

# A SIMPLE AND ACCURATE NONLINEAR METHOD FOR RECOVERING THE SURFACE WAVE ELEVATION FROM PRESSURE MEASUREMENTS

Philippe Bonneton, CNRS / Bordeaux University, [philippe.bonneton@u-bordeaux.fr](mailto:philippe.bonneton@u-bordeaux.fr)

Arthur Mouragues, CNRS / Bordeaux University, [arthur.mouragues@u-bordeaux.fr](mailto:arthur.mouragues@u-bordeaux.fr)

David Lannes, CNRS / Bordeaux University, [David.Lannes@math.u-bordeaux.fr](mailto:David.Lannes@math.u-bordeaux.fr)

Kevin Martins, Bath University, [K.Martins@bath.ac.uk](mailto:K.Martins@bath.ac.uk)

Hervé Michallet, CNRS / Grenoble University, [Herve.Michallet@legi.cnrs.fr](mailto:Herve.Michallet@legi.cnrs.fr)

Near-bottom-mounted pressure sensors have long been used for measuring surface wave in the nearshore. The commonly used practice is to recover the wave field by means of a transfer function based on linear wave theory (e.g. Guza and Thornton, 1980; Bishop and Donelan, 1987). However, wave nonlinearities can be strong in the shoaling zone, especially in the region close to the onset of breaking, and thus the use of a linear theory can be questioned. Martins et al. (2017) and Bonneton (2017, 2018) have shown that the linear reconstruction fails to describe the peaky and skewed shape of nonlinear waves prior to breaking, with wave height errors up to 30%. Such measurement errors are problematic for many coastal applications. For instance, studies on wave overtopping and submersion require accurate measurements of the highest wave crests. Furthermore, a correct description of wave asymmetry and skewness is of paramount importance for understanding sediment dynamics. Finally, an accurate description of the wave elevation field is also crucial for the validation of the new generation of fully-nonlinear phase-resolving wave models.

Bonneton et al. (2017) recently derived a fully dispersive nonlinear reconstruction method, which writes:

$$\zeta_{NL} = \zeta_L - \frac{1}{g} \partial_t (\zeta_L \partial_t \zeta_L) \quad (1)$$

where  $\zeta_L$  and  $\zeta_{NL}$  are the linear and nonlinear elevation approximations respectively,  $g$  the gravity and  $\partial_t$  the time derivative. Comparisons with numerical Euler solutions and laboratory data showed that this simple nonlinear method provides much better results than the classical linear approach  $\zeta_L$  (the so-called transfer function method).

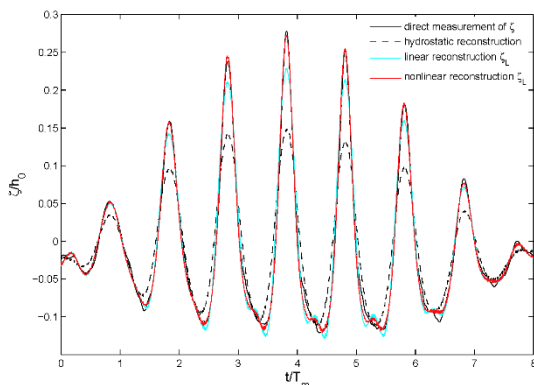


Figure 1 - Surface elevation reconstruction of nonlinear bichromatic waves propagating over a gently movable bed;  $f_1=0.55$  Hz,  $f_2=0.62$  Hz ( $T_m=2/(f_1+f_2)$ ),  $h_0=0.326$  m. Data from Michallet et al. (2017).

In the present paper, this novel approach is evaluated on laboratory and field data of shoaling waves near the

breaking point. Unlike classic linear methods, the nonlinear formula (1) is able to reproduce the peaked and skewed shape of nonlinear waves at the individual wave scale (see Figure 1). Improvements in the frequency domain are also observed, as the new method is able to accurately predict surface wave elevation spectra over four harmonics (see Figure 2). For the bichromatic wave case, presented in Figures 1 and 2, the classical linear method strongly underestimates, by 25%, the wave skewness. The nonlinear formula (1) significantly improve the results with a skewness error of 3%. We will show that our nonlinear formula represents an economic and easy to use alternative to direct wave elevation measurement methods (e.g. acoustic surface tracking and LiDAR scanning).

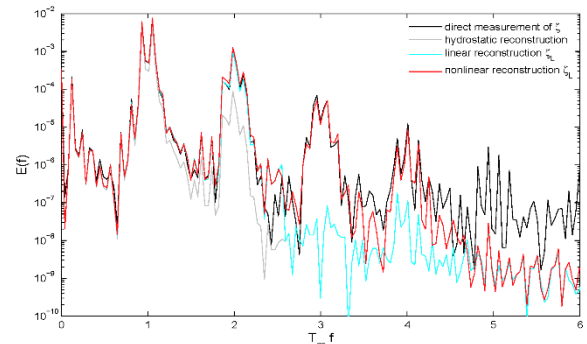


Figure 2 - Surface elevation energy density spectra,  $E(f)$ , for nonlinear bichromatic waves propagating over a gently movable bed;  $f_1=0.55$  Hz,  $f_2=0.62$  Hz ( $T_m=2/(f_1+f_2)$ ),  $h_0=0.326$  m. Data from Michallet et al. (2017).

## REFERENCES

- Bishop, C. T., and Donelan, M. A. (1987). Measuring waves with pressure transducers. *Coastal Engineering*, **11**(4), 309-328.
- Bonneton, P., and Lannes, D. (2017). Recovering water wave elevation from pressure measurements. *Journal of Fluid Mechanics*. **833**, 399-429.
- Bonneton, P., Lannes, D, Martins, K. and Michallet, H. (2018). A nonlinear weakly dispersive method for recovering the surface wave elevation from pressure measurements. In press in *Coastal Eng.*
- Guza, R. T., and Thornton, E.B. (1980). Local and shoaled comparisons of sea surface elevations, pressures, and velocities, *J. Geophys. Res.*, **85**(C3), 1524-1530
- Martins, K., Blenkinsopp, C.E, Almar, R., Zang, Z., (2017). On the influence of swash-based reflection on surf zone hydrodynamics: a wave-by-wave approach. *Coastal Eng.*, **122**, 27-43.
- Michallet, H., Barthélemy, E., Lammens, A., Marin, G. and Vaudelin, G. (2017). Bed motion under waves: plug and sheet flow observations. *Coastal Dynamics 2017*.