CHAPTER 295

A Numerical Model of Sheet Flow Sediment Transport

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Abstract

Governing equations are derived for a numerical model to simulate the sediment motion under sheet flow condition. They consist of equations for the conservation of the horizontal momentum, the turbulence energy and the sediment mass. Values of coefficients in the governing equations are determined using the experimental results of Horikawa *et al.* (1982). The mean sediment transport rates during half a period of sinusoidally oscillatory flow are calculated and the computed values agree well with the experimental results of Sawamoto and Yamashita (1986). Computations are also made for the net transport rate in unsinusoidally oscillatory flow with and without superposed steady flow, showing a good agreement with the measurements of Dibajnia and Watanabe (1992). Finally the thickness of the sheet flow moving layers is computed and related to grain-flow parameters.

Introduction

The first author conducted a detailed experiment on the sheet flow sediment transport in an oscillatory flow tank with sinusoidal velocity variations, measured the velocity as well as concentration of the sediment, and evaluated the mean transport rate during half a period of sinusoidal flow (Horikawa *et al.*, 1982). After this experiment that remarkably advanced our understanding of the sheet flow transport, many studies have been performed until now. According to recent studies on the sediment transport, it has been widely recognized that the sheet flow predominates in the surf zone not only during storms but also even under moderate waves in the field (e.g., Watanabe *et al.*, 1991, Dibajnia *et al.*, 1994).

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Hence not a few numerical models for the sheet flow sediment movement have been presented so far, but the validty and applicability of most of them are not sufficient enough.

This study aims to develop governing equations and a numerical model that can reproduce time-space changes in the velocity and concentration of the sediment as well as the net transport rate under the sheet flow condition.

Governing Equations Derived for A Numerical Model

Equations (1) to (3) are newly proposed to describe the sheet flow sediment motion.

For the conservation of the horizontal momentum:

$$\partial u/\partial t = (\rho_w/\rho)\left[\partial U/\partial t + \partial\left(K_u \partial u/\partial z\right)/\partial z\right] \tag{1}$$

$$K_u = \begin{cases} \nu + \kappa_{ul} d k^{1/2} + \kappa_{uc}\nu F(c) & (-\infty < z < z_5) \\ \nu + [\kappa_{ul} d + \kappa_{uh}(z - z_5)]k^{1/2} + \kappa_{uc}\nu F(c) & (z_5 < z < \infty) \end{cases}$$

$$F(c) = [1 + 0.183c/(1 - 0.878c)]^2 \text{ - - - - - - - - - - - - - (Eilers, 1941)}$$

For the conservation of the turbulence energy:

$$\partial k/\partial t = K_{kp}\left(\partial u/\partial z\right)^2 + \partial\left(K_{kd}\partial k/\partial z\right)/\partial z - \varepsilon \tag{2}$$

$$K_{kp} = \begin{cases} 0 & (-\infty < z < z_5) \\ \left[\nu + \kappa_{kp}(z - z_5)\, k^{1/2}\right](c_5 - c) & (z_5 < z < \infty) \end{cases}$$

$$K_{kd} = \begin{cases} 0 & (-\infty < z < z_{10}) \\ (\nu + \kappa_{kdl} d k^{1/2})(1 - c) & (z_{10} < z < z_5) \\ \left\{\nu + [\kappa_{kdl} d + \kappa_{kdh}(z - z_5)]\, k^{1/2}\right\}(1 - c) & (z_5 < z < \infty) \end{cases}$$

$$\varepsilon = \kappa_{k\varepsilon}\, g F(c) k^{1/2}$$

For the conservation of the sediment mass:

$$\partial c/\partial t = \partial\left(K_c \partial c/\partial z\right)/\partial z + \partial(wc)/\partial z \tag{3}$$

$$K_c = \begin{cases} \nu + \kappa_{cl} d k^{1/2}|_{z=z_5} & (-\infty < z < z_5) \\ \nu + [\kappa_{cl} d + \kappa_{ch}(z - z_5)]k^{1/2} & (z_5 < z < \infty) \end{cases}$$

$$w = w_0(1 - c)$$

Here U is the main flow velocity, u the velocity of the fluid-sediment mixture, k the turbulence energy, c the volumetric sediment concentration($0 \le c \le 1.0$), t the time, z the vertical distance measured upward from an initial bed surface, ρ_w the fluid density, ρ the density of the fluid-sediment mixture, ν the fluid kinematic viscosity, d the sediment grain size, w_0 the free fall velocity of sediment particles, z_5 and z_{10} are z where $c = c_5 = 0.5$ and $c = 0.99$, respectively, and g is the acceleration of gravity. All the coefficients κ with subscripts are constants.

