ABSTRACT

The aim of this paper is to investigate experimentally and numerically the breaking limit, breaking and post-breaking wave deformation due to three different types of submerged structures such as bottom-seated, non-bottom-seated fixed and tautly-moored structures. Based on laboratory experiments, the breaking limits have been formulated for three different types of the submerged structures. Moreover, a modified SOLA-VOF method with the non-reflective wave generation method has been proposed and found to be very effective in evaluating the wave breaking process and post-breaking wave characteristics.

INTRODUCTION

An accurate prediction of the wave deformation due to a submerged structure is very important for the nearshore sea environment. Most of the foregoing researches have discussed wave breaking and breaking wave deformation only due to bottom-seated submerged structures. On the other hand, the breaking limit and breaking wave deformation for other types of submerged structures have been little investigated. Recently, a research and development of numerical computation techniques has been highlighted to evaluate the wave breaking process with a strong energy dissipation (for example; Miyata et al., 1988, Takigawa et al., 1991, Park and Miyata, 1994, and van Gent et al., 1994). A reliable numerical analysis model, however, has not been established yet to compute the wave deformation after breaking.

This paper is aimed to discuss experimentally and numerically the breaking limit, length of breaker zone, and post-breaking wave deformation due to three different submerged structures such as bottom-seated, non-bottom-seated, and tautly-moored structures. First of all, breaking limits, breaker
types and length of breaker zone are experimentally investigated for three different submerged structures in a two-dimensional regular wave field. Next, a numerical analysis model which combines the SOLA-VOF method (Hirt and Nichols, 1981) with the non-reflective numerical wave source (Brorsen and Larsen, 1987) has been developed to evaluate the wave breaking and its post deformation in the regular wave field. The validity of the present numerical calculation method is proved by comparing the computed results with the experimental ones.

LABORATORY EXPERIMENTS

Two-dimensional laboratory experiments were carried out using a wave tank at Nagoya university. The still water depth $h$ was 40, 50 and 60cm. The structural types employed in this experiment are bottom-seated (Type I), non-bottom seated (Type II) and tautly-moored structures (Type III), as shown in Fig. 1. The non-dimensional structural width $B/L$ ($B$: width of the structure, $L$: incident wavelength) for Type I ranged from 0.025 to 1.2, and the non-dimensional structural height $D/h$ ($D$: height of the structure) was 0.4, 0.6 and 0.8. The length and height of the structures (Type II and III) were 68cm and 23 or 34cm, respectively. The initial angle $\theta_0$ of the mooring line to the bottom was 45 degrees in case of Type III. The submerged water depth $h_d$ was 6, 9 and 12cm. The regular waves with periods $T=0.6 \sim 1.68s$ and steepness $H/L=0.02 \sim 0.13$ ($H$:incident wave height) were generated. The total number of experimental runs was about 540. For each experimental run, the water surface profile $\eta$ and the water particle velocities $u$ and $w$ were measured with capacitance-type wave gages and electromagnetic type velocimeters, respectively. Also, the wave breaking process was recorded using a video camera.

NUMERICAL ANALYSIS

The numerical analysis model which combines the SOLA-VOF method with the non-reflective numerical generator has been proposed. The governing equation consists of the continuity equation Eq.(1), Navier-Stokes equations Eqs.(3) and (4) for incompressible fluid, and the advection equation Eq.(5)
which represents the behavior of the free-surface. These equations involve the source term because the wave generation source is placed within the computational domain.

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = q(x, z, t) = \begin{cases} 
q^*(z, t) & : x = x_s \\
0 & : x \neq x_s
\end{cases}
\]  

(1)

\[
q^*(z, t) = \begin{cases} 
1 - \exp(-0.5t/T) \cdot 2U & : t/T \leq 3 \\
2U & : t/T > 3
\end{cases}
\]  

(2)

\[
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial (uw)}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + uq
\]

(3)

\[
\frac{\partial w}{\partial t} + \frac{\partial (wu)}{\partial x} + \frac{\partial w^2}{\partial z} = -g - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + wq + \frac{1}{3} \nu \frac{\partial q}{\partial z}
\]

(4)

\[
\frac{\partial F}{\partial t} + \frac{\partial (F' u)}{\partial x} + \frac{\partial (F' w)}{\partial z} = Fq
\]

(5)

where the Cartesian coordinate system \((x, z)\) is employed, and \(u\) and \(w\) are the velocity components in the respective directions of \(x\) and \(z\), \(t\) the time, \(\rho\) the pressure, \(\rho\) the fluid density, \(\nu\) the kinematic viscosity, \(g\) the gravitational acceleration, and \(q\) the wave generation source with \(q^*\) as the source strength which is only located at \(x = x_s\). The wave generation source \(q^*\) with the horizontal velocity \(U\) corresponding to the third-order Stokes wave is gradually intensified, as given in Eq.(2), in order to produce a regular Stokes wave train and \(T\) is the incident wave period. The VOF function \(F\) represents the volume fraction of the cell occupied by the fluid; the cell with \(F=0\) is the air cell (the empty cell), the cell with \(0 < F < 1\) is the air and water mixture cell (the surface cell) and \(F=1\) is the water cell (the full cell).

