This study proposes a new parameter, the wave front steepness parameter, to better describe the wave slamming coefficient. Four cases of wave breaking were employed to study the relationship between wave front steepness and wave slamming coefficients. Based on model results with OpenFOAM, an empirical formula was derived to relate the slamming coefficient to the wave front steepness.

Keywords: wave slamming, OpenFOAM, monopile, numerical model, wind turbine foundation

INTRODUCTION

In relatively shallow water regions, incoming waves can break and lead to wave slamming impacts upon structures. The effect of these wave slamming impacts on offshore wind turbine foundations has significant implications for structural stability and associated construction costs. This is because wave slamming impacts often lead to a larger magnitude of wave force than in-line forces, e.g. Morison wave force. The duration of these slamming forces is extremely short, which makes it difficult to analyse the breaking wave forces. In structural or stability analyses, the slamming force, $F_S$, must be included in Morison’s equation, as an additional part of the total wave force.

Engineering practice to predict the wave slamming impact upon a structure generally involves the use of a slamming coefficient, $C_s$, and hydrodynamic conditions. A few researchers have investigated wave slamming forces on single vertical and/or inclined piles over the past 100 years. Specifically, von Karman (1929) proposed a method based on the assumption that the slamming force can be deduced by considering the breaking wave as a vertical wall of water that hits the cylinder. The force on the plate can be calculated by considering the potential flow below the plate and integrating the pressures calculated by the Bernoulli equation. This method is too conservative as the breaking waves usually hit the cylinder with limited touch area. Wagner (1932) introduced a model for the so-called pile-up effect. This effect causes earlier immersion of the pile, which leads to decreased duration of the impact and higher line force. Goda et al. (1966) proposed that the height of the impact area should be equal to the curling factor, multiplied with the maximum elevation of the wave at breaking. Sarpkaya (1978) proposed a time-dependent slamming coefficient in terms of the fluid velocity, immersion depth of the cylinder and time. Wienke and Oumeraci (2005) investigated wave slamming forces on cylinders in a large scale model set-up, and obtained curling factors for different inclinations of the pile. Ros Collados (2011) also investigated the slamming factor on a vertical cylinder. His findings revealed a slamming factor of 4.3 and a triangular vertical force distribution. However, all findings above present large uncertainties on the value of the slamming coefficient, $C_s$, ranging from about 1.0 to 6.4 (Wienke and Oumeraci, 2005). They are also dependent on a curling factor, $\lambda$, and duration of impact, $\tau$. This causes uncertainties in the dimension of structures exposed to these kinds of forces, and are therefore still under investigation.

ISO 21650 (2007) is an International Standard which sums up the different theories, but it does not recommend the slamming coefficients values, nor formula to calculate the duration of impact. The large variability associated with $C_s$ in the literature is indeterminate and therefore inappropriate for industrial and/or commercial use. The OpenFOAM wave model offers an alternative and more practical way to study wave breaking and the resultant impacts upon a cylinder (e.g. Christensen et al., 2005). The main objective of this paper is to study wave slamming impacts upon offshore wind turbine foundations using the OpenFOAM model, and, subsequently, derive an empirical formula, both in terms of wave shape and nonlinearity, that ascertains a robust choice of slamming coefficient for engineering applications.
METHODOLOGY

Parameter definition

Below is a summary of existing empirical formulae for wave slamming impacts on structures. In an oscillatory flow with flow velocity $u(t)$, the Morison equation gives the inline force on slender cylinders parallel to the flow direction

$$F = F_D + F_M = \frac{1}{2} \rho C_d D u |u| + \rho C_m D |\dot{u}|,$$  \hspace{1cm} (1)

where $\rho$ is the flow density, $C_d$ and $C_m$ are the drag coefficient and the inertia coefficient respectively, $D$ is the cylinder diameter, $u$ is the flow velocity and $\dot{u}$ represents the flow acceleration.

