CHAPTER 292

Sheet Flow Modelled as Pure Convection

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Abstract

A considerable amount of experimental data regarding the transport of sand under violent wave conditions (specifically the sheet flow regime) has been gathered in recent years. Historically, modelling of the concentration distributions of the suspended sediments immediately above the sheet flow layer has been undertaken using the diffusion equation. A physically more appropriate model is the combined convection diffusion approach. However, analysis of the experimental results to date indicate that a pure convection which is far simpler than either of the other two is adequate.

Introduction

The transport of sand under violent wave conditions with Shield's parameter values greater than one is often described by the term sheet flow because no bedforms survive through the wave cycles and a layer (sheet) of sand, many grain diameters thick, can be seen to move back and forth with the wave motion.

A considerable amount of experimental details of the process have been gathered in recent years by Ribberink and co-workers (1992). However, modelling of the process is still inadequate. This paper concentrates on the "upper" or suspended part of the sediment distribution. Traditionally, concentration distributions have been modelled using forms of the diffusion equation. As will be presented below, analysis of experimental results indicates that a combined

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convection diffusion model or in some cases a pure convection model appears to be adequate.

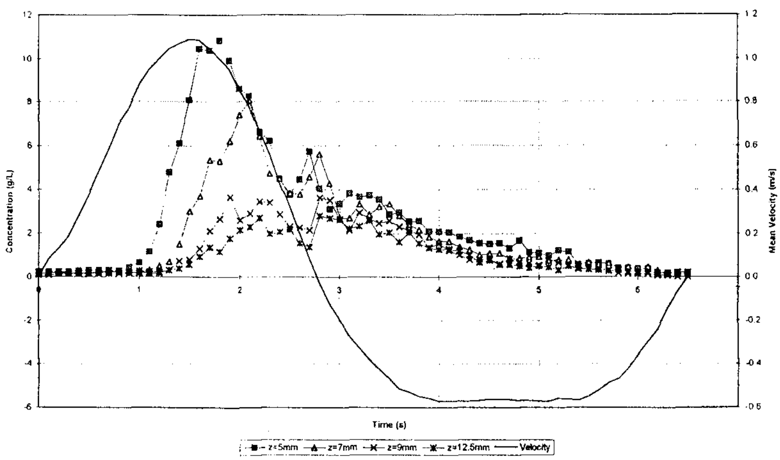
Experimental Data

A series of experiments have been conducted in the Large Oscillating Water Tunnel (LOWT) of Delft Hydraulics which have focussed on both time averaged and time dependant near bed and within bed sediment concentrations and sediment transport rates.

Flow conditions investigated have ranged from purely sinusoidal to regular, asymmetric waves over both rippled and plane beds. Sediments investigated have been predominantly sands with a D_{50} = 0.2 mm. An additional set of experiments have been conducted using a finer sand D_{50} = 0.12 mm.

Figure 1 provides an example of the results obtained through this procedure (Ribberink et al, 1992).

Figure 1: Experiment C8 Results

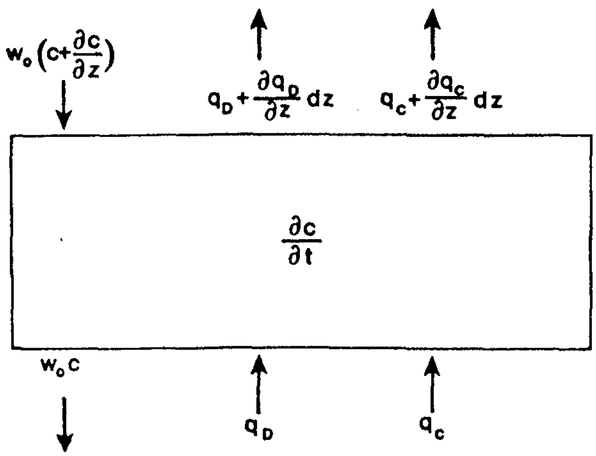


Suspended Sediment Distribution Models

Natural suspended sediment processes are a combination of both convective and diffusive processes. The process by which sediment (heavier than water) is driven upward has traditionally been described in terms of pure gradient diffusion. This process is due to driven by vertical mixing and consists of a large number of small, random vertical steps. Alternatively, the scale over which this mixing takes place is much smaller than the scale of the concentration profile.

Analysis of recent experimental data indicates that the pure gradient diffusion model is inadequate and that in many cases, a larger scale or convective process is reflected by many suspensions as well. These results reflect a process whereby the mixing scale is of the same order as the concentration profile.

Considering a horizontally uniform sediment concentration field $c=c(z,t)$ and a correspondingly uniform sediment velocity field $u_s=u_s(z,t)$, the vertical sediment flux $q=cu_s$ is considered to consist of three components. These components are a downward component $-w_0c$ due to gravitational settling and an upward flux which can be of convective (subscript C) or diffusive (subscript D) nature or a combination of both. This process is illustrated in Figure 2 (Nielsen, 1992).



The total vertical flux is thus written as:

$$q_z = -w_0c + q_D + q_C$$

enabling the conservation equation to be written as

$$\frac{\partial c}{\partial t} = w_0\frac{\partial c}{\partial z} - \frac{\partial q_D}{\partial z} - \frac{\partial q_C}{\partial z}$$

The forms of convective and diffusive functions considered for this paper are as below:

Convective Flux, $q_c(z,t) = p(t - \frac{z}{w_c})F(z)$

Diffusive Flux, $q_D(z,t) = -\varepsilon_s \frac{\partial c}{\partial z}$

The pickup function $p(t)$ is a non-negative function describing the instantaneous pick up rate at the bed and the dimensionless convective distribution function $F(z)$ determines the fraction of the entrained sand which travels (convectively) beyond the level z above the bed.