Equation (1) is a turbulence model for the conservation of the horizontal momentum in the bottom boundary layer. Here it is assumed that the fluid and sediment move together with the same horizontal velocity u. The density of the fluid-sediment mixture $\rho(z,t)$ is defined as

$$\rho = (1-\lambda)c\rho_s + [1-(1-\lambda)c]\rho_w \tag{4}$$

where λ is the porosity of the fluid-sediment mixture at c=1.0, ρ_s the density of the sediment particles. A main point of Eq. (1) is how to express the eddy viscosity K_u. According to Kolmogorov (1941) and Prandtl (1945), K_u is determined as $K_u \sim \ell k^{1/2}$, where ℓ is the length scale of turbulence and k is the tubulence energy. As observed in one of the experimental results of Horikawa *et al.* (1982) in Fig. 1, the vertical distribution of concentration has an inflection point around the vertical distance z_5 where concentration c is c_5. This implies that the length scale in the inner layer is very different from that in the outer layer. Then we assume that the length scale in the inner layer is governed by the grain size d, because the concentration in this layer is very high. On the other hand, the length scale in the outer layer will be set as $z - z_5$, because more free turbulence field is formed in this layer. The grain size $d, z - z_5$, and the turbulence energy k are thus included in the expression of the eddy viscosity K_u. In addition, we employ an equation proposed by Eilers (1941) to describe the chaotic collision of grains.

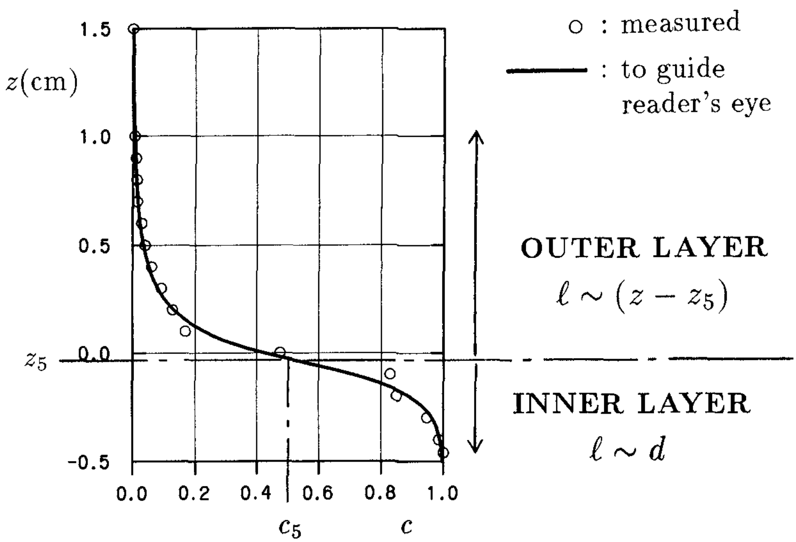


Fig. 1 Distribution of sediment concentration.
(d=0.2mm, T=3.64s, $\hat{U}$=127cm/s, $\theta = 90°$,i.e., $U=\hat{U}$)

Equation (2) shows that time change in the turbulence energy k is determined by three terms; the production, diffusion and dissipation. Regarding coefficients K_{kp} and K_{kd} in Eq. (2), similar formation is made in terms of $\nu, z - z_5, k$ and d for the same reasons mentioned above. A main difference is that there are layers where no production or diffusion of the tuburence energy takes place. This assumption is acceptable, because the concentration in the vicinity of the bed surface is very high and it is nearly impossible for the fluid-sediment mixture with high concentration to form the strong turbulence field. Another point is that the effect of concentration is simply expressed by $(\mathrm{c}_5 - \mathrm{c})$ or $(1 - c)$ and that the dissipation rate ε is described in a very simple way. On these assumptions, Eq. (2) becomes a one-equation model of the turbulence energy which is much simpler to solve than two-equation models such as k-ε models.

