CHAPTER 237

Cross-shore Sediment Transport and Beach Deformation Model

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Abstract

Based on a large amount of published laboratory results, reliable model is developed for computing beach profiles under regular wave actions. The sediment transport is separated into suspended load and bed load. The suspended load is computed as the product of the time-averaged suspended concentration and the time-averaged velocity. The bed load is developed following the similar step as Watanabe (1983) but the applied area is different. The wave model of Dally et al. (1985) is modified and used to compute wave height transformation. The beach profile change is computed from the conservation of sediment mass. The beach deformation model is verified with small scale and large scale experiments. Reasonably good agreement is obtained between measured and computed beach profiles.

I. Introduction

In the previous research works, most of the models were developed based on data with the limited experimental conditions. Therefore their validity is limited according to the range of experimental conditions which were employed in the calibration or examination. The evidence is that there are so many models exist. At this moment, the experimental results obtained by many researchers have been accumulated and a large number of experimental results have become available. It is a good time to develop a model based on the large amount and wide range of experimental results. The present model is developed based on 1138 data sets of 24 sources of published experimental results covering both small and large scale experiments, as shown in table 1.

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Table 1. Sources and number of collected data for present sediment transport study.

Sources	$c(z)$	$u(z)$	H	$h(x,t)$	Others	Experiment
Bosman and Steetzel (1986)	3					SE
Deigaard et al. (1986)	6					SE
Dette and Uliczka (1986)	11					LE
Duncan (1981)					A,12	SE
Hansen and Svendsen (1984)		4	1			SE
Hayakawa et al. (1983)	4					SE
Horikawa and Kuo (1966)			213			SE
Horikawa et al. (1982)	7					SE
Irie et al. (1985)	27					SE
Kajima et al. (1983)	149	219	79	91		LE
Kraus and Larson (1988)				69		LE
Nadaoka et al. (1982)		11	2		h_{ot},2	SE
Nagayama (1983)			12			SE
Nakato et al. (1977)	3					SE
Nielsen (1979)	44					SE
Okayasu et al.(1988)		44	9		h_{ot},9	SE
Okayasu et al.(1989)					h_{ot},47	SE
Sato et al. (1988, 1989)			5			SE
Sato et al. (1990)	14					SE
Sawamoto et al. (1981)	4					SE
Shibayama and Horikawa (1985)			10	11		SE
Skafel and Krishnappan (1984)	8					SE
Sleath (1982)	4					SE
Vongvisessomjai (1986)	4					SE
Total	288	278	331	171	70	

where $c(z)$ is the time-averaged sediment concentration, $u(z)$ is the time-averaged fluid velocity, H is the wave height, $h(x,t)$ is the beach profile, A is the cross section area of surface roller, h_{ot} is the still water depth at transition point, SE is the small scale experiment, and LE is the large scale experiment.

1.1 Governing equation

The cross-shore change in local water depth, h, can be calculated by solving the conservation of sediment mass as the following

$$\frac{\partial h}{\partial t} = \frac{1}{(1-\Lambda)} \frac{\partial q_t}{\partial x} \quad \text{.....} \quad (1)$$

where t is the time, x is the horizontal coordinate in cross-shore direction, Λ is the porosity, and q_t is the total transport rate per unit width.

The sediment transport rate is usually expressed as

$$q_t = \int_{-\delta}^{h} c(z)u(z)\,dz \quad \text{.....} \quad (2)$$

where δ is the level above which there is no effective movement of sand particles, z is the vertical coordinate measured upward from the bed, $c(z)$ is the time-averaged sediment concentration, and $u(z)$ is the time-averaged fluid velocity.

In order to compute the transport rate, q_t, the sediment concentration and fluid velocity should be known first. The sediment concentration profile was described in Shibayama and Rattanapitikon (1993). The following sections will describe about velocity profile, sediment transport, wave model, and beach deformation model, respectively.

II. Time-Averaged Velocity Profiles

Following Okayasu et al. (1988, pp. 93-94), the vertical distribution of time-averaged velocity profile is calculated based on the assumption of eddy viscosity model. By considering time-averaged values, the eddy viscosity model can be expressed as

$$\tau = \rho \nu_t \frac{\partial u}{\partial z} \quad \text{.....} \quad (3)$$

where τ is the time averaged shear stress, ρ is the fluid density, ν_t is the time-averaged eddy viscosity coefficient, u is the time-averaged velocity, and z is the upward vertical coordinate from the bed.

To solve the eddy viscosity model, one boundary condition of velocity should be given and the expression of τ / ν_t should also be known. In this study, the vertical-averaged value of velocity, u_m, is used as the boundary condition for Eq. 3 and will be described in the following subsection.

