PROBABILISTIC SHORELINE CHANGE MODELING AND RISK ESTIMATION OF EROSION

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Prediction of long-term shoreline changes is a key task in planning and management of coastal zones and regional sediment management. Due to complex natural features of offshore waves, sediments, and longshore sediment transport, quantifying uncertainties of shoreline evolution and risks of extreme shoreline changes (erosion and accretion) is of vital importance for practicing uncertainty- or risk-based design of shorelines. This paper presents probabilistic shoreline change modeling to quantify uncertainties of shoreline variations by using numerical-model-based Monte-Carlo simulations. A shoreline evolution model, GenCade, is used to simulate longshore sediment transport and shoreline changes induced by random waves from offshore. A probability density function with a modified tail distribution is developed to capture stochastic features of wave heights under fair weather and storm conditions. It produces a time series of wave heights including small and extreme waves based on their probabilities (or frequencies of appearance). Probabilistic modeling of shoreline change is demonstrated by computing spatiotemporal variations of statistical parameters such as mean and variance of shoreline changes along an idealized coast bounded by two groins. Maximum shoreline changes in return years with a confidence range are also estimated by using maximum likelihood method. Reasonable results of obtained probabilistic shoreline changes reveal that this model-based Monte-Carlo simulation and uncertainty estimation approach are applicable to facilitate risk/uncertainty-based design and planning of shorelines.

Keywords: Shoreline Change Modeling, Erosion Risk Estimation, Monte-Carlo Simulation, Coastal Sediment Transport

INTRODUCTIONS

Understanding of long-term shoreline changes is essential in planning and management of coastal zones and regional sediments along coasts and in inlets, and estuaries. Due to complex spatiotemporal variations of physical forcing such as waves, currents, and sediment transport, design and planning of shoreline protection such as installation of structures and beach nourishments rely on prediction of long-term trends of shoreline position driven by waves and sediment transport processes, as well as human activities on beaches. Ability to predict statistical features of shoreline changes and to quantify uncertainties and risks of extreme shoreline erosion is a key for practicing risk/uncertainty-based management of coastal zones.

Uncertainty in shoreline changes may be generated by different sources from randomness of input wave data, model parameters, model (system) errors, data observation errors, etc. It is relatively difficult to quantify simulation model errors, particularly for nonlinear numerical models. For assessing uncertainty of input wave data and model parameters, Monte-Carlo (MC) simulation has been widely used to produce data samples of shoreline changes based on simulations of shoreline evolution (e.g. Vrijling and Meijer 1992, Dong and Chen 1999, Ruggiero et al. 2006, Scheel et al. 2014, Ding et al. 2017). From those simulated data samples, calculations of statistic parameters such as mean shoreline positions and covariance of changes are straightforward. In probabilistic shoreline modeling, stochastic processes are assumed to be generated by randomness or probabilistic distributions of multiple independent variables such as offshore wave parameters (Dong and Chen 1999, Ding et al. 2017), waves and sediment transport flux (Vrijling and Meijer 1992, Ruggiero et al. 2006), or sediment properties (Scheel et al. 2014). It was found that input wave parameters (wave height, direction, and period) play a significant role in producing most uncertainty or error in shoreline simulation.

An alternative to MC simulations is to develop and solve a stochastic differential equation (e.g. a Fokker-Planck evolution equation) to directly find solutions of statistical parameters (i.e. mean and covariance) and the probability distribution functions of beach positions (Dong and Wu 2013, Wu and Dong 2015). But a presumption of the white noise on wave height and other variables as well as an over

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simplified shoreline change model restricts its applicability to assess uncertainty of shoreline changes in coasts, due to nonlinearity in wave dynamics and sediment transport.

The MC simulations based on numerical models have an advantage of taking into account complex inputs from model variables to reproduce rational random signals including nonlinear effects from waves, currents, and sediment transport. Analytical or semi-analytical solutions based on linearized one-line models are efficient simulation models (e.g. Vrijling and Meijer 1992, Dong and Chen 1999, Reeve et al. 2014). But oversimplification in the models is a concern when using those models in Monte Carlo simulation for practical uncertainty-based design and planning due to lack of capabilities for handling complex coastal processes and engineering practice (structure constructions, beach filling, dredging, etc). Therefore, a one-line model for general engineering applications is needed for using the MC method to predict probability of shoreline changes which reflects natural physical and engineering conditions on coasts. For example, Ruggiero et al. (2006) applied a numerical shoreline change model (Unibest) for an uncertainty assessment of shoreline change along the Long Beach Peninsula, WA. Ding et al. (2018a) developed a MC simulation model for the U.S. Army Corps of Engineers (USACE) one-line model, GenCade, to simulate mean and covariance of shoreline change along the coast at Duck, NC.

Moreover, reliability of MC simulation results is dependent on random input data into the model which represents statistical characteristics of those stochastic variables in the field. For instance, if using a Rayleigh distribution to generate a time series of a limited number of wave heights, large wave heights which represent storm events may be missing due to the narrow-band feature of this function. A broad-band distribution for wave heights is needed to increase the appearance of higher waves which represents storm conditions at sea. Correlation properties of wave directions and periods are also needed, which usually are determined by field observation data.

This paper presents the GenCade-based MC simulations to quantify uncertainties of shoreline changes. A probability density function is developed to capture stochastic features of wave heights under both fair weather and storm conditions by modifying the tail distribution to appropriately increase the occurrence of extreme wave heights. This probabilistic shoreline change modeling is demonstrated by computing spatiotemporal variations of statistical parameters such as mean and variance of shoreline changes along an idealized coast bounded by two groins. Two test cases with two mean wave angles exhibit spatiotemporal distributions of mean and standard deviation of shoreline changes. By the means of maximum likelihood method, extreme shoreline erosion in return years with uncertainties are estimated with using an extreme probability distribution model, the Weibull function.