Equation (3) for the concervation of sediment mass takes similar formulation except that the diffusion coefficient K_c for the inner layer is determined by the turbulence energy k at $z = z_5$, because sediment particles are expected to diffuse even in the inner layer where the magnitude of turbulence energy is very small. In nother words, the reduction of concentration in the inner layer is assumed to take place at the phase of acceleration of the main flow through loosening of the grain arrangement due to shear stresses around $z = z_5$. The reduction rate of fall velocity w with the increase of sediment concentration can be approximately expressed as $(1 - c)$ according to Katori and Homma (1984).

Boundary conditions are

$$\left.\begin{array}{l} u = 0 \\ k = 0 \\ \mathrm{c} = 1 \end{array}\right\} \quad \text{at} \quad z < z_{10} \qquad \left.\begin{array}{l} u = U \\ k = 0 \\ \mathrm{c} = 0 \end{array}\right\} \quad \text{at} \quad z = z_\infty \tag{5}$$

Here z_{10} is the vertical distance where $c = 0.99$, and z_∞ is that of the outer edge of the bottom boundary layer. It has been reported that the thickness of the bottom boundary layer becomes much larger when fluid contains granular sediment particles. For this reason, z_∞ should be set as large as possible in computation. According to trial and error, $z_\infty = 5$cm is large enough for grain diameter smaller than several milimeters. In addition it has been found that the concentration around $z = z_{10}$ sometimes exceeds 1.0 at the phase of sediment falling and in the worst cases the computaion fails by oscillation or divergence of solutions. We employ the following procedure to avoid such computaional failure. At each time step of calculation of the sediment concentration by Eq. (3), we detect the top vertical distance z_B where the concentration becomes or exceeds 1.0 by searching downward from z_∞, set the concentration below z_B as 1.0, and make the following adjustment for the upper region so as to maintain the conservation of total sediment mass.

$$\begin{array}{l} \mathrm{c} \Leftarrow \mathrm{c}\ [(V_o - z_\mathrm{B})/(V_i - z_\mathrm{B})] \qquad \text{at } z > z_\mathrm{B} \\ V_i = \int_{z_{-\infty}}^{z_\infty} \mathrm{c}\ dz \end{array} \tag{6}$$

where V_o is the volume of sediment per unit horizontal area at the begining of calculation, and $z_{-\infty}$ is the vertical distance of the lower boundary of a calculation domain.

Determination of Coefficients κ

In the governing eqations, there are several coefficients κ with subscripts to be determined. These coefficients are determined using the experimental data presented by Horikawa *et al.* (1982). Figure 2 shows another example of their measurements. The experimental conditions are: d=0.2mm, ρ_s=2.66, T=3.64s and $\hat{U}$=127cm/s. In Fig. 2 the phase θ of the main flow is defined as $\theta = 0°$ for $U = 0$ and $\theta = 90°$ for $U = \hat{U}$. Open circles in Fig. 2 are the measurement data of the horizontal velocity u and concentration c. As for the experimental data of the turbulence energy k, Yamashita *et al.* (1985) measured the velocity by