Analytical Solution - Pure Convection

The vertical convective sediment flux, q_c is generally written in the following form

$$q_c(z,t) = p(t - \frac{z}{w_c})F(z)$$

where w_c is the average vertical velocity with which the sand is convected upwards.

Assuming that the distribution function $F(z)$ is an exponential of the form

$$F(z) = e^{-z/L}$$

the general form of the time dependant sediment concentration equation may be expressed in terms of periodic functions as:

$$c_n(z,t) = C_n f(z) e^{in\omega t}$$

becoming

$$c_n(z,t) = C_n e^{-\beta_n z/L} e^{in\omega t}$$

where

$$C_n = \frac{P_n}{w_0} \frac{1 + in\omega L / w_c}{1 + in\omega L / w_c + in\omega L / w_0}$$

and

$$\beta_n = 1 + \frac{in\omega L}{w_c}$$

If the real portion of c_n is considered, have

$$Re\{c_n(z,t)\} = Re\{C_n\} e^{-z/L}$$

Analytical Solution - Pure Gradient Diffusion

As with the pure convection description, with the general form of the time dependant sediment concentration equation as

$$c_n(z,t) = C_n f(z) e^{in\omega t}$$

for the pure gradient diffusion solution, this becomes

$$c_n(z,t) = C_n e^{-\alpha_n z w_0/\varepsilon_s} e^{in\omega t}$$

where

$$C_n = \frac{P_n}{w_0 \alpha_n}$$

and

$$\alpha_n = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{in\omega\, \varepsilon_s}{w_0^2}}$$

Note that for the pure gradient diffusion solution, the length scale, L of the convective solution is replaced by the sediment diffusivity over the still water settling velocity ε_s / w_0.

There are two important differences between the analytical solutions for pure gradient diffusion and pure convection. Firstly, all the harmonic concentration components decay as $e^{-z/L}$ for the convective solution and secondly, the phase lag relative to the pick up function grows at the same rate for all frequencies. This indicates that in a process dominated by convection, defined concentration peaks will travel upwards through the water column at a constant rate. However, for a process dominated by diffusion, the phase lags will grow with elevation above the bed, leading to "blurring" of successive peaks.

Data Analysis

The experimental results previously illustrated may be expressed in terms of simple periodic functions via a Fourier Analysis. This has been carried out on the available data to the tenth harmonic. It has been found to date that six harmonics provide an appropriate level of accuracy.

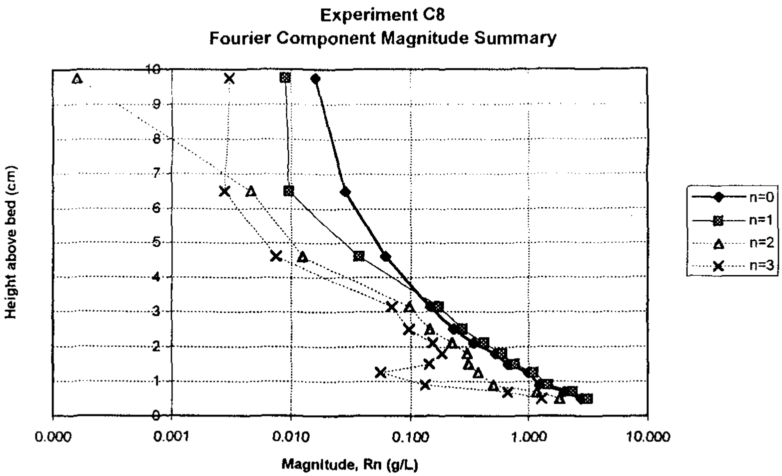


Figure 3 provides an example of the relative magnitudes of the primary harmonics obtained for the data set presented in Figure 1. From these results, parameters representing the mixing length (pure convection solution) and diffusivity (pure gradient diffusion solution) can be derived.

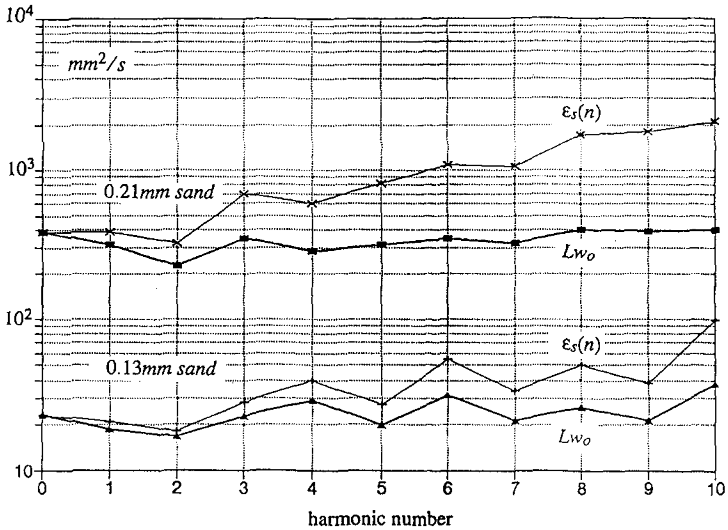


Figure 4 presents the results of this process. Note that the length scale (corresponding to pure convection) is approximately constant across all harmonics. However, the diffusivities show an increasing "drift" as the harmonic number increases. As could be expected, this is more pronounced for the sand of larger grain size.

Conclusions

The primary conclusion which may be drawn from this analysis is that the experimental results analysed are more appropriately represented by a purely convective model. This is convenient as computationally, the solution of the equations corresponding to pure convection is much more straightforward than for the second order partial DE for the pure diffusion (and combined convection diffusion) equation.

References

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