2.1 Vertically averaged velocity

Using the concept of Svendsen (1984), the vertically averaged velocity, from bed to wave trough, u_m, consists of two components. One is caused by the wave motion, u_w, and the other is caused by the surface roller, u_r.

$$u_m = u_w + u_r \quad \text{.....} \quad (4)$$

Various formulas for computing u_w and u_r have been suggested by the previous researchers. From the previous studies, of Duncan (1981), Svendsen (1984), Stive and Wind (1986), Deigaard et al. (1991), and Fredsoe and Deigaard (1992), we can conclude that there are three formulas for computing u_w and three formulas for computing u_r (see in table 2).

Table 2. Formulas for computing u_m and u_r

	Formula 1	Formula 2	Formula 3
u_w	$\dfrac{B_o \sigma H^2 \coth(kh)}{h}$	$\dfrac{B_o c_w H^2}{h^2}$	$\dfrac{c_w H}{h}$
u_r	$\dfrac{H^3}{(0.7Th^2)}$	$\dfrac{0.9H^2}{(Th)}$	$\dfrac{0.1c_w H}{h}$

where $B_o = (1/T)\int_0^T (\xi/H)dt$, c_w is the phase velocity, H is the wave height, h is the mean water depth, T is the wave period, and ξ is the water surface elevation measured from mean water level.

The general form of u_m may be written as

$$u_m = au_w + bu_r \quad \text{(5)}$$

where a and b are the constants.

The proper combination of u_w and u_r and the constants a,b can be found from multi-regression analysis with the observed u_m.

After the regression analysis and include the effect of transition zone, the final equation for computing u_m can be written as

$$u_m = 0.77\frac{B_o \sigma H^2 \coth(kh)}{h} + b_1 0.1\frac{c_w H}{h} \quad \text{(6)}$$

where b_1 is the constant and expressed as

$$b_1 = \begin{cases} 0 & \textit{offshore zone} \\ \left(1/\sqrt{H} - 1/\sqrt{H_b}\right)/\left(1/\sqrt{H_t} - 1/\sqrt{H_b}\right) & \textit{transition zone} \\ 1 & \textit{inner zone} \end{cases}$$

where H_b and H_t is the wave height at breaking point and transition point.

Example of examination results of u_m along the cross-shore direction are shown in Fig. 1.

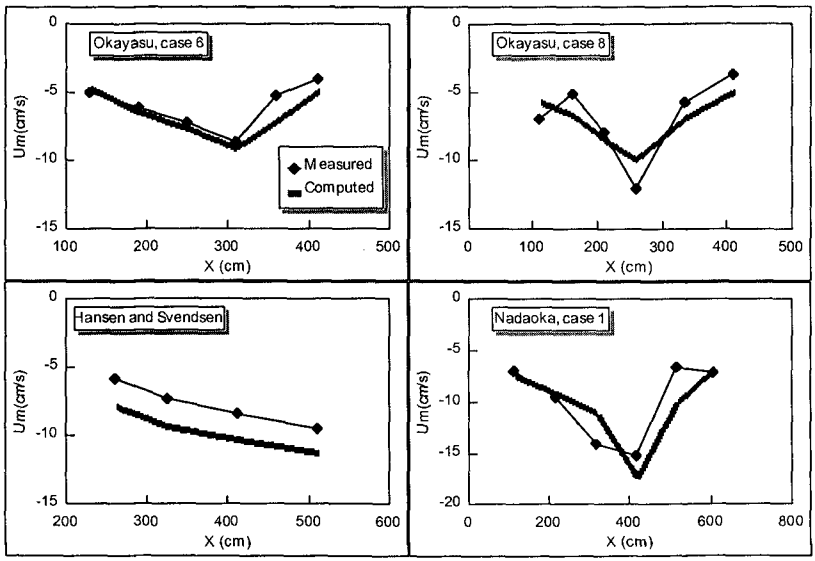

Figure 1. Example of cross-shore variations of measured and computed vertical averaged velocity, u_m.

2.2 Vertical distribution of shear stress and eddy viscosity coefficient

Based on dimensional analysis, Okayasu (1989, pp. 93-94) proposed a formulas for computing τ / ν_t as follows

$$\frac{\tau}{\nu_t} = \rho^{1/3} D_B^{\ 1/3} \left[\frac{k_3}{d_t} + \frac{k_4}{z} \right] \quad \text{.....} \quad (7)$$

where k_3 and k_4 are the constants, d_t is the water depth at wave trough, and D_B is the energy dissipation. From the bore model (Thornton and Guza, 1983),

$$D_B = \frac{\rho g H^3}{4Th} \quad \text{.....} \quad (8)$$

By inserting Eq. 7 into Eq. 3 and then, after an integration of Eq. 3 and use u_m as the boundary condition, the analytical solution of u can be expressed as

$$u = \rho^{1/3} D_B^{\ 1/3} \left[k_3 (\frac{z}{d_t} - \frac{1}{2}) + k_4 (\ln \frac{z}{d_t} - 1) \right] + u_m \quad \text{.....} \quad (9)$$

As mentioned in previous research works, the energy dissipation process at the breaking point is not in the same manner as in the inner zone (e.g., Okayasu, 1989). To incorporate this process, Eq. 9 may be written as follows

$$u = k_5 \rho^{1/3} D_B^{\ 1/3} \left[k_3 (\frac{z}{d_t} - \frac{1}{2}) + k_4 (\ln \frac{z}{d_t} - 1) \right] + u_m \quad \text{.....} \quad (10)$$

where k_5 is the constant and equal to 1 at the inner zone.

