CHAPTER 191

WAVE TRANSMISSION PAST VERTICAL WAVE BARRIERS

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Abstract

Three theories for predicting regular wave transmission past vertical wave barriers are evaluated using three sets of experimental data. The three theories are: (1) the power transmission theory of Wiegel (1960), (2) and modified power transmission theory that includes effects of wave reflection, and (3) the eigenfunction expansion theory of Losada et al. (1992). Under deep and near-deep water conditions - which are typical of most design conditions - the theory of Wiegel is found to over predict wave transmission under most circumstances. The modified power transmission theory provides better agreement with the data. The eigenfunction method provides good agreement for deep wave barrier drafts but overestimates transmission for shallow drafts.

Introduction

For more than 30 years, estimates of wave transmission past vertical wave barriers (vertical wall breakwaters, sometimes called wave screens, wave fences, skirt breakwaters, or curtain walls) have been based primarily on the theory and experimental data published by Wiegel (1960). The "Wiegel Theory" has been adopted as the recommended practice by both the Army Corps of Engineers (1984) in the *Shore Protection Manual* and by the Naval Facilities Engineering Command (1982) in the *Coastal Protection Design Manual 26.2*. While other theories have been published since then, none has been as widely adopted for design purposes and none has been accompanied by new experimental data.

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Despite the widespread use of the Wiegel theory, questions have arisen recently[3] concerning its accuracy. Wiegel himself recognized that the theory was not physically rigorous and he considered it a first approximation based on some limiting assumptions (Wiegel, 1995, personal communication). In general, a comparison of Wiegel's theory to his own data shows that his theory tends to overpredict wave transmission under deep water conditions and underpredict transmission as the relative water depth became more shallow. Since most wave barriers are built in deep or near-deep water conditions, use of this theory in design may produce deeper wave barrier drafts, at greater cost, than would actually be required to achieve some desired level of wave transmission.

Recently, several more sophisticated theories for wave transmission have been proposed based on numerical solutions of the boundary value problem for waves interacting with a vertical barrier. Liu and Abbaspour (1982) developed a numerical solution based on the boundary integral equation method, while both Losada, Losada, and Roldan (1992) and Abul-Azm (1993) developed numerical solutions based on the method of eigenfunction expansion. Compared to Wiegel's simple theory, these theories are more difficult to apply because they require complex matrix solutions. In addition, the numerical solutions have not been widely or rigorously verified through comparison to measurements.

In this paper, we evaluate three different theories for predicting the transmission of regular waves past wave barriers and we then compare these theories to some new experimental data for wave transmission past vertical wave barriers. The three theories that are evaluated include: (1) the original power transmission theory of Wiegel, (2) the eigenfunction expansion theory of Losada et al. and Abul-Azm, and (3) a modified power transmission theory which was developed in the course of this study. This modified theory is, like the original Wiegel theory, based on the wave power transmission past the wave barrier and, like the Wiegel theory, predicts the wave transmission is a simple closed-form equation. Unlike the original derivation, however, the modified theory accounts for the effects of partial wave reflection from the barrier and this results in different (lower) transmission than is predicted by the Wiegel theory

These theories are then evaluated using laboratory data from three different sources. This includes the original Wiegel data as well as additional laboratory tests data published by Peratrovich, Nottingham & Drage, Inc. (1992) based upon tests conducted by the British Columbia Research Corporation (BCRC). The third set of data was then collected as part of this study and is based on experiments conducted at the U.S. Naval Academy Hydromechanics Laboratory (NAHL).

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[3] Based on discussions at a Wave Barrier Design workshop, held by Peratrovich, Nottingham & Drage, Inc. in Seattle, Washington, on April 24-25, 1995

Definition of a Vertical Wave Barrier

A definition sketch of a vertical wave barrier is shown in Figure 1. The wave barrier consists of an impermeable vertical wall with a draft or penetration, w, in water of depth d. The wave field consists of incident regular waves with height H_i and frequency σ, along with transmitted waves of height $H_t=K_tH_i$, and reflected waves of height $H_r=K_rH_i$, where K_t and K_r are the transmission and reflection coefficients. The water depth is assumed uniform on both sides of the wall so that the wave length, L, and the wave group velocity, Cg, are equal on both sides. If we define the wavenumber as $k = 2\pi/L$, then the wave transmission is fundamentally dependent on two dimensionless variables: the relative water depth kd and the relative barrier penetration kw.

