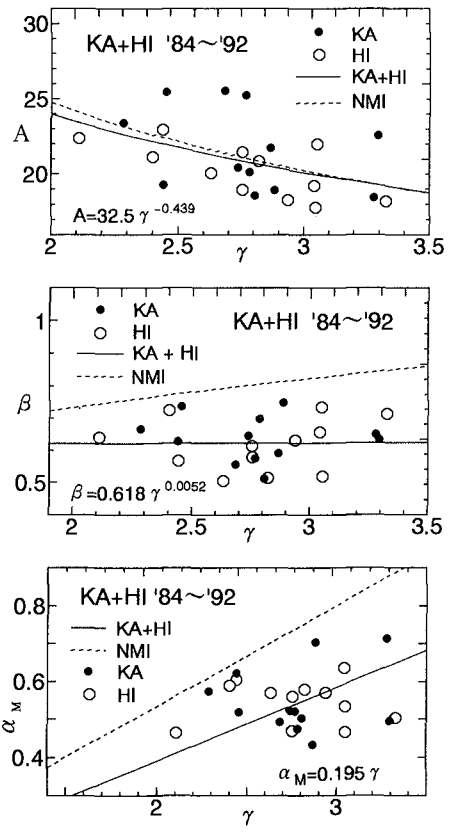

Fig. 6 Plots of A, β, α_M against γ. Monthly data of HI and KA.

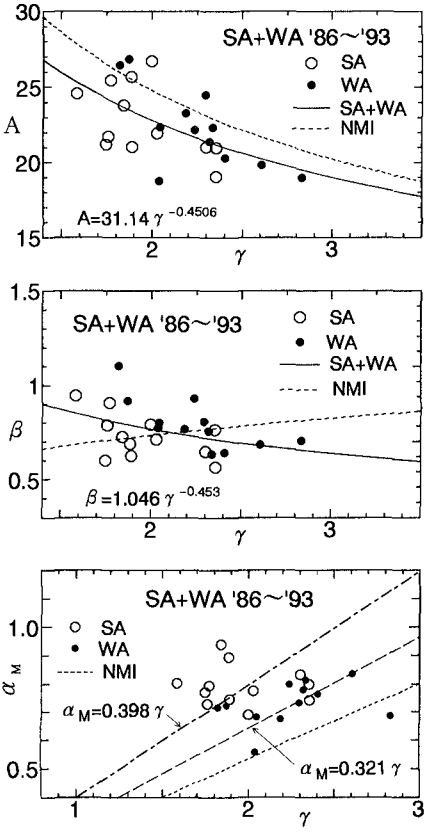

Fig.7 Plots of A, β, α_M against γ. Monthly data of SA and WA.

For reference we show α_g and α_l for SA and WA:

$$\alpha_g = 0.398\,\gamma\,(H_s'/\bar{H}_s)^{0.185} \quad \text{(SA)} \quad (27a)$$

$$\alpha_l = 0.396\,\gamma\,(H_s'/\bar{H}_s)^{-0.137} \quad \text{(SA)} \quad (27b)$$

$$\alpha_g = 0.355\,\gamma\,(H_s'/\bar{H}_s)^{0.176} \quad \text{(WA)} \quad (28a)$$

$$\alpha_l = 0.355\,\gamma\,(H_s'/\bar{H}_s)^{-0.232} \quad \text{(WA)} \quad (28b)$$

It is noted that the magnitude of α_M in the Pacific coast is smaller than that of the NMI method, while it is larger in the Sea of Japan coast as seen from the bottom ones in Figs. 6 and 7. The variability of the estimates by the NMI method are discussed further in the later section.

4. Applicability of Mathiesen's model

Mathiesen (1994) found that the average absolute rate of change of significant wave height S could be expressed by Eq.(15). Figure 8 shows an example of $S(H)$ calculated from the wave data of HI. We can see that the relation of Eq.(15) holds well. Although for the same site the constants q and R in Eq.(15) slightly vary month to month, we neglected the monthly changes similarly to Mathiesen. The mean values of q and R for each location are shown in Table - 1.

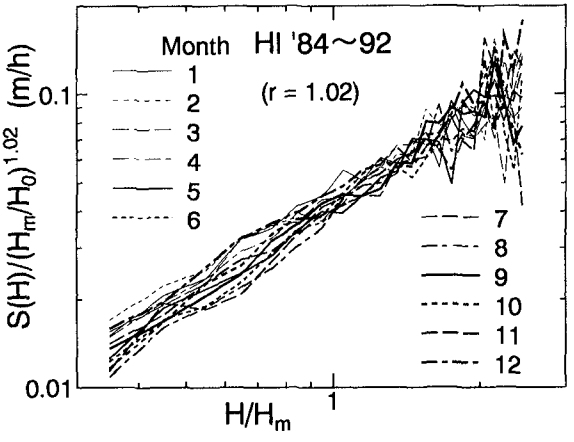

Fig.8 Average absolute rate of change of significant wave height at HI.