From the multi-regression analysis and assume linear distribution of energy dissipation in the transition zone, Eq. 10 become

$$u = b_2 \rho^{1/3} D_B^{\ 1/3} \left[b_3 (\frac{z}{d_t} - \frac{1}{2}) - 0.22 (\ln \frac{z}{d_t} - 1) \right] + u_m \quad \text{.....} \quad (11)$$

where b_2 and b_3 are the constants and expressed as,

$$b_2 = \begin{cases} 0.3 + 0.7(x_b - x)/(x_b - x_t) & \textit{transition zone} \\ 1.0 & \textit{inner zone} \end{cases}$$

$$b_3 = \begin{cases} (x_b - x)/(x_b - x_t) & \textit{transition zone} \\ 1.0 & \textit{inner zone} \end{cases}$$

where x is the position in cross-shore direction, x_b is the position at the breaking point, and x_t is the position of the transition point.

The comparison between measured u and computed u from Eq. 11, using measured u_m, are shown in Fig. 2. From Fig. 2 we can judge that the form of τ / ν_t in Eq. 7 are accurate enough to be used for computing the velocity profiles. Examples of measured and computed velocity are shown in Fig. 3.

2.3 Formula verification

The prototype scale experiment of Kajima et al. (1983) is used to examine the validity of present formula. By using the same formula as that used for small scale wave flume (Eq. 11), Fig. 4 shows examples of verification results in the term of averaged velocity of each section along the cross-shore direction.

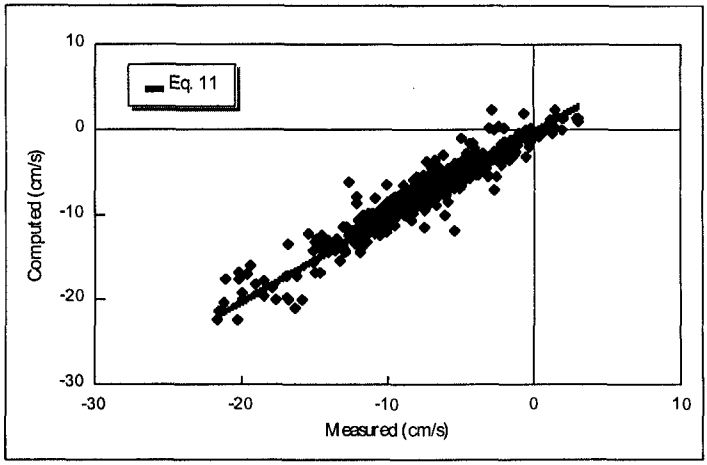

Figure 2. Comparison between measured and computed u (using measured u_m).

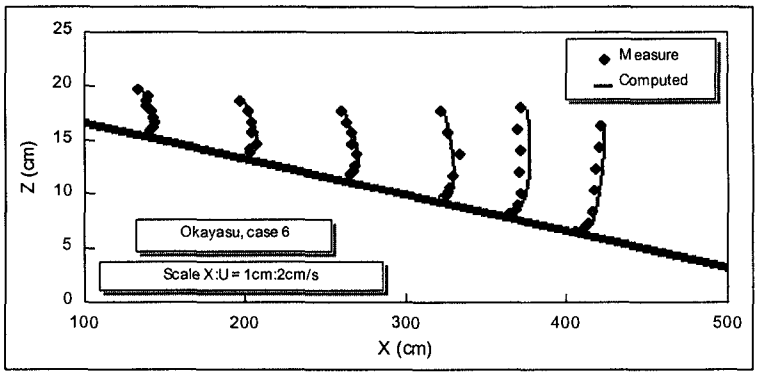

Figure 3. Comparison between measured and computed velocity profiles (laboratory data from Okayasu et al., 1988, case 6).

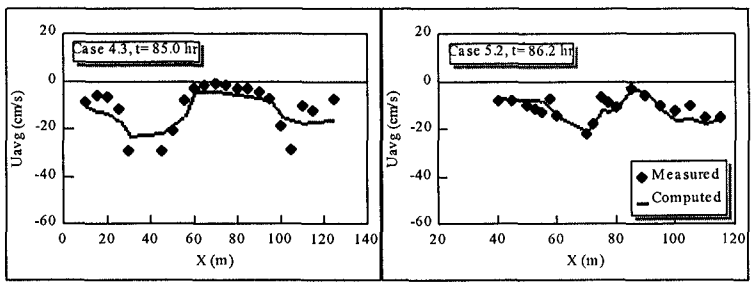

Figure 4. Example of verification results in term of averaged velocity along the cross-shore direction (laboratory data from Kajima et al., 1983).