Once wave particle velocities are larger than the phase speed, waves will start breaking at the free surface. This part of the water volume has larger particle velocities and kinematic wave energy, thus, imposes a significant impact force on the structures. The total force from breaking waves is then described as:

$$F = F_D + F_M + F_S.$$  \hspace{1cm} (2)

The slamming force, $F_s$, is commonly written as:

$$F_S = \frac{1}{2} \rho w C_S D C_b^2 \lambda \eta_b,$$  \hspace{1cm} (3)

where $C_s$ is the slamming factor, $C_b$ is the wave celerity close to breaking point, $\lambda$ is the curling factor and $\eta_b$ is the maximum surface elevation of the wave at breaking. Refer to Wienke and Oumeraci (2005) for sketches and definitions of these parameters.

Based on extensive model testing Stansberg (2008) indicated that the most serious wave impact load events are closely related to properties in the actual individual wave events. Thus, they are often related to energetic and steep waves. Peng et al. (2013) concluded that due to a steeper front face of irregular waves, a wave group can cause a larger wave load on the foundation than regular waves, considering a regular wave height equal to the maximum individual wave height. Therefore, in this study the wave slamming impact will be related to the so-called wave front steepness, $S_f=\eta/L_\eta$, defined in Figure 1. Given the same wave steepness ($H/L$), a cnoidal wave has a peaky crest and flat trough compared to a sinusoidal wave. In contrast, wave front steepness is able to describe this kind of wave shape characteristics but also describe the wave nonlinearity.

![Figure 1](image_url)  

Figure 1 An example of surface elevation with a steep wave front and the definition of wave front steepness, $S_f=\tan(\alpha)=\eta/L_\eta$.

Model description

OpenFOAM (www.openfoam.org) is a free, open source Computational Fluid Dynamics (CFD) software package developed by OpenCFD Ltd and distributed by the OpenFOAM Foundation. In the field of fluid mechanics, it solves the Reynolds averaged Navier-Stokes equations, and captures the free surface via the extensively used Volume of Fluid (VOF) method. OpenFOAM computes the waves with
adjustable time steps, which are determined based on courant number. A snapshot of a wave impact on a wind turbine foundation by OpenFOAM is shown in Figure 2.

![Figure 2 Snapshot of wave slamming on the monopile by OpenFOAM.](image1)

A toolbox of waves2Foam (Jacobsen et al. 2011) was used to generate and absorb free surface water waves. Together with VOF, Waves2Foam applies the relaxation zone technique (active sponge layers) to the ‘InterFoam’ multiple phase solver. This toolbox is able to present a large range of wave theories, as well as develop user target wave conditions. In order to successfully absorb the reflected waves from the computational domain inside the relaxation zone, a width of relaxation zone larger than one wavelength is recommended. However, this will require significant computational time in its operation.

In this study, monopiles were employed, which are symmetric relative to wave propagation direction. Only half of the monopile and associated computational domain were employed in order to reduce the computational time. This assumes no 3-D effect due to the blockage of waves, which is acceptable when the monopile is slender relative to the incoming wavelength.

Computational mesh used by Peng et al (2013) was employed here to ensure the numerical stability during simulations: \(\Delta x/\Delta z<2.5; \Delta x=\Delta y=\min(D/10, L/100)\), where \(D\) is structure diameter and \(L\) is wave length. Figure 3 shows an example of computational mesh surrounding the pile.

![Figure 3 Example of computational mesh surrounding the monopile](image2)

**MODEL VALIDATION**

In order to investigate the capability of the present model to effectively simulate wave breaking, laboratory measurements from Ting and Kirby (1994)’s were used to validate the wave model. The experiments of Ting and Kirby (1994) were conducted in a two-dimensional wave tank located in the Ocean Engineering Laboratory, at the University of Delaware. The tank was 40m long, 0.6m wide and 1.0m deep. A plywood false bottom was installed in the wave tank to create a uniform slope of 1 in 35. Refer to Ting and Kirby (1994) for a schematic diagram of the experimental design employed here. A model domain was subsequently constructed to reproduce the laboratory experiments over the same
scale. As there is a relaxation zone arranged at the inlet and outlet of the numerical domain, the incoming wave conditions were reproduced in front of the toe of the slope, with a constant water depth.