The values of A and β calculated from Mathiesen's Eqs.(16) and (17) are compared with those of the NMI method and its improved (data-fitted) equation in Fig.9 (HI:upper, SA:lower) in which the monthly average

significant wave heights H_m are also shown. The suffix M and NMI denote Mathiesen's and NMI method, respectively and no suffix the improved equation method. It is noted that A_M are in general different from A_{NMI} and A, and especially too small in summer.

Some examples of the estimated mean durations by the three methods for

Table-1 Constants in Eq.(7)

Station	q	r
HI	0.045 m/h	1.02
KA	0.045	0.98
SA	0.062	0.85
WA	0.055	0.89

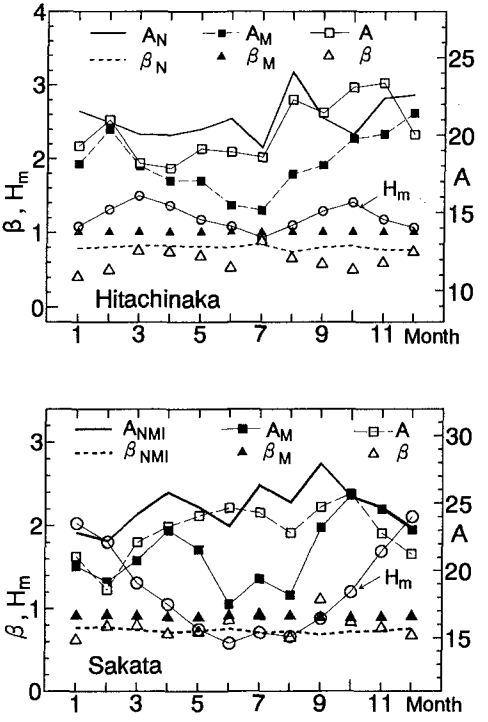

Fig.9 A and β for three method; NMI, improved NMI and Mathiesen at HI (upper) and SA (lower). **Suffix** N or NMI: NMI method, M: Mathiesen's, no suffix: Improved NMI.

exceedance (GE) and non-exceedance (LT) waves are shown together with those calculated directly from the measured data in Fig.10.

Contrary to our expectation Mathiesen's method generally did not show the better agreement with the measured data than NMI method. Improved NMI method which utilized the relations like Eq.(4) yielded the best agreement.

We tried to find the reason why Mathiesen's model showed rather worse results than the NMI method. The relation of Eq.(9), which Mathiesen used

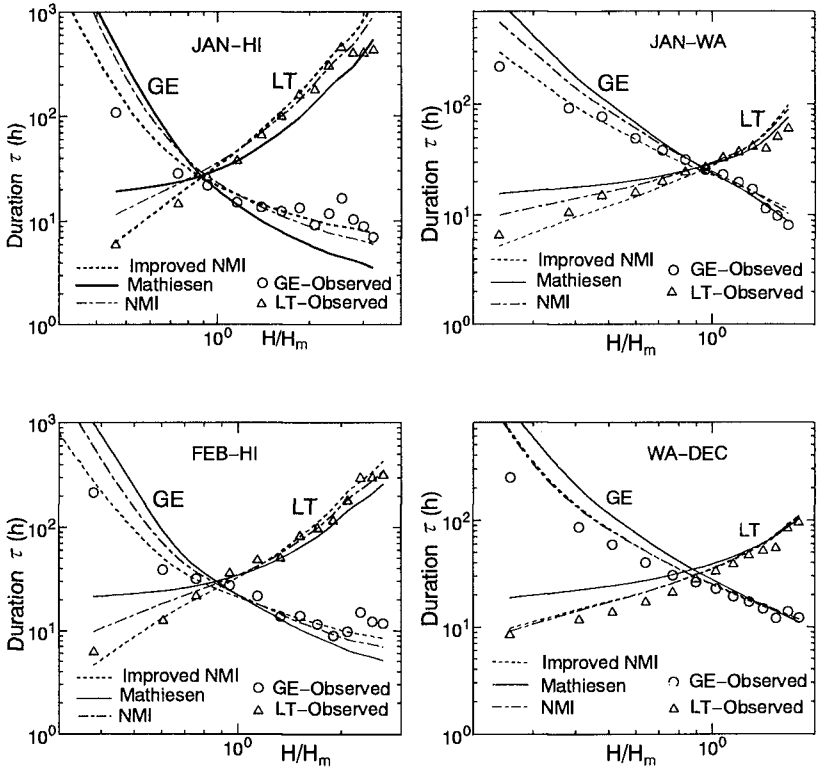

Fig. 10 Comparison of estimates by Mathiesen's method with NMI and improved NMI method. (Mean duration for exceedance and non-exceedance waves for HI and WA.)

as the foundation of his model, is originally valid to the time series of surface displacement of water waves and $\nu(0)$ coincides with a reciprocal of the zero up-cross wave period. With regard to the time series of wave eight which we used, it was found that the validity of Eq.(9) became poor at the wave height apart from the mean height. It was also found that the degree of accuracy of Eq. (10) was reduced depending on the month.