III. Sediment Transport Rate

The sediment transport rate is separated into bed load and suspended load. The suspended load is expressed as the product of sediment concentration and fluid velocity. The bed load formula is derived in the similar procedure as Watanabe (1983) but the assumption on the time level and region of application are not the same. After calibrate the coefficient of bed load, the total load can be written as

$$q_t = \int_{\delta_s}^{d_t} c(z)u(z)\,dz + 2.0(\psi - \psi_c)\sqrt{\psi}\, w_s d \quad \text{.....} \quad (12)$$

where ψ is the Shields parameter, ψ_c is the critical Shields parameter, w_s is the falling velocity, and δ_s is the bottom boundary layer which computed from Jonsson (1966) formula.

The comparison of computed suspended load, bed load and total load and the measured total load are shown in Fig. 5. It should be note that the separated equation for computing transport direction is not necessary in the present model. The transport direction is depended on the combination of bed load and suspended load. In the present model, bed load is the dominant transport for accretion beach (on-shore directed transport), and suspended load is the dominant transport for erosion beach (off-shore directed transport). These transport directions correspond well with the measured total load transport as shown in Fig. 5.

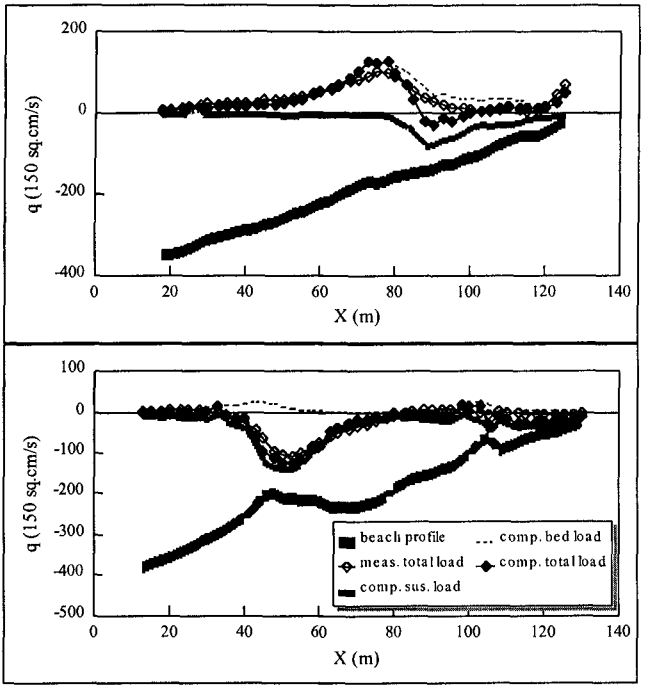

Figure 5. Comparisons between measured and computed sediment transport (laboratory data from Kajima et al., 1983, case 2.2, and 4.3).

IV. Wave Model

For computing beach transformation, the wave model should be kept as simple as possible because of the frequent updating of wave field for accounting the variability of mean water surface and the change of bottom profiles. In the present study, wave height transformation in cross-shore direction will be computed from the energy flux conservation.

$$\frac{\partial E c_g}{\partial x} = -D_B \quad \text{.....} \quad (13)$$

where E is the wave energy density, c_g is the group velocity, and D_B is the energy dissipation rate which is zero outside the surf zone.

4.1 Energy dissipation rate

Widely used formulas for computing energy dissipation rate are Bore model and Dally et al. (1985) model. The Bore model is shown in Eq. 8, and Dally model is

$$D_B = \frac{0.15 c_g \rho g}{8h}\left[H^2 - (\Gamma h)^2\right] \quad \text{.....} \quad (14)$$

The advantage of Dally model is that it is able to reproduce the pause (or stop breaking) in the wave breaking process at a finite wave height on a horizontal bed or in the recovery zone while the Bore model gives a continuous dissipation due to wave breaking. However, for the condition when waves continuously break both two models give nearly the same result of energy dissipation rates (for example see Fig. 6). Based on the above considerations, Dally model is selected for calculation in the present research.

The published experimental results of wave height transformation inside the surf zone have been collected to test the ability of Dally model as shown in the first column of table 3. Total 9 sources of published experimental results, covering 331 data sets, are used in this section. The experiments cover wide range of wave and bottom topography conditions, including both small scale and large scale wave flume experiments. Most of the experiments were performed under fixed bed conditions, except data of Kajima et al. (1983) and Shibayama and Horikawa (1985) which were performed under movable bed conditions.

The verification results are presented in term of error function, ER, as used by Dally et al. (1985), which is defined as

$$ER = 100\sqrt{\frac{\sum_{i=1}^{tn}\left(H_{ci} - H_{mi}\right)^2}{\sum_{i=1}^{tn} H_{mi}^{\,2}}} \quad \text{.....} \quad (15)$$

where i is the wave height number, H_{ci} is the computed wave height of number i, H_{mi} is the measured wave height of number i, and tn is the total number of measured wave height. Small value of ER expresses a good prediction.