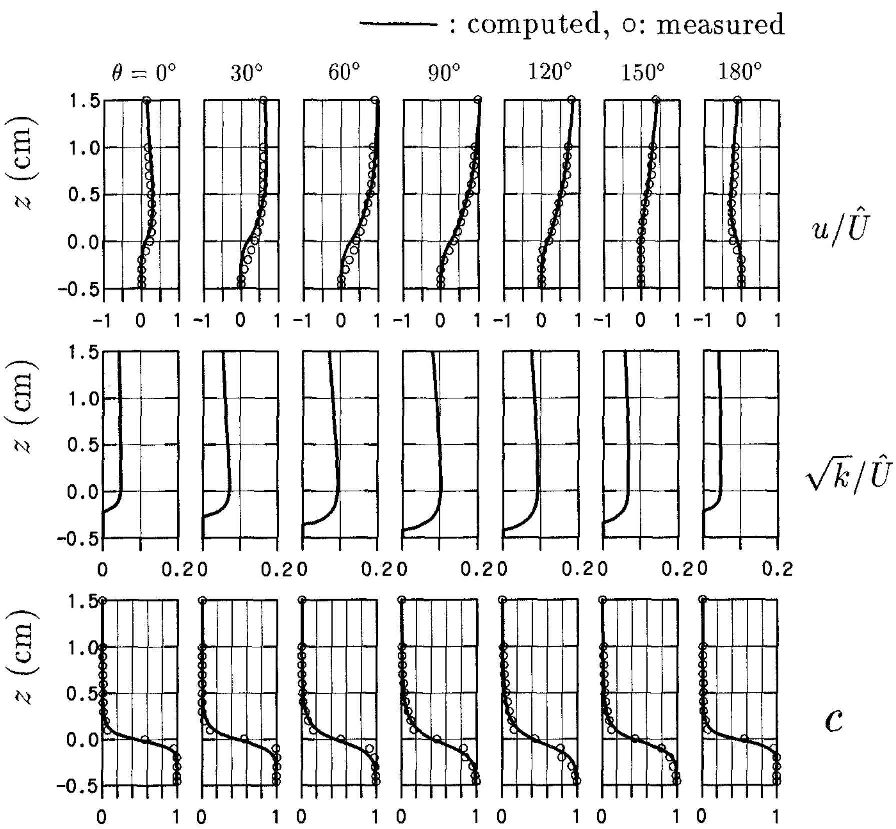


Fig. 2 Comparison of the horizontal velocity u and concentration c between the measurements and the computations.

using an optic fiber velocity meter and reported that the maximum magnitude of square root of the turbulence energy is about 1/10 of the main flow velocity. The computations shown by solid lines in Fig. 2 show a remarkably good agreement with the measurments when we set values of the coefficients κ as follows:

$$\begin{aligned}&\kappa_{ul}=1.8, \quad \kappa_{uh}=0.045, \quad \kappa_{uc}=10.0\\&\kappa_{kp}=0.984, \quad \kappa_{kdl}=45.0, \quad \kappa_{kdh}=18.0, \quad \kappa_{k\varepsilon}=0.006\\&\kappa_{cl}=0.833, \quad \kappa_{ch}=0.09\end{aligned} \tag{7}$$

It should be emphasized that these constants have given a good agreement between the computations and the measurements also for all the other cases of Horikawa *et al.* (1982), and that they will be commonly used in the following calculations.

In the numerical computation the initial conditions are u=0, k=0, and

$$\begin{aligned}&c=0 \qquad z>0\\&c=1 \qquad z\leq 0\end{aligned} \tag{8}$$

The time interval Δt and the vertical grid interval Δz are set as follows:

$$\begin{aligned}&\Delta t = T/2400\\&\Delta z = \min(\sqrt{2q\Delta t}, d)\end{aligned} \tag{9}$$

where q is the mean sediment transport rate during half a period estimated by the following formulas presented by Sawamoto and Yamashita (1986).

$$\Phi \equiv (1-\lambda)q/w_0 d = 2.2\ (u_*/w_0)^3 \tag{10}$$

in which Φ is the nondimensional mean transport rate, u_* the friction velocity, and w_0 the free fall velocity. To estimate the friction velocity u_*, we employ the following wave friction factor f_w proposed by Jonsson (1966).

$$\begin{aligned}&\frac{1}{4\sqrt{f_w}}+\log\frac{1}{4\sqrt{f_w}}=-0.08+\log(a_m/\xi) \qquad (a_m/\xi>1.57)\\&f_w=0.3 \qquad (a_m/\xi<1.57)\end{aligned} \tag{11}$$

where a_m is the amplitude of main flow orbital movement, and ξ is the bottom roughness, which is set as ξ=d in this study.