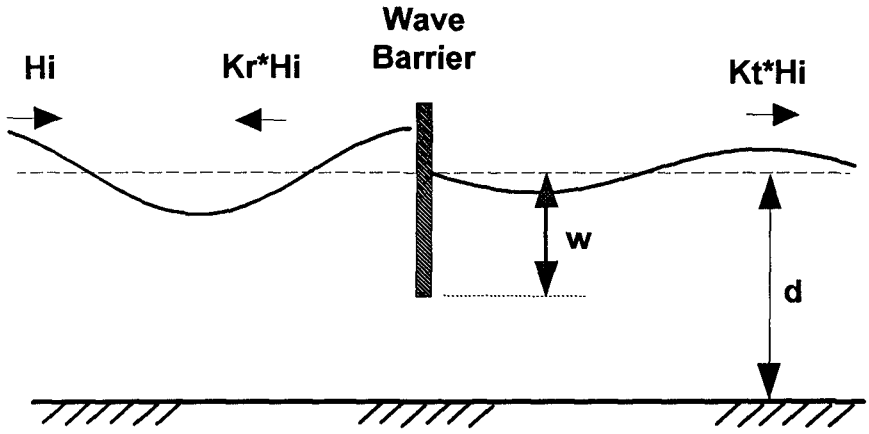

Figure 1. Definition sketch of wave interaction with a vertical wave barrier.

Wiegel Power Transmission Theory

The Wiegel theory is based on the concept that wave motions behind the wall (downstream) are related to the wave power transmission below the wall. Wave power is computed as the depth-integrated product of wave induced dynamic pressures, p, and wave-induced horizontal fluid velocities, u, time-averaged over one wave cycle. Wiegel then assumed that the net wave power transmitted behind the wall (over the full depth) was equal to the fraction of the incident wave power below the bottom of the wall as

$$\frac{1}{T}\int_0^T\int_{-d}^{0} p_t u_t \, dz \, dt = \frac{1}{T}\int_0^T\int_{-d}^{-w} p_i u_i \, dz \, dt \tag{1}$$

Substituting the expressions for dynamic pressure and horizontal fluid velocity from linear wave theory and carrying out the integration then leads to the solution for the transmission coefficient given by Wiegel (1960) as

$$K_t = T_F^{1/2} \tag{2}$$

where we introduce the *transmission function*, T_F, which is given by

$$T_F = \frac{2k(d-w) + \sinh 2k(d-w)}{2kd + \sinh 2kd} \tag{3}$$

Modified Power Transmission Theory

In the modified power transmission theory, the same basic approach is used but the effects of wave reflection are also considered. This time, the dynamic pressures below the barrier are assumed to be given to first order by the sum of incident and reflected pressures, $p_i + p_r$. Because pressures are additive, the net pressure acting on a vertical plane below the wave barrier is greater than that assumed by Wiegel. In a similar way, the horizontal fluid velocities below the wave barrier are modified by reflection. However, the effective velocity is $u_i - u_r$ and is reduced from that assumed by Wiegel.