The verification results of Dally model are shown in the third column of table 3. From the third column of table 3, we can see that Dally model gives quite good estimation of wave height inside the surf zone.

Although, Dally model gives good estimation to the experimental results, it still has some error. We may be able to improve Dally model.

Considering Dally model, the measured Γ can be computed from the measured wave height and water depth by using the following formula (rewriting Eq. 14).

$$\Gamma = \frac{1}{h}\sqrt{H^2 - \frac{\partial(Ec_g)}{\partial x}\frac{8h}{0.15c_g\rho g}} \quad \text{..... (16)}$$

The right hand side term of Eq. 16, RS, can be computed if we have the profile of wave height transformation. If we plot RS with any wave parameters, it will show a horizontal line (since Γ is constant).

Fig. 7 shows the relation between RS and the various dimensionless parameters, i.e., h/L_o, h/L, $h/\sqrt{LH}$. From Fig. 7 we can see that the parameter Γ is not a constant. Comparison among Fig. 7a-7c, the relation between Γ and $h/\sqrt{LH}$, in Fig. 7c, shows more consistent results than the others. A formula for parameter Γ, from Fig. 7c, can be expressed as

$$\Gamma = \exp\left[-0.36 - 1.25\frac{h}{\sqrt{LH}}\right] \quad \text{..... (17)}$$

The next question is how much difference of computed wave height is resulted when we compare the results calculated by using constant Γ and calculated by using Γ from Eq. 17. Table 3 shows the error function, ER, when wave height is computed from Dally model by using either constant Γ or Γ from Eq. 17.

From table 3 we can see that the computed results show not much difference, but for most cases the results of computed wave height are improved. The selection of any type of model depends on each researcher. In the present study, the better estimation formula (Eq. 17) will be used.

Table 3. Verification results in term of error function parameter, ER.

Sources	No. of data sets	$\Gamma = 0.4$	Γ from Eq. 17
Hansen and Svendsen (1984)	1	16.12	6.95
Horikawa and Kuo (1966),slope=0	101	13.30	11.65
Horikawa and Kuo (1966),slope=1/80-1/20	112	20.58	17.67
Nagayama (1983)	12	9.19	8.62
Kajima et al. (1983)	79	18.36	16.37
Nadaoka et al. (1982)	2	11.97	10.81
Okayasu et al. (1988)	10	14.18	11.3
Sato et al. (1988)	3	11.36	7.74
Sato et al. (1989)	2	31.83	19.78
Shibayama and Horikawa (1986)	10	16.23	17.69

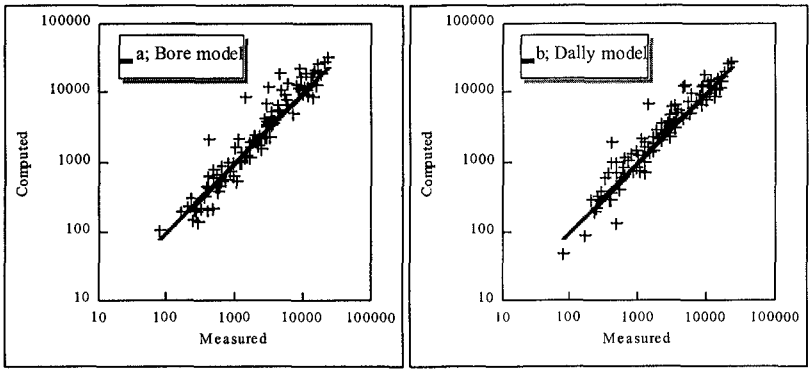

Figure 6. Comparison between measured and computed energy dissipation rate from a: Bore model, b: Dally model (laboratory data from Hansen and Svendsen, 1984, Okayasu et al., 1988, and Sato et al., 1988 and 1989).