The data measured at \((x-x_b)/h_b = 7.462\) and at \((x-x_b)/h_b = 10.528\) will be used to validate the numerical model, where \(x_b\) and \(h_b\) are the coordinate and depth at the wave breaking point respectively. These two measurement locations are in front of the wave breaking point and represent steep waves due to wave shoaling. Figure 4 shows that the normalised surface elevations, \(\frac{\eta}{\eta_\text{ref}}\), computed by the model agree well with the measurements of Ting and Kirby (1994). In particular, the modeled free surface captures the front face very well. Deviations on the back face shown in Figure 4 may be due to inaccurate turbulence dissipation, but in the present study the front face steepness is what we focus on.

**MODEL RESULTS**

**Wave breaking**

In order to investigate the wave slamming impact on the monopile, wave breaking has to be generated. There are multiple ways to generate wave breaking with numerical models, and this study makes use of following two methodologies:

- shoaling, with a sloping beach in the computational domain;
- wave groups associated with extreme waves, e.g. 0.1% of the largest waves;

Since a 3D computational domain will consume too much running time, a 2D computation domain is employed instead to study the wave breaking here, before considering the effect of the structure. The presence of a monopile structure will subsequently lead to 3D effects, e.g. refraction, run-up and diffraction, upon the incoming waves. However, without the monopile structure, the waves travelling in the 2D domain shall still exhibit similar behavior in a 3D domain, if the side wall effect is negligible. Moreover, the 2D computational domain is far more effective at studying wave breaking than the 3D computational domain.

Wave breaking times and locations will be captured during the 2D simulations. After recording the wave breaking information, it is now to incorporate the monopile in 3D domains. Wienke and Oumeraci (2005) found that the maximum wave slamming impact occurs when waves just break in front of the pile or on the pile. Therefore, the structure will be placed at the breaking point, and the computational time will be long enough to cover the process of wave breaking.
Wave front steepness

Given the same wave trains propagating in a 3D domain, as those in a 2D domain, the 3D wave travels toward the structure, leading to wave reflection, wave run-up, wave breaking, refraction and diffraction. The free surface surrounding the structure is therefore disturbed. In order to study the wave front steepness, waves were extracted at approximately one diameter away from the center of the structure, in a lateral direction, are used. These wave trains are assumed to be undisturbed, which is acceptable when the pile is slender relative to the incoming wave length.

Wave front steepness is defined as the ratio of surface elevation to wavelength associated with the peak of surface elevation, as shown in Figure 1. This definition relies on the snapshot of surface elevations, rather than the time series of surface elevation at a specific location. This is because random wavelength is difficult to be calculated from the current theory as individual wave periods differ from each other.

The standard procedure to calculate wave front steepness is:
1. Plot the time series of total wave forces on the pile.
2. Identify the timestamp of peaks of wave forces, particularly those surrounding the maximum wave forces.
3. Extract surface elevation snapshots from the 3D output data;
4. Identify the surface elevations at the timestamp of peaks in step 2;
5. Extract the maximum surface elevation and the time period associated with the maximum surface elevations obtained from step 4;
6. Calculate wave front steepness.

An example of the type of results acquired from the above procedures is displayed in Figure 5.