Based on the above arguments, the transmitted wave power downstream can be equated to the net wave power transmitted under the barrier as

$$\frac{1}{T}\int_0^T\int_{-d}^{0} p_t u_t \, dz \, dt = \frac{1}{T}\int_0^T\int_{-d}^{-w} (p_i+p_r)(u_i-u_r) \, dz \, dt \tag{4}$$

Substituting the expressions for dynamic pressures and horizontal fluid velocities from linear wave theory and canceling common terms then gives:

$$K_t^2 = (1-K_r^2)\ T_F \tag{5}$$

By including the effect of the reflected wave, the solution given above contains two unknowns and cannot be solved without introduction of another relationship between K_t and K_r. Since equation (5) implicitly assumes that there are energy losses in the system, the necessary relationship is one that guarantees conservation of fluid mass, or continuity of the fluid velocities, below the wave barrier as $u_t = u_i - u_r$. Ignoring any phase shifts that may occur across the wall, and assuming that velocities are described by linear wave theory, the matching condition requires that

$$K_t = 1 - K_r \tag{6}$$

Substitution of equation (6) into equation (5) then gives the following solution for transmission coefficient from the modified power transmission theory:

$$K_t = \frac{2\,T_F}{1 + T_F} \tag{7}$$

where the transmission function T_F is defined in equation (3). Because of the inclusion of the effects of wave reflection, equation (7) predicts wave transmission coefficients that are smaller than those predicted by the Wiegel theory in equation (2). This may be seen most readily in equation (5) where it is clear that the effect of wave reflection (with reflection coefficient K_r greater than zero but less than one) is to decrease the wave transmission compared with that predicted by Wiegel.

The modified power transmission theory is, like the Wiegel theory, an approximation of the actual wave transmission process. From a theoretical standpoint, the method appears inconsistent because the usual balance of incident, reflected, and transmitted wave energy is not preserved. In addition, the modified theory, like the Wiegel theory, produces inconsistent results when taken in shallow water limit. However, the method is intended as a simple engineering solution and, as will be shown, it provides significantly better results when compared to measured wave transmission than the Wiegel theory for most conditions of interest.

Eigenfunction Solution

Because of the theoretical limitations of the power transmission theories, it is next of interest to consider mathematically exact solutions for linear water wave interaction with a thin vertical barrier. Such a solution has been given by both Losada et al. (1992) and Abul Azm (1993) based on eigenfunction expansion methods and their solution will be further considered here. In this paper, the method will only be presented in a summary form and the reader is referred to the original papers for a more thorough description of the method.

The eigenfunction expansion method involves solution for the velocity potentials on the upwave side (wavemaker or incident wave side) and on the downwave side (transmitted wave side) of the wave barrier. These upwave and downwave potentials must then be appropriately matched at the location of the wave barrier ($x=0$). Following Dalrymple and Martin (1990), these potentials must be harmonic in time with frequency σ and must have a spatial dependence (in x and z) given by

$$\Phi_{up} = Z_1 e^{-ik_1 x} + \sum_{n=1}^{N} R_n Z_n e^{ik_n x} \qquad \Phi_{dn} = Z_1 e^{-ik_1 x} - \sum_{n=1}^{N} R_n Z_n e^{-ik_n x} \qquad (8)$$

Equation (8) automatically satisfies the requirement that the velocities must be matched at all elevations on and below the barrier. In this form, the first term in each velocity potential is the incident progressive wave mode while the terms in the summation includes both the scattered progressive wave ($n=1$) and the evanescent wave modes ($n>1$), all with unknown complex amplitudes R_n.

The functions Z_n in equation (8) describe the depth-dependence of the wave modes and are given by

$$Z_n = \frac{i g H_i}{2\sigma} \frac{\cosh k_n(d+z)}{\cosh k_n d} \qquad (9)$$

The wavenumbers k_n are given by the solution of the dispersion equation

$$\sigma^2 = g k_n \tanh k_n d \qquad (10)$$

where the first (real) root is the linear progressive wavenumber, $k_1 = k$, and where there are then an infinite set of imaginary roots for $n>1$.

The solution for the complex amplitudes R_n must satisfy two additional physical requirements: (a) the velocities must be zero on both sides of the barrier in the upper region where $-w<z<0$, and (b) the velocity potentials (or equivalently the dynamic pressures) must match in the gap below the barrier where $-d<z<-w$. As a result, two distinct equations (from upper and lower regions) are obtained - the so-called dual-series relationships noted by Dalrymple and Martin (1990) - and both must be satisfied simultaneously to find the unknowns R_n.