Sediment Transport Rate

Computations of the sediment transport rate are conducted to verify the validity and applicability of the present model. Sawamoto and Yamashita (1986) have carried out a series of experiments in a U-shape tube with a rectangular cross section. In the tube, quasi-sinusoidal flow is generated by releasing the initial inbalance of water heads between the tube ends. The period of the oscillatory flow is fixed to be 3.8s owing to the natural frequency of the fluid motion in the tube. The range of the amplitude of main flow velocity is 44.3-125.3 cm/s. They have used three kinds of quartz sands and a coal powder as the bed materials to measure their transport rates. Imitation pearl pellets have also been used to observe the sediment motion through a side glass wall. Figure 3 shows the comparison between Φ_{cal} and Φ_{meas}, where Φ_{cal} is the calculated nondimensional mean transport rate during half a period and Φ_{meas} is that of the measurement data by Sawamoto and Yamashita (1986), Horikawa *et al.* (1982) and Abou-Seida (1965). Although a little scatter is observed especially for the case of Abou-Seida, the overall agreement between Φ_{cal} and Φ_{meas} is fairly good.

In addition Sawamoto and Yamashita (1986) proposed a mean transport rate formula during half a period of sinusoidal flow as already shown by Eq. (10). Figure 4 shows that the computed values of Φ shown by symbols agree well with thier formula except for the range of small u_*/w_0, where the sediment movement approaches to its threshold of sediment motion. Here the computation conditions are almost the same with those of Sawamoto and Yamashita, while the range of main flow velocity amplitude $\hat{U}$ has been extented to 0.2-4.0m/s.

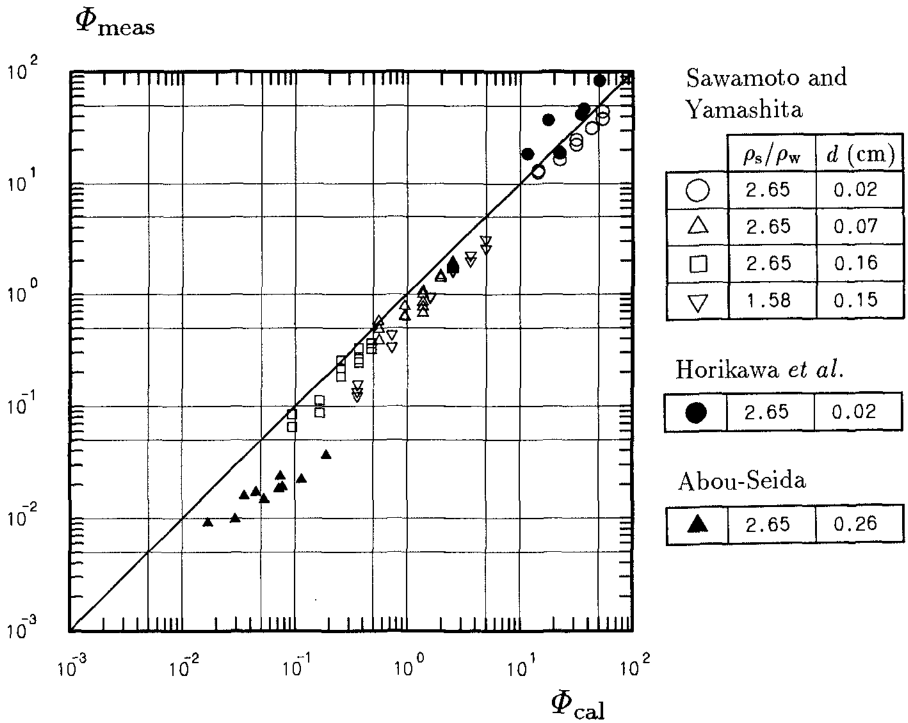
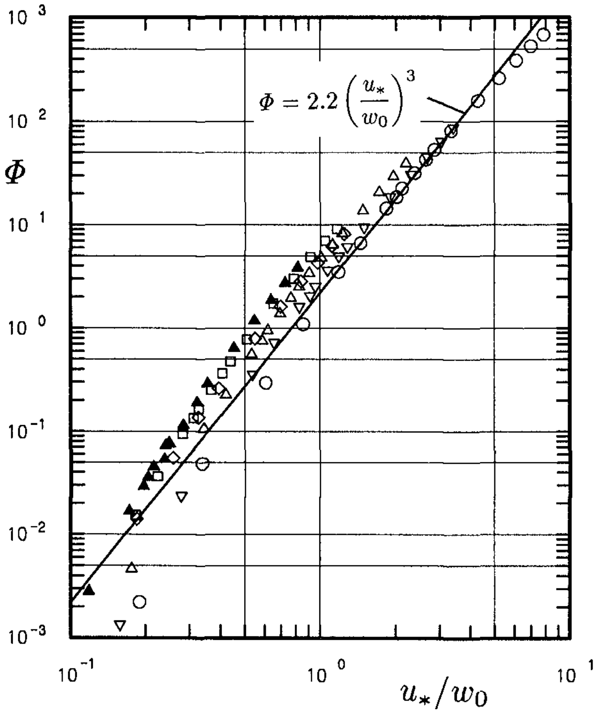