Slamming coefficient

In this study, four cases were selected to investigate the wave slamming coefficients (Table 1). These 4 cases consist of two regular waves and two wave groups, the former breaks due to the shoaling on the beach and the latter breaks due to nonlinear wave interactions.
Table 1 Overview of selected cases for investigating wave slamming impact

<table>
<thead>
<tr>
<th>Case No.</th>
<th>$H_s$ (m)</th>
<th>$T_p$ (s)</th>
<th>$h$ (m)</th>
<th>Wave theory</th>
<th>Radius (m)</th>
<th>Info</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.6</td>
<td>4.2</td>
<td>4</td>
<td>Jonswap, $\gamma=3.3$</td>
<td>0.35</td>
<td>Wave group 1</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>4.2</td>
<td>4</td>
<td>Jonswap, $\gamma=3.3$</td>
<td>0.35</td>
<td>Wave group 2</td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
<td>2</td>
<td>0.5</td>
<td>Rienecker-Fenton</td>
<td>0.1</td>
<td>Wave breaking on beach</td>
</tr>
<tr>
<td>4</td>
<td>0.13</td>
<td>5</td>
<td>0.4</td>
<td>Cnoidal wave</td>
<td>0.1</td>
<td>Ting&amp;Kirby</td>
</tr>
</tbody>
</table>

Considering Case 1 as an example, Figure 6 presents the relationship between total force on the pile and the wave front steepness. Wave forces increase as wave front steepness increases. A second-order regression fit was derived using the least squares method. With the same undisturbed wave conditions, the Morison forces were then calculated, both using linear wave theory and Rienecker & Fenton’s (1981) stream function wave theory. Results show that for small wave front steepness, total wave forces are close to the predictions of the Morison equation, with both theories. However, for large wave front steepness, total wave forces are much larger than the predictions of the Morison equation. This is likely due to the wave slamming impact from a steep wave front, which the Morison equation does not account for. The predicted force equated with the Morison equation, with Rienecker & Fenton’s (1981) stream function wave theory, is larger than that with linear wave theory. This is mainly due to the high nonlinearity present in Rienecker & Fenton’s (1981) stream function wave theory, leading to peaky wave crests and large particle velocities as a result. Results shown in Figure 6 confirm that wave slamming impact is of key importance when wave front steepness gets larger.

![Figure 6 Relationship between total force on the pile and the wave front steepness for Case 1](image)

Recall the purpose of this study, wave front steepness should be connected to wave slamming coefficients, in order to determine the value of the slamming coefficient to be used in equation 3 (above). Given the effect of wave front steepness on wave forces on the pile, four cases of wave slamming, mentioned in Table 1, were selected to derive the relationship between wave slamming coefficients and wave front steepness. Figure 7 shows a quadratic relationship between wave slamming coefficients and wave front steepness. An empirical formula relating the slamming coefficient to the wave front steepness was derived using least squares regression as below:

$$C_s = 253S_f^2 - 195S_f + 38$$

where $C_s$ is the slamming coefficient and $S_f$ is the wave front steepness.
This formula is not definitive and further improvement of the relationship between slamming coefficients and wave front steepness is still needed in the future, primarily because Equation 4 is derived from limited cases only. Also note, this study is only concerned with the slamming impact ‘force’, rather than slamming impact pressure, which aligns with different peak magnitudes and phases.

For practical engineers, two methods could be employed to determine the wave front steepness.

- Metocean design criteria can be used to generate a wave train, select wave groups associated with 0.1% of the largest waves, and then apply these wave groups to a 3D numerical model for nonlinear wave-structure interactions;
- Wave front steepness could be described by wave asymmetry, defined as the lack of symmetry of wave profile relative to vertical axis. Peng et al. (2009) and Zou and Peng (2011) derived the relationship between the Ursell number and wave asymmetry, which could be connected to wave front steepness.

![Figure 7 Relationship between slamming impact and wave front steepness](image)

**SUMMARY**

The numerical model OpenFOAM was employed to study wave slamming impacts on cylindrical wind turbine foundations. A new parameter, wave front steepness, was proposed to describe the wave slamming coefficient. Four cases of wave breaking were employed to study the relationship between wave front steepness and wave slamming coefficients. Based on numerical model results, an empirical formula relating the slamming coefficient to the wave front steepness and hydrodynamic conditions was derived. This study enhances our understanding of the underlying physics governing wave slamming impact on structures. Moreover the newly derived formula can assist engineers in the acquisition of a slamming coefficient during the concept design phase.
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