It may then be shown that two equivalent methods may be used to satisfy the matching conditions: one through a least squares procedure and the other through a more direct procedure. In the first approach, used by Losada et al. (1992) and Abul-

Azm (1993), the matching conditions are first applied locally, retaining the vertical (z) dependence. The resulting dual series relationships are then combined and re-written as one mixed boundary condition which must equal zero over the full depth. This combined function is then solved in a least-squares sense in which the square of the function is minimized. In a second approach, used in this paper, the matching conditions are applied and are again combined into one mixed boundary condition. This combined function is then, however, multiplied by the orthogonal functions Z_n and depth-integrated over the full depth. This results in a single matrix equation that can be solved directly without the need a least-squares solution.

Following the second method, the mixed boundary condition to be satisfied, denoted *G(z)* as in Losada et al. (1992), is defined for the upper and lower regions as follows. In the upper region, the horizontal velocities ($u=\partial\Phi/\partial x$) are set equal to zero at the wave barrier ($x=0$) resulting in

$$G(z) = -k_1 Z_1 + \sum_{n=1}^{N} R_n k_n Z_n = 0 \qquad -w < z < 0 \tag{11}$$

In the lower region, the velocity potentials from equation (8) are matched directly under the wall ($x=0$) as

$$Z_1 + \sum_{n=1}^{N} R_n Z_n = Z_1 - \sum_{n=1}^{N} R_n Z_n \tag{12}$$

Following cancellation of the leading terms, and after multiplying by k_1 to make equation (12) dimensionally consistent with equation (11), the remaining portion of the mixed boundary condition is obtained as

$$G(z) = 2 k_1 \sum_{n=1}^{N} R_n Z_n = 0 \qquad -d < z < -w \tag{13}$$

The mixed boundary condition *G(z)* can then be satisfied in the usual way by employing the orthogonality properties of the depth-dependent eigenfunctions Z_n from equation (9) as

$$\int_{-d}^{0} G(z) Z_m \, dz = 0 \tag{14}$$

This yields the following set of matrix equations which must be solved for the unknown amplitudes R_n

$$\sum_{n=1}^{N} R_n \left(2 k_1 X_{nm} + k_n Y_{nm} \right) = k_1 Y_{1m} \tag{15}$$

where the functions X_{nm} and Y_{nm} are defined by Losada et al. (1992) and are given by

$$X_{nm} = \int_{-d}^{-w} Z_n Z_m \, dz \tag{16}$$

$$Y_{nm} = \int_{-w}^{0} Z_n Z_m \, dz \tag{17}$$

Once the matrix in equation (15) is solved for the unknowns R_n, the transmission coefficient is obtained from the first term, R_1. The reflection and transmission coefficients for the progressive wave modes are given by

$$K_r = | R_1 | \qquad K_t = | 1 - R_1 | \tag{18}$$

Numerical computations have shown that the solution obtained by employing the orthogonality properties of the eigenfunctions is numerically equivalent to that obtained by Losada et al. (1992) using the least-squares solution procedure. It is noted, however, that the matrix in equation (15) is somewhat easier to solve than that given by Losada et al. in that it has stronger diagonal dominance.

Comparison to Data

The three theories for wave transmission past vertical wave barriers are now compared to available laboratory data for regular waves. Two sources of published data were considered: (1) the data given by Wiegel (1960) and (2) the data given by Peratrovich, Nottingham, and Drage (1992) from tests conducted at the British Columbia Research Corporation (BCRC). These data were then supplemented by additional data collected in the Naval Academy Hydromechanics Laboratory (NAHL).

The NAHL tests were conducted in a wave tank 120 feet (36.6 m) long, 8 feet (2.43 m) wide, and 5 feet (1.52 m) deep. A thin rigid wall (2 inches or 5 cm thick) was placed about 60 feet (18.3 m) from the wavemaker. The wall was supported at the sides of the tank and was also backed by an aluminum frame to prevent deflection of the wall. With the fixed water depth, four wall penetrations were tested producing four values of the relative wall penetration, w/d, was 0.4, 0.5, 0.6, and 0.7.