Fig. 3 Comparison between Φ_{cal} and Φ_{meas} in sinusoidally oscillatory flow.



	ρ_s/ρ_w	d (cm)
○	2.65	0.02
△	2.65	0.07
□	2.65	0.16
▽	1.58	0.15
▲	2.65	0.26
◇	1.60	0.50

Fig. 4 Relation between $\Phi_{\rm cal}$ and u_*/w_0 in sinusoidally oscillatory flow.

Dibajnia and Watanabe (1992) have measured the net rate of sheet flow sediment transport in unsinusoidally oscillatory flow with and without superposed steady flow. The experiments have been carried out in a loop-shape oscillatory/steady flow water tunnel. The flow is driven by a piston to be oscillation with arbitrary temporal variations and by a pump to superpose a steady current on the oscillation. The bet material is the sand with a grain diameter of 0.2mm. They have selected five oscillation periods ranging from 1s to 4s and four conditions of the asymmetry index $U_{\rm max}/\hat{U}$=0.5 to 0.8, where $U_{\rm max}$ is the maximum value of the oscillatory flow velocity U and $\hat{U}$ is the total amplitude of U. In addition the steady flow with four velocities ($\bar{V} \simeq$ -20, -10, 10, 20cm/s) have been superposed on each of the oscillations. In Fig. 5 their data are compared with computations, which shows a good agreement for both the positive and the negative transport rate. Here the results are plotted for the two oscillation periods of 3s and 4s, because the other experimental results for T= 1.0s, 1.5s and 2.0s show large scatter. The reason for such large scatter is to that Dibajnia and Watanabe have not measured the flow velocities directly in coxistent oscillatory

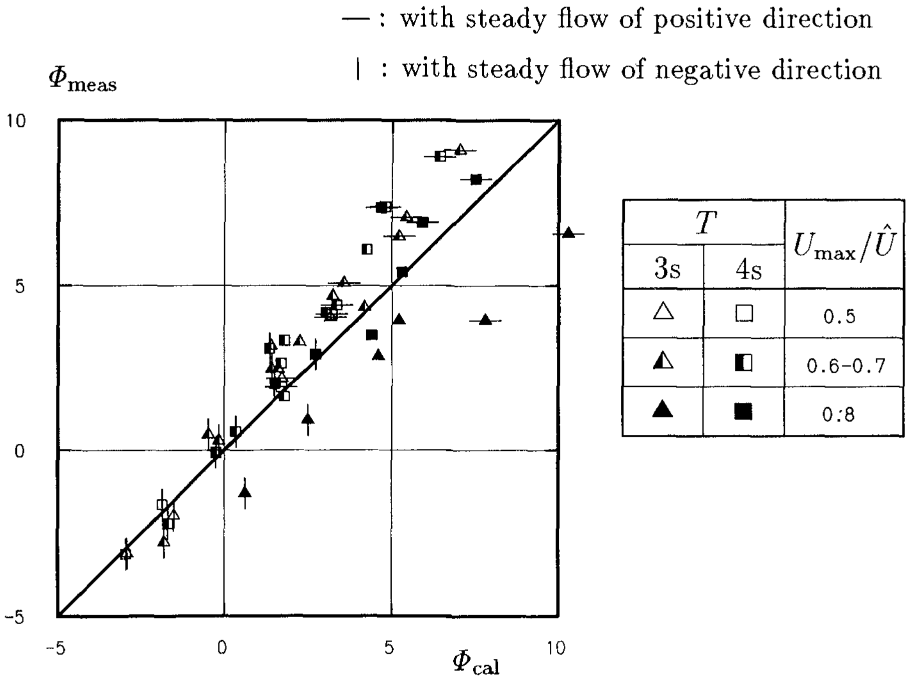


Fig. 5 Comparison between Φ_{cal} and Φ_{meas} in unsinusoidally oscillatory flow with and without superposed steady flow.

and steady flow. The expected flow conditions may not have been obtained when the period of oscillatory flow is short and the interaction between oscillatory flow and steady flow becomes significant in their water tunnel.