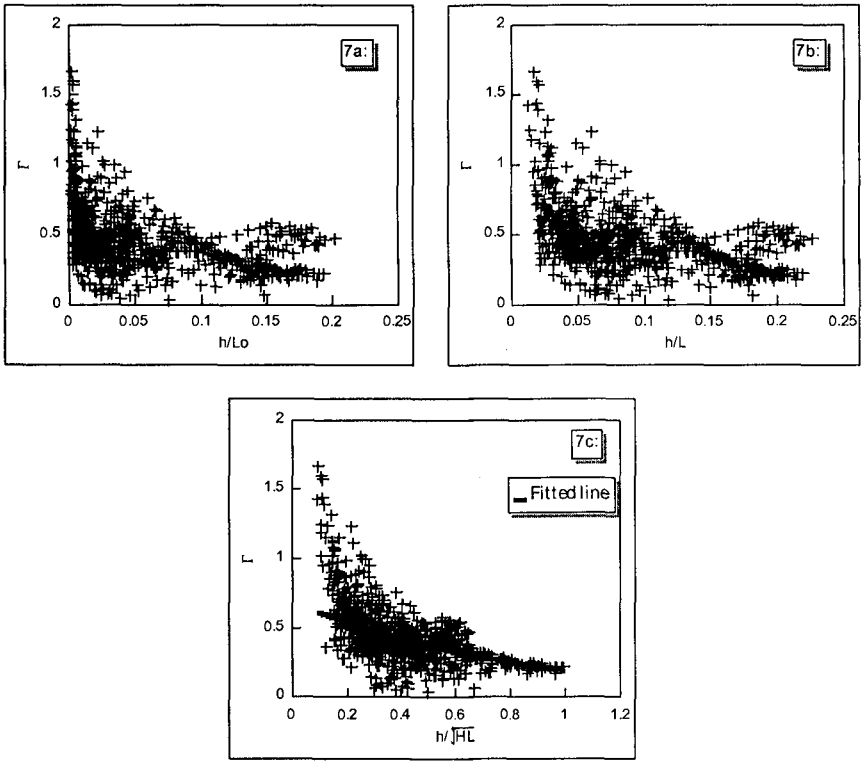

Figure 7. Relations between Γ and a: h/L_o, b: h/L, c: $h/\sqrt{LH}$ (laboratory data from Kajima et al., 1983).

V. Beach Deformation Model

Wave model is used to compute wave transformation from offshore boundary to shoreline boundary. From the computed wave height, sediment transport rate can be computed. Then new beach profile can be computed from the mass conservation equation (Eq. 1). The new beach profile will feed-back into the wave model and causes wave height changes. This yields the loop of dynamic beach deformation and the beach profile at any desired time can be computed.

The shoreward boundary is defined at the wave runup height. Following Larson and Kraus (1989, pp. 170-172), the sediment transport inside the swash zone is assumed to decrease linearly from the end of surf zone to zero at the runup limit. If the local beach slope exceeded the repose angle, avalanching concept will be used.

5.1 Model verification

A number of simulations are performed in order to examine the capability of the present model. All coefficients in the model are kept to be constant for all cases in the verification. Model results are compared with laboratory data of small scale experiment of Shibayama and Horikawa (1985) and large scale experiment of Kajima et al. (1983) and Kraus and Larson (1988).

The verification is performed for all cases, total 45 cases. The verification of these independent data sources and wide range of experiment conditions are expected to clearly demonstrate the accuracy, and stability of present model. The main input data of the model is the incident wave dimensions and initial beach profile. The model was run on workstations (HP 9000 series 700 computers); the total CPU time for simulation about 1000 hr (45 cases) is about 6 min. Examples of examination results are shown in Figs. 8-10. The examination results of all cases, including four profiles per case, are shown in Rattanapitikon (1995, pp. 173-196). Interesting points of the comparison results are described as follows

1. The global shapes of measured profiles are generally well predicted by the model. The predicted profiles are smoother than the measured ones. Small fluctuation of measured profiles can not be predicted by the present model (see Kajima, case 3.1 and Kraus, case 510 in Fig. 8).

2. The agreements of predicted beach profiles of the prototype scale are better than those of small scale wave flume. The fluctuation of measured beach profiles in small scale experiment are more than the prototype scale experiment and the present model can not predict those fluctuations.

3. The model is able to simulate a breaker bar of either the growth of breaker bar (see case 5.2 in Fig. 10) or the reduction of breaker bar (see case 1.2 in Fig. 9).

VI. Conclusions

Cross-shore beach deformation model is developed based on a large amount of published laboratory results. The model contains description of time-averaged concentration profile, velocity profile, sediment transport, wave height, and beach deformation. The validity of model is confirmed by small scale and large scale experiments. The main merit of this model is that it requires very short CPU time and the results are reasonably well.