Employing the modified SOLA-VOF method with the wave generation source, the velocity components \((u \text{ and } w)\) and the pressure \(p\) at the next time step are determined by using the continuity and momentum equations (Eqs.(1), (3) and (4)). The staggered mesh was adopted for discretization of the calculational domain. The flow chart of this numerical scheme is shown in Fig. 2, where \(D = \partial u/\partial x + \partial w/\partial z - q\). Regarding the boundary conditions, Sommerfeld radiation condition Eq.(6), where \(Q\) is a quantity representing the velocities \(u, w\) and \(C\) is the wave celerity, is applied for the open boundaries. The non-slip condition is used on both the structure surface and the sea bed.

\[
\frac{\partial Q}{\partial t} + C \frac{\partial Q}{\partial x} = 0
\]

(6)

The computational domain is taken as 500cm times 70cm in the respective directions of \(x\) and \(z\). As shown in Fig. 3, the wave generation source is located at an appropriate location determined according to the wavelength, and the origin of \(x\) coincides with the wave generation source. While, the positive direction \(x\)-axis is taken toward the structure and the vertical axis \(z\) is taken positive upward with its origin being on the still water level. The cell
length $\Delta x$ and $\Delta z$ are $1/50L$ and $1/40h$ in respective directions of $x$ and $z$, and the successive time interval $\Delta t$, initially $\Delta t_i=0.01s$, is determined at every time step so that the Courant condition Eq. (7) is satisfied. Here, $\gamma=0.5$ in this study, and $|u|_{\text{max}}$ and $|w|_{\text{max}}$ are, respectively, maximum velocity of $u$ and $w$.

$$
\Delta t < \gamma \cdot \min \left\{ \frac{\Delta x}{|u|_{\text{max}}}, \frac{\Delta z}{|w|_{\text{max}}} \right\}
$$

Figure 4 shows one example of the time history of the calculated water surface profile $\eta(t)$ (solid line) in which the theoretical value of third-order Stokes wave (open circle) is simultaneously shown for comparison. It is obvious that the calculated water surface profile becomes stable and regular after the fifth generated wave. Moreover, both the calculated and theoretical results

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**Fig. 2** Flow chart of Numerical Calculation

**Fig. 3** Definition Sketch
agree well and, hence, the validity of the wave generation by the wave source method is proved.

RESULTS AND DISCUSSIONS
I. Laboratory Experiment

a) Breaking Limit: Analyzing the laboratory experiments, the breaking limit for three different structures is, first of all, discussed. Figures 5(a) and (b) show the relationship between the wave steepness $H/L$ and the non-dimensional submergence depth $h_d/L$ in cases of Type II and III structures, respectively. The critical wave steepness $(H/L)_b$ in case of Type II structure can be represented only by one curve, regardless of the values of $h_d/h$, while the critical value $(H/L)_b$ in case of Type III structure takes a peak value at certain value of $h_d/L$ according to $h_d/h$, as shown in Fig. 5(b). This took place under the resonance condition that the natural period of the motion of the tautly-moored structure is close to the wave period. Employing the concept of breaking limit of partial standing wave (Iwata and Kiyono, 1985), the breaking limit for Type II and III structures can be formulated as

$$
\left( \frac{H}{L} \right)_b = \xi \left[ 0.0845 \left\{ 1 - \exp \left( -1.675 \frac{h_d}{L_0} \right) \right\} \left( \frac{1 - K_R}{1 + K_R} \right) 
+ 0.218 \tanh \left( \frac{2\pi h_d}{L_b} \left( \frac{2K_R}{1 + K_R} \right) \right) \right]
; \xi = \beta / \left[ \alpha (T_n - T)^2 + 1 \right] + 1
$$

(8)

where, $(H/L)_b$ is the critical wave steepness, $K_R$ the reflection coefficient, $L_0$ the wavelength at deep water, $T_n$ the natural period of the tautly-moored structure, and $\alpha$ and $\beta$ are numerical constants (see Fig. 5(b)) which are zero for fixed structures.