Thickness of Moving Layers

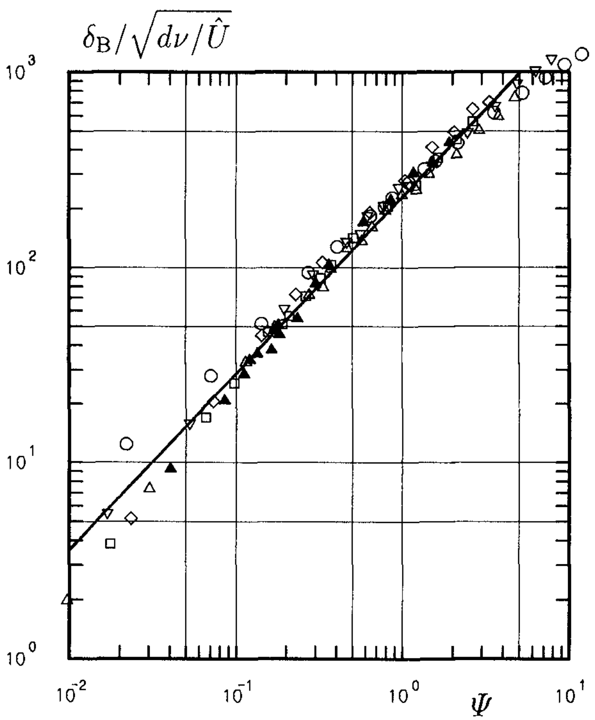
The thickness of moving layers is another major concern about the sheet flow sediment transport. In this study, the moving layer thickness δ_B is defined as the maximum change in the elevation of the bed surface, where the sediment particles never move during the whole period of the oscillatory flow. The quantity δ_B is not only important to be used as a boundary condition for analytical treatment of the sheet flow transport but also useful for rough evaluation of the transport rate. It is possible to make such evaluation by multiplying δ_B by the mean concentration and the mean near-bed velocity. Although experimental studies on δ_B have been performed (e.g., Li *et al.*, 1993, Yamashita *et al.*, 1993), most of them are observations through side glass walls where the movement of sediment particles is greatly affected by the side boundary layer. For this reason, we only show computations without comparison with the measurements. Figure 6 shows the computed results, indicating a high correlation with grain-flow parameters. The solid line fitted for the computations reads

$$\delta_B/\sqrt{d\nu/\hat{U}} = 230\ \Psi^{0.891} \tag{12}$$

where d is the grain size, ν the fluid kinematic viscosity, $\hat{U}$ the amplitude of main flow velocity, $\Psi(=u_*{}^2/sdg)$ is the Shield number, and $s = (\rho_s/\rho_w - 1)$ is the immersed specific gravity of sediment.

Conclusions

This paper has presented a numerical model for the sheet flow sediment transport. The model consists of governig eqations for the conservation of horizontal momentum, the turbulence energy and the sediment mass. Comparisons have been made between the computations and laboratory measurements for vertical distributions of the horizontal velocity and sediment concentration as well as the



	ρ_s/ρ_w	d (cm)
○	2.65	0.02
△	2.65	0.07
□	2.65	0.16
▽	1.58	0.15
▲	2.65	0.26
◇	1.60	0.50

Fig. 6 Relation between $\delta_B/\sqrt{d\nu/\hat{U}}$ and Ψ in sinusoidally oscillatory flow.

sediment transport rate under a variety of main flow conditions, showing a remarkably good agreement for any of them. It should be emphasized that every coefficient in the governing equations has been set to be a constant respectively common for all the computations. These results indicate the validity and applicability of the present model for estimating the net transport rate of the sheet flow as well as time-space changes in its velocity and concentration over a very wide range of conditions.

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