In these experiments, 80 tests were performed with regular waves. The wave periods varied from 0.9 sec to 2.5 sec and wave heights ranged from 1 in (2.5 cm) to almost 9 inches (23 cm). Combinations of these parameters were used to obtained values of wave steepness, H_i/L, between 0.01 and 0.06 with most tests being in the range of 0.02 to 0.04. Incident waves were measured with a fixed wave gage near the wavemaker while transmitted waves were measured with the fixed gage located about 15 feet (4.6 m) behind the wall. In all tests, a series of 5 to 10 waves of uniform height was generated and measurements were limited to exclude any effects of wave reflection from the beach landward of the wall or from the wavemaker.

Figures 2a through 2h present comparisons of measured and predicted transmission coefficients for regular waves. Each figure represents a specific relative water depth, d/L. Transmission coefficients are then plotted as a function of the relative wall penetration, w/d. Note that in some cases where more than one source of data is used, the relative water depths were approximated in order to compare the various theories and data sets on the same graph. As an example, in Figure 2b, tests conducted by Wiegel used a relative depth of 0.68, while tests conducted by BCRC used a relative depth of 0.73. Theoretical results were based on an approximate average value of $d/L=0.70$.

Figures 2a, 2b, and 2c show measured and predicted wave transmission for deep water conditions where the relative depth is about 0.5 or higher. In these figures, it is evident that the Wiegel theory overestimates the wave transmission while the modified theory provides much better predictions at all values of relative depth and wave barrier penetration. For those cases where the penetration reached one-third to one-half of the water depth, wave reflection from the wave barrier is expected to have been most pronounced. It is for these conditions that the differences between the two theories are particularly large and the modified theory, which includes the effects of wave reflection, provides a significant improvement. The eigenfunction solution overestimates the transmission for small wall penetrations but then agrees with the modified theory and predicts the transmission quite well for deeper wall penetrations.

All other cases shown in Figure 2 represent intermediate or transitional water depths. In Figures 2d, 2e, and 2f it is evident that the modified theory yields improved prediction of wave transmission for most wave barrier penetrations when compared to the Wiegel theory. For barrier penetrations of 0.2 or less (wave barrier penetrations just below the wave trough level), both theories underpredict the measured wave transmission. For cases of deeper penetration, the Wiegel theory again tends to overestimate transmission while the modified theory is more accurate. In these cases, the eigenfunction solution again overestimates transmission for small wall penetrations and agrees with the modified power transmission theory - and with the data - at larger wall penetrations.

In Figures 2g and 2h, the relative depth is fairly shallow (though still greater than the normal limit between intermediate and shallow water wave conditions) and results of the three theories are mixed. Of the two power transmission theories, the Wiegel theory generally provides better predictions than the modified theory. Unlike the deep water conditions, the Wiegel theory no longer overestimates transmission and, in fact, has a tendency to underestimate transmission. The modified theory consistently underestimates the transmission more severely. In these "shallow water" conditions, the eigenfunction theory tends to overestimate the transmission at all values of wall penetration until the wall penetration is over 90 percent of the water depth.

Conclusion and Discussion

Results presented in this paper indicate that for most conditions of interest in engineering design - for deep water conditions with large wave barrier penetration (typically to mid-depth or deeper) - the Wiegel theory generally overestimates wave transmission while both the modified power transmission theory and the eigenfunction expansion theory produce more accurate predictions with little bias toward over or under-prediction of wave transmission. Neither the modified power transmission theory nor the eigenfunction expansion method produce improved estimates of wave transmission under all conditions, however. In near-shallow water, the modified theory tends to underestimate wave transmission while, for these same relative depth conditions, the eigenfunction method tends to overestimate wave transmission.