Acknowledgment

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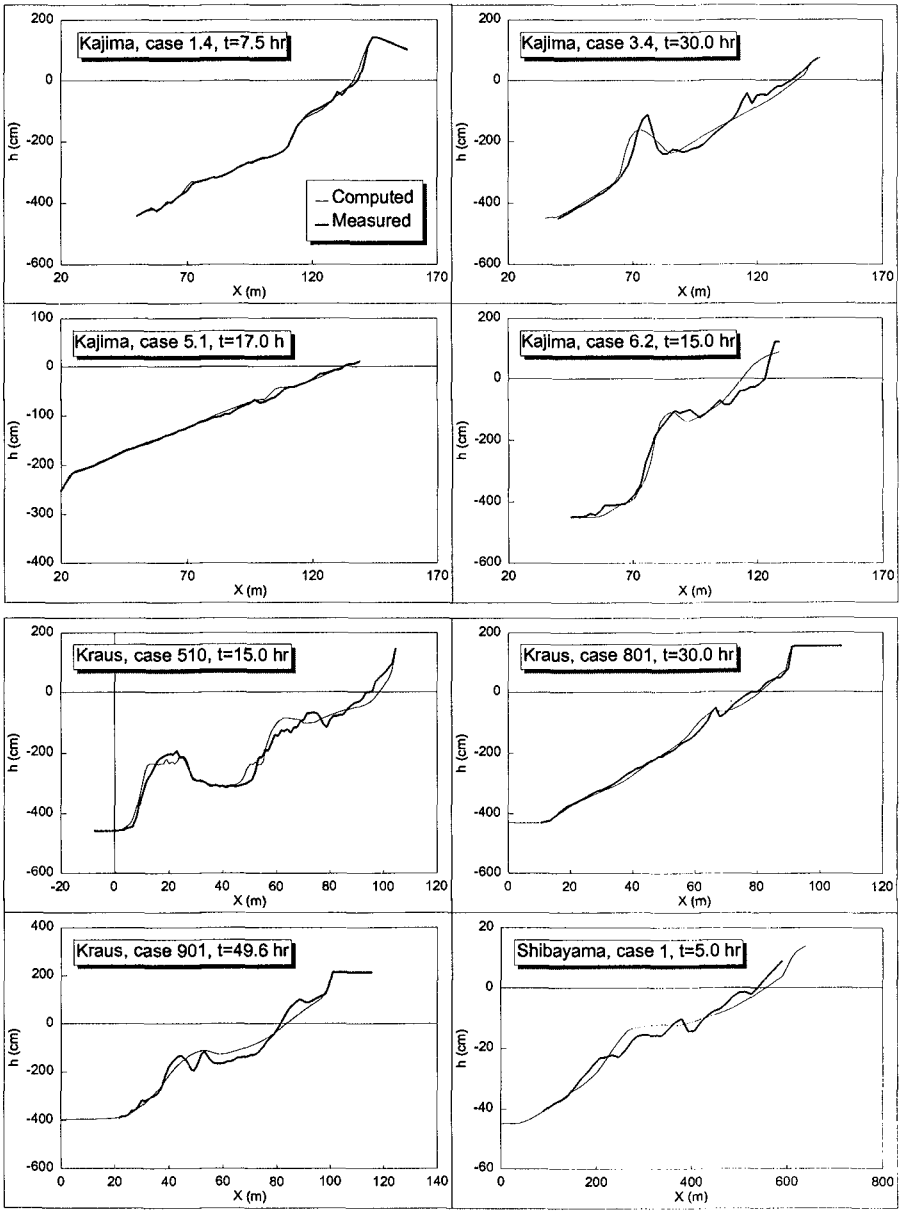

Figure 8. Examples of comparison between measured and computed beach deformation (laboratory data from Kajima et al., 1983, Kraus and Larson, 1988, and Shibayama and Horikawa, 1985).

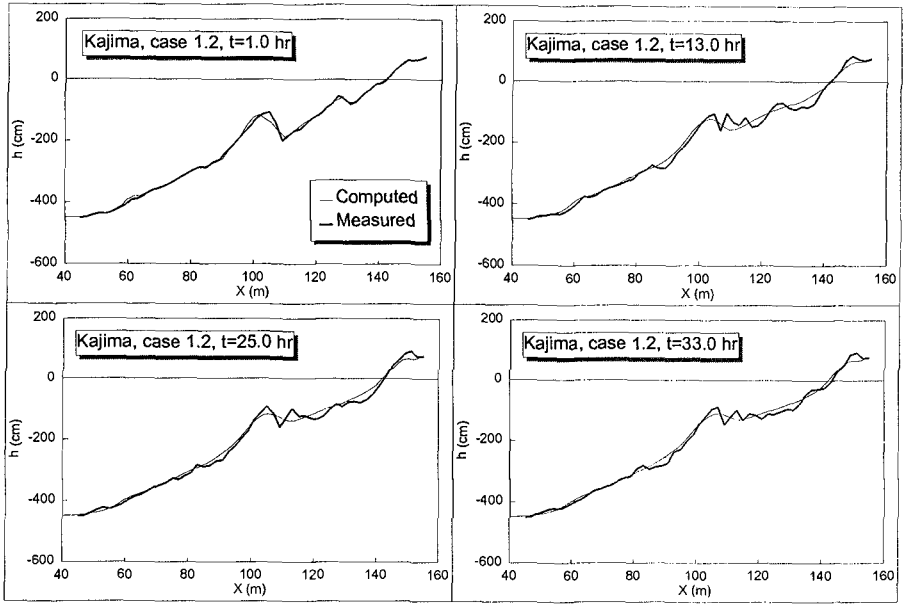

Figure 9. Comparison between measured and computed beach deformation (laboratory data from Kajima et al., 1983, case 1.2).