Equation (8) agrees well with the experimental values, as shown in Fig. 5. The comparison between Figs. 5(a) and (b) shows that the breaking limit $(H/L)_b$ in case of Type III structure is slightly larger than that in case of Type II structure. In other words, the non-bottom-seated structure is more effective to break waves than the tautly-moored structure.

Equation (8) can be also applied to define the breaking limit for Type I structure. In addition, the breaking limits $(H/h_d)_b$ for Type I structure can be
formulated, as shown in Fig. 6, in terms of $B/L$ for $h/L = 0.13$ and 0.4 without using the reflection coefficient $K_R$. The breaking limits for Type I structure are formulated as follows:

\[
\begin{align*}
(H/h_d)_b &= 0.3656 + 0.3668 \exp(-10B/L) \\
&\quad ; 0.1 \leq B/L \leq 0.6 \text{ for } h/L = 0.13 \\
(H/h_d)_b &= 0.3641 + 0.2313 \exp(-10B/L) \\
&\quad ; 0.1 \leq B/L \leq 0.6 \text{ for } h/L = 0.2 \\
(H/h_d)_b &= 0.4417 + 0.2252 \exp(-10B/L) \\
&\quad ; 0.1 \leq B/L \leq 1.0 \text{ for } h/L = 0.4
\end{align*}
\]  

\[(9)\]

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**Fig. 5** Breaking limit

(a) Non-Bottom-Seated Structure (Type II)

(b) Tautly-Moored Structure (Type III)
b) **Breaker Type**: As shown in Figs. 6 and 7, the change in the breaking limit and the breaker type become very small for $B/L > 0.5$ in case of $h/L=0.2$ and 0.4, and for $B/L > 0.3$ in case of $h/L=0.13$. The breaker type changes from Spilling breaker to S-P breaker (an intermediate type between Spilling and Plunging breaker) and finally to Double breaker (Katano et al., 1992) as $H/h_d$ increases. It is also revealed from Fig. 7 that the critical values of $H/h_d$ for the wave breaking and the breaker type classifications in case of $h/L=0.4$ is larger than other cases. Also, the non-bottom-seated structure, among three different types of the submerged structures, is found to be the most effective one to break waves because a strong circulating flow is usually formed around the structure.

c) **Breaking Position**: As shown in Fig. 8, the non-dimensional breaking position $x_b/L$ ($x_b$: distance from the front of the structure to the breaking position) shifts to the offshore side of the structure by an increment of $H/h_d$ and is almost constant at $x_b/L=0.05$ near the antinode position of the partial standing wave in front of the submerged structure for $H/h_d \geq 1.0$. Inspection of Figs. 7 and 8 reveals that $x_b/L$ shifts to the offshore side of the structure according to the change of the breaker types from Spilling to S-P, and to Double breakers.

d) **Breaker Zone Length**: Non-dimensional breaker zone length $L_b/L$ is plotted versus $H/h_d$ in Fig. 9 with parameters of $h_d/h$ and $B/L$, where $L_b$ is the distance from the breaking point to the location where the breaking wave-caused turbulence with air entrainment disappears. As $H/h_d$ increases, $L_b/L$ becomes larger regardless of $h_d/h$. Further, $L_b/L$ for $h_d/h=0.2$ is larger than that for $h_d/h=0.4$ because of the stronger non-linear interaction between the wave and the structure.

e) **High Harmonic Components**: Figure 10 shows the spatial distribution of the non-dimensional wave height spectrum $2A(f)/H$ around the submerged structure, where $A(f)$ is the amplitude spectrum for frequency $f$ and B.P. is the breaking point. It is found from Figs. 10(a) and (b) that the fundamental harmonic component increases in front of the structure and decays toward the
onshore side of the structure because of the wave energy dissipation due to the wave breaking. On the other hand, higher frequency components grow up above the structure, especially the second harmonic component becomes larger around the submerged structure. This is accounted by the wave energy transfer from the fundamental harmonic component to the second harmonic component due to the non-linear interaction between the wave and the structure. Figures

Fig. 7 Breaking limit and breaker type (Type I)  
Fig. 8 Breaking Position (Type I)
Fig. 9  Breaker Zone Length (Type I)