One puzzling feature of the results in Figure 2 is the degree to which the modified power transmission theory agrees with the complete eigenfunction solution for certain conditions, namely for deep water and large wall penetrations. This can be explained by considering the first term in the eigenfunction expansion. From equation (15), if $n=m=1$, the leading-order behavior of the eigenfunction solution is

$$R_1 = \frac{Y_{11}}{2X_{11} + Y_{11}} \tag{19}$$

The integrals Y_{11} and X_{11} may be expressed in terms of the transmission function, T_F, in equation (3) and it can be shown that $X_{11} \approx T_F$ and $Y_{11} \approx 1 - T_F$. As a result, the leading-order wave transmission from the eigenfunction solution can then be obtained from equation (11) in the following form

$$K_t = 1 - R_1 = \frac{2T_F}{1 + T_F} \tag{20}$$

This result is identical to that obtained from the modified power transmission theory in equation (7). The modified power transmission theory is therefore consistent with

the leading-order behavior of the progressive wave modes in the eigenfunction solution and is valid for conditions where the evanescent wave modes are not expected to be significant.

For conditions where the evanescent modes are important (small wave barrier penetrations), the eigenfunction solution tends to overestimate wave transmission and is less accurate than would be expected. The reason for this appears to be that frictional losses become important due to flow separation at the bottom of the wave barrier. In the NAHL tests, both dye studies and velocity measurements using an acoustic doppler velocimeter confirmed the presence of a large vortex at the bottom of the wall. The inclusion of friction in the eigenfunction solution dramatically improves its predictive ability and this will be the subject of future work.