5. Discussion

Figure 11 shows the monthly values of γ for 4 sites analyzed in this study. As seen from this figure, the values of γ for the two sites (SA and WA) in the Sea of Japan are, except the months in winter, smaller than those of the two sites (HI and KA) in the coast of Pacific Ocean.

In the figures 3, 6 and 7, *etc.* from which the expressions of A and β were

obtained in terms of γ, the point were scattered considerably. So it may give an impression that the estimates by NMI method or improved NMI method include a great deal of errors. The figure such as Figs. 2 and 5 is not suitable to grasp the general degree of agreement. In order to see the accuracy of the estimated values compared with the observed values, we prepared Fig.12. It shows the cumulative frequency (times/month) of the non-exceedance waves ($H \leqq 1.0$m) and exceedance waves ($H \geqq 1.5$m) for durations of 24 and 72 hours for HI in the Pacific coast and WA in the Sea of Japan. Triangle points

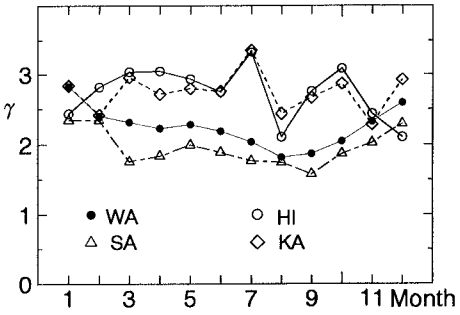

Fig. 11 Monthly variations of γ for 4 stations.

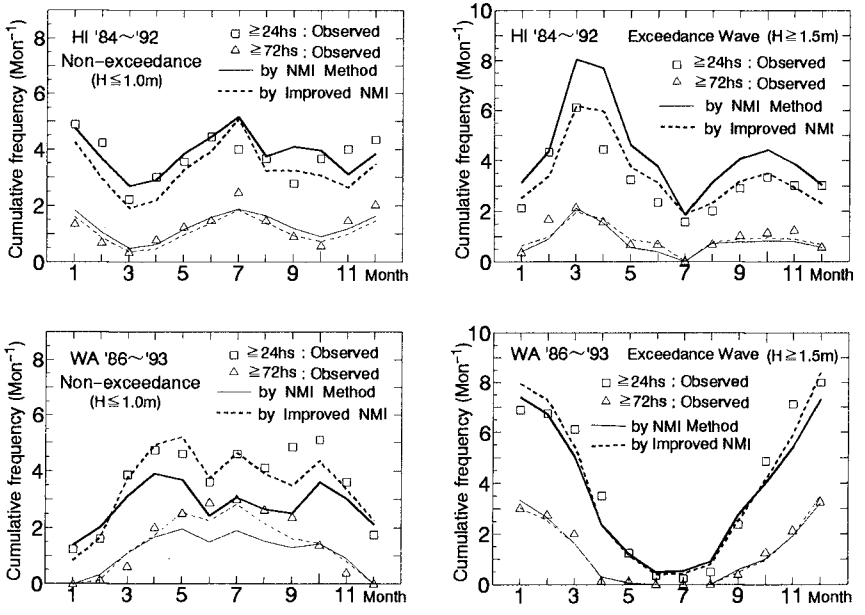

Fig.12 Estimates of wave persistence by NMI and improved NMI method in comparison with observed data at HI (upper) and WA (lower).

represent the observed values for 24 hours and square points 72 hours. Solid and dotted lines denote the estimates by NMI and improved NMI method, respectively.

From these figures it is seen that the increasing or decreasing tendencies of the estimated results are in good agreement with those of the observed values, and it may indicate the excellent feature of the NMI method in which the estimates of wave height persistence statistics are obtained from cumulative probability distributions of significant wave height $Q(\geqq H_s)$ and a single parameter γ .

6. Conclusions

The NMI method was found to give reasonably good estimates, though it yielded slightly deviated result for each site. Modification like Eqs.(4) and (7) was effective. Although considerable scatters were found in evaluating the optimum NMI parameters for the different data sets, it was found from the monthly data analysis that the NMI method yielded good tendencies of variations of wave persistence statistics in comparison with the real values calculated directly from the measured wave data.

It was found contrary to our expectations that Mathiesen's model did not give any better estimates than the NMI method for the mean durations of both exceedance and non-exceedance waves. Some possible reasons were discussed. .

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