Fig. 10  Spatial Distribution of Non-dimensional Wave Height Spectrum (Type I)
10(b) and (c) reveal that the spatial distribution of $2A_f/H$ depends on $h_d/h$ and that larger wave energy dissipation takes place with decreasing $h_d/h$. It is obvious from Fig. 10(d) (case of a weak Spilling breaker) that the second harmonic component is seen to change periodically after breaking. According to Massel (1983) who calculated the wave transformation due to submerged structures for non-breaking waves, the beat length $\lambda_2$ has been formulated in Eq. (10), where $k_1$ and $k_2$ are the wave numbers for the fundamental and second harmonic components, and $\sigma$ is the angular frequency of the fundamental component. Applying Eq. (10) to the case presented in Fig. 10(d), $\lambda_2/L \approx 0.6$ is obtained and found to coincide with the corresponding experimental one. Therefore, it can be thought that the free second harmonic component wave is generated even under the breaking wave condition.

\[
\lambda_2 = \frac{2\pi}{(k_2 - 2k_1)} ; \quad \sigma^2 = gk_1 \tanh k_1 h_d, \quad 4\sigma^2 = gk_2 \tanh k_2 h_d \quad (10)
\]

II Numerical Calculation

The post-breaking wave deformation due to a submerged structure is computed using the modified SOLA-VOF method. The non-dimensional water surface profiles $\eta/H$ measured at $x=174$ cm offshore side and $x=290$, 314 and 338 cm onshore side of the Type II structure are plotted versus $t/T$ in Figs. 11(a)~(d) in case of a spilling breaker. In Fig. 11, laboratory experimental values are also shown for comparison. It is found that higher frequency component waves are generated as mentioned above and, hence, the time history

![Fig. 11 Comparison of Calculated and Experimental Water Surface Profile](Type II ; $H/L=0.04$, $h/L=0.33$)
of the water surface profile has a complex form in the onshore region of the structure after breaking. From Fig. 11, the calculated values are found to be in good agreement with experimental ones. Good agreement between the calculated values and laboratory experiments is also confirmed for the water particle velocity, as shown in Fig. 12. Therefore, it is demonstrated that the present numerical calculation method can reproduce well the water surface profile and the water particle velocity before and after breaking due to an impermeable submerged structure.

Figures 13(a)~(d) show the spatial variations of the water surface profiles and the particle velocities around the structure at \( t = 7.55, 7.60, 7.72 \) and \( 7.83 \)s, respectively, under the same condition stated in Fig. 11. Figures 13(a) and (b) show that the wave passing over the submerged structure breaks with an overturning wave front, and Figs. 13(c) and (d) show that the broken wave is deformed into several wave components with two or three peaks in one wavelength. According to Fig. 14, in case of Type I structure, large vortex is found to be formed on the onshore side of the submerged structure after the wave passes over it. In addition, analysis of video tape recorder confirms that the breaker type and wave deformations evaluated with the numerical calculations are very similar to those measured at the laboratory. However, the air-bubbles entrainment due to wave breaking, which was observed by the video tape recorder and the visual observations, cannot be simulated in the numerical calculations.

As shown in Fig. 15, both the numerical and experimental results show that the high mean onshore velocity is generated near the free-surface onshore side of the structure and the high mean downward velocity is observed to occur near \( x = 340 \)cm. The mean offshore velocity, i.e. the return flow, is also generated to compensate the onshore mass transport caused by wave breaking around the still water level. The return flow is, in particular, strong below the structure because the flow cross sectional area below the structure abruptly diminishes from the onshore side of the structure, which could accelerate the flow velocity. According to Fig. 15(a), the upward velocity takes place in front of the structure where the antinode of the partial standing wave seems to appear, and the offshore steady flow takes place just above the crown of the structure. However, it is expected that there is a strong onshore steady flow above the wave trough (i.e. around the still water level) over the structure. Judging from
Fig. 13 Results of Numerical Calculation (Type II)
\(H/L=0.04, h/L=0.33\)
the above-stated, it can be said that a circulating flow is formed around the submerged structure. Judging from Figs. 15(a) and (b), the proposed numerical calculation scheme is shown to agree well with the measured mean wave velocity field in laboratory experiments.

Fig. 14  Results of Numerical Calculation (Type I)  
\((H/L=0.077, h/L=0.4)\)

(a) Calculation

(b) Experiment

Fig. 15  Spatial Distribution of Mean Velocity around Structure (Type II)  
\((H/L=0.04, h/L=0.33)\)
CONCLUSIONS

The breaking limit, breaking and post-breaking wave deformation due to three different types of submerged structures have been discussed experimentally and numerically. Main conclusions in this study are summarized as follows:

1) Breaking limits have been formulated experimentally for three different types of submerged structures.
2) Breaker types have been also formulated experimentally for two types of submerged fixed structures.
3) Among the three types of structures, the non-bottom-seated structure has been found to be the most effective one to break the waves.
4) A numerical model using the modified SOLA-VOF method has been found to evaluate well the wave deformation before and after breaking in case of a spilling breaker.

REFERENCES