Acknowledgments

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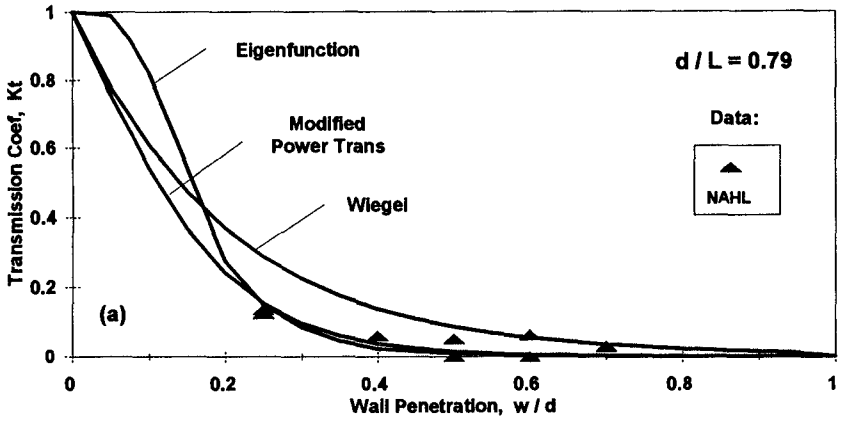
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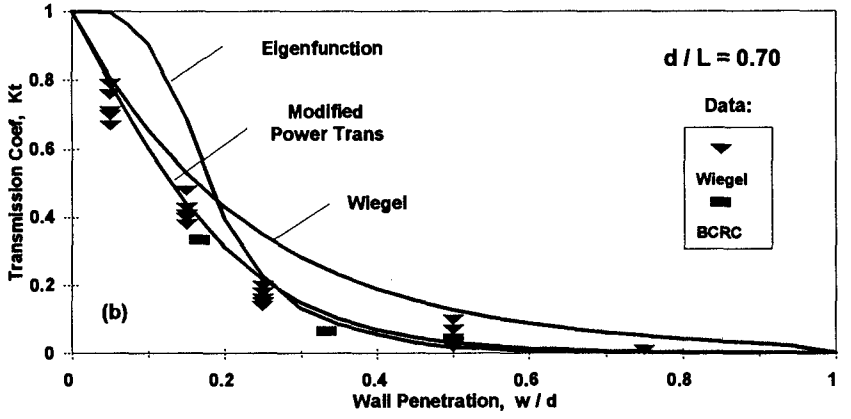
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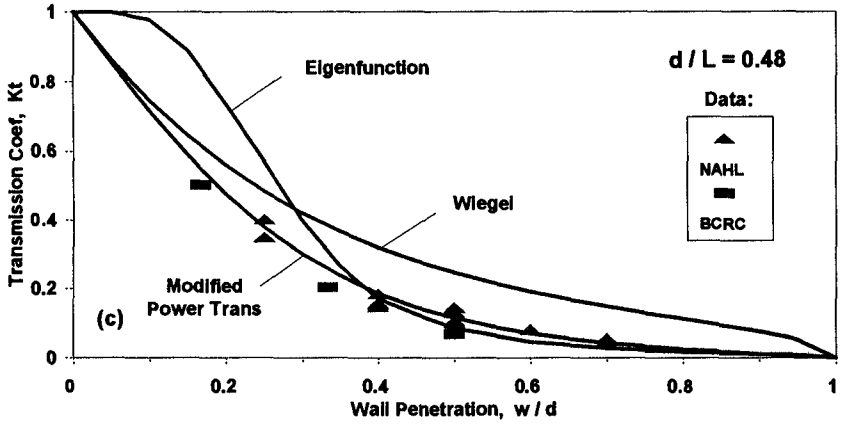
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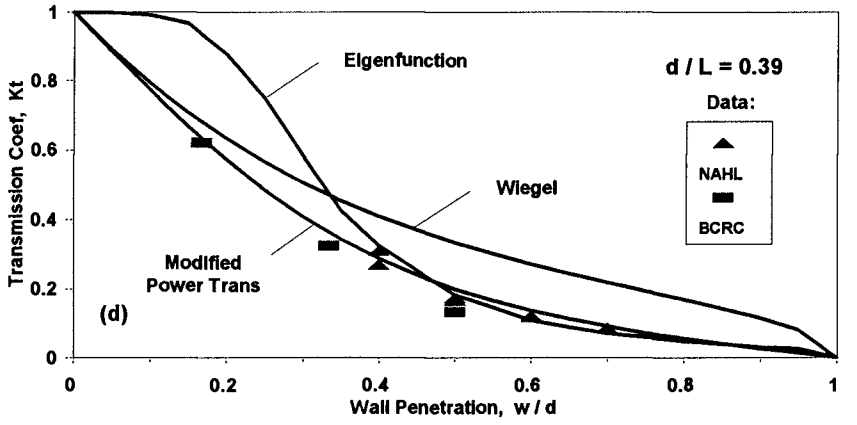
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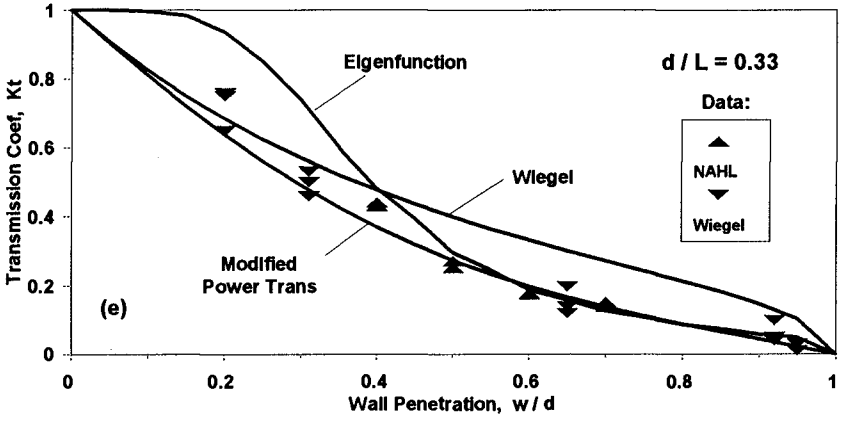