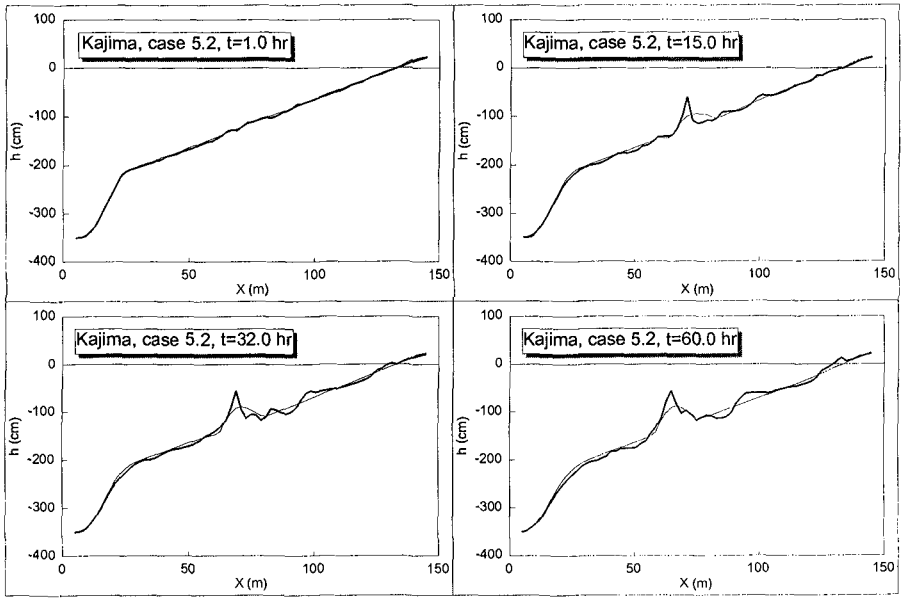

Figure 10. Comparison between measured and computed beach deformation (laboratory data from Kajima et al., 1983, case 5.2).

References

Dally, W.R., Dean, R.G., and Dalrymple, R.A. (1985): Wave height variation across beachs of arbitrary profile, J. of Geo. Res., Vol. 90, No. C6, pp. 11917-11927.

Deigaard, R., and Fredsoe, J. (1991): Modelling of undertow by a one-equation turbulence model, Coastal Engineering, No. 15, pp. 431-458.

Duncan J.H. (1981): An experimental investigation of breaking waves produced by a towed hydrofoil, Proc. R. Soc. London, Ser. A 377, pp. 331-348.

Horikawa, K., and Kuo, C.T. (1966): A study of wave transformation inside surf zone, Proc. 10th Coastal Engineering Conf., ASCE, pp. 217-233.

Kajima, R., Shimizu, T., Maruyama, K., and Saito, S. (1983): On-offshore sediment transport experiment by using large scale wave flume, Collected data No. 1-8, Central Research Institute of Electric Power Industry, Japan (in Japanese).

Kraus, N.C., and Larson, M (1988): Beach profile change measured in the tank of large waves, 1956-1957 and 1962., Technical Report CERC-88-6, Coastal engineering research center, US Army Eng. Waterways Exp. Station.

Larson, M. and Kraus, N.C. (1989): SBEACH: Numerical model for simulating storm-induced beach change, Report 1, Technical Report CERC-89-9, Coastal engineering research center, US Army Eng. Waterways Exp. Station.

Nadaoka, K., Kondoh, T., and Tanaka, N. (1982): The structure of velocity field within the surf zone revealed by means of laser-doppler anemometry, Report of The Port and Harbour Research Institute, Vol. 21, No. 2, pp. 50-102 (in Japanese).

Okayasu, A., Shibayama, T., and Horikawa, K. (1988): Vertical variation of undertow in the surf zone, Proc. 21st Coastal Eng. Conf., ASCE, pp. 478-491.

Okayasu, A. (1989): Characteristics of Turbulence Structure and Undertow in Surf Zone, D. Eng. Dissertation, Civil Eng., University of Tokyo, Japan.

Rattanapitikon, W. (1995): Cross-shore Sediment Transport and Beach Deformation Model, D. Eng. Dissertation, Civil Eng., Yokohama National University, Yokohama, Japan.

Sato, S., Isayama, T., and Shibayama, T. (1989): Long-wave component in near -bottom velocity under random waves on a gentle slope, Coastal Engineering in Japan, JSCE, Vol. 32, No. 2, pp. 149-159.

Shibayama, T., Horikawa, K. (1985): Numerical model for two-dimensional beach transformation, Proc. of JSCE, No. 357/II-3, pp. 167-176.

Shibayama, T., and Rattanapitikon, W. (1993): Vertical distribution of suspended sediment concentration in and outside surf zone, Coastal Engineering in Japan, JSCE, Vol. 36, No. 1, pp. 49-65.

Stive, M.J.F., and Wind, H.G. (1986): Cross-shore mean flow in the surf zone, Coastal Engineering, No. 10, pp. 325-340.

Svendsen, I.A. (1984): Mass flux and undertow in surf zone, Coastal Engineering, No. 8, pp. 347-365.

Thornton, E.B. and Guza, R.T. (1983): Transformation of wave height distribution, J. Geophys. Res., Vol. 88, pp. 5925-5983.

Watanabe, A. (1983): Lecture-1; Fundamental of littoral transport, Seminar on Sediment Problems in Beachs and Estuaries, Manila and Jakarta.