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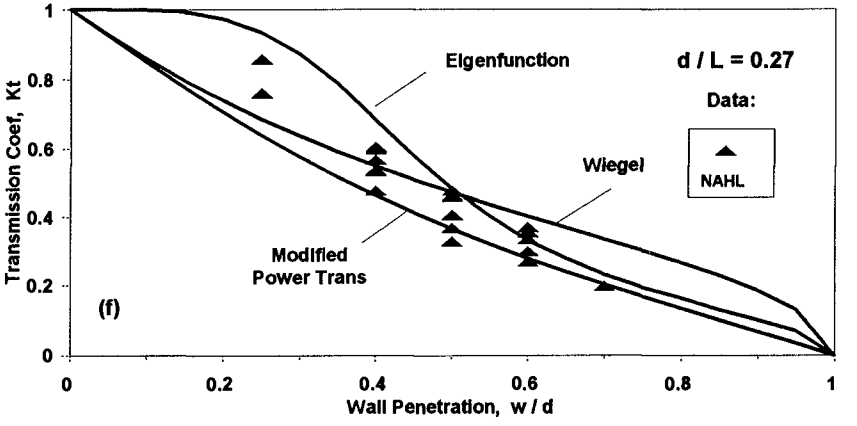
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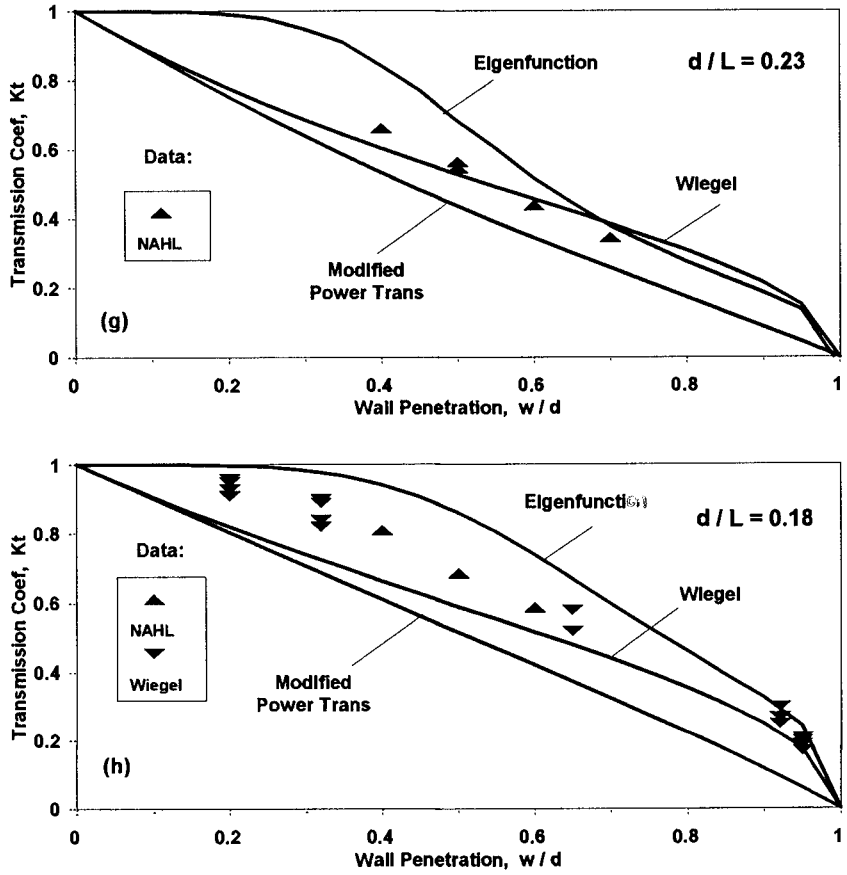

Figure 2a-2h. Comparisons of three theories to measured data for regular wave transmission past vertical wave barriers: (a) $d/L=0.79$, (b) $d/L=0.70$, (c)$d/L=0.48$, (d) $d/L=0.39$, (e) $d/L=0.33$, (f) $d/L=0.27$, (g) $d/L=0.23$, and (h) $d/L=0.18$.