The GenCade model is developed to simulate shoreline changes driven by coastal processes and to assess the effects of engineering design and planning (e.g. coastal structures, beach nourishment, inlet sediment transport, etc). Therefore, these GenCade-based MC simulations are expected to be an engineering application tool for assessing uncertainty and risk of shoreline changes and facilitating uncertainty-based coastal design.

PROBABILISTIC SHORELINE CHANGE MODELING

Shoreline Evolution Simulation Model (GenCade)

Generally, shoreline evolution simulation model or one-line model is based on the mass conservation of sediment transport in beach system including nearshore zone, surf zone, and subaerial beach. As illustrated in Figure 1, one-line model assumes that the beach profile is displaced parallel to itself in the cross-shore direction, which is the assumption of parallel contour lines. Therefore, shoreline changes can be calculated by a conservation equation of sediment volume. Based on the USACE shoreline evolution simulation model, GenCade (Frey et al. 2012), several new capabilities considering longshore and cross-shore sediment transport, as well as shoreline retreat due to sea level rise and subsidence have been developed. The new governing equation in GenCade for calculating shoreline change is given as follows:

$$\frac{\partial y}{\partial t} = \frac{1}{D_S}\left(-\frac{\partial Q_l}{\partial x} + q_s + \phi\right) - \frac{R+S}{\tan\beta} \tag{1}$$

where y = cross-shore coordinate and represents the shoreline position, t = time, x = the alongshore coordinate, Q_l = longshore transport rate, D_s = total closure depth ($= d_C + d_B$, d_C is the closure depth,

d_b is the berm height) , q_s = line source or sink of sediment, ϕ = cross-shore transport rate, R = sea level rise rate, S = subsidence, $\tan\beta = D_s/W_*$, an average beach slope, and W_* = width of the active profile (approximately the width of longshore sediment transport zone).

The shoreline retreat rate $\frac{R+S}{\tan\beta}$ due to long-term effect of sea level rise (R) and land subsidence (S) is calculated on the assumption of the equilibrium beach profile (Dean and Dalrymple 2002, Bruun 1962).

Eq. (1) represents a new governing equation to simulate shoreline evolution driven by longshore/cross-shore transport, sea level change, and land subsidence. It has been incorporated into GenCade, so that all the newly-developed capabilities are applicable to simulate long-term shoreline changes for practical coastal engineering projects.

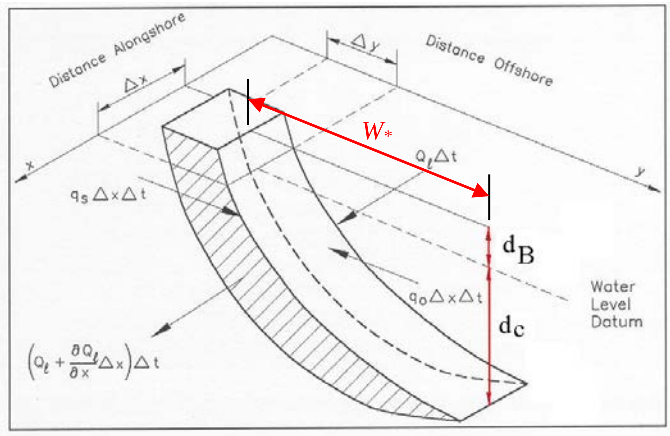

Figure 1 Shoreline change model on equilibrium beach profile

For longshore transport, the CERC (Coastal Engineering Research Center) formula (Frey et al. 2012) is used to estimate Q_l, which is dependent on alongshore wave breaking energy flux related to breaking wave height (H), direction (α), group velocity(C_g), as well as sediment property. A newly-implemented cross-shore sediment transport rate is calculated by considering the net sediment transport in cross-shore direction driven by waves, currents, and gravity (Ding et al. 2018b).

Wave Climate Generation and Monte-Carlo Simulation

Generating incident wave parameters to represent wave climate in a random sea state is vital to study probabilistic features of shoreline changes by means of model-based MC simulations. There are different approaches to create a time series of wave heights, which is a key wave parameter, provided wave excitation follows stochastic processes. One approach is to split the random variation of wave heights into two parts, a mean of wave height and a time-dependent white noise. A Gaussian white noise process can be used to create the random signals by giving a mean of zero and a variance (Vanem et al. 2012). Dong and Wu (2013) utilized this approach with a stochastic differential equation to directly predict stochastic variables of shoreline changes such as mean value and variance (or standard deviation).

However, on the basis of long-term field study of wave dynamics at random sea, coastal scientists and engineers have provided systematic knowledge and procedures to estimate statistical wave parameters in terms of multidirectional wave spectral analysis. Those spectra for wave heights are much closer to the reality of random waves than the zero-mean white noise. We should utilize those spectral distributions to generate wave climate. Moreover, using model-based MC simulation also allows to define different probability distributions for wave parameters (wave heights, directions, and periods), according to spectra established by observation data.

Therefore, for predicting probabilistic shoreline changes driven by the above-mentioned multiple processes due to wave actions, sea level rise, land subsidence, and cross-shore transport, we use wave spectra to generate stochastic wave climate for the following probabilistic shoreline change modeling. In present research, we assume that only wave action is a stochastic process; all other long-term processes such as sea level rise and subsidence are not random, but pre-determined. Hereafter, two variables, i.e. wave height and wave angle, are given as the stochastic variables, which comply with their probabilistic characteristics (or spectra) observed at sea. According to observations of random wave heights in deep water, the stochastic features of wave heights can be approximately described by the Rayleigh probability density function *r* (Longuet-Higgins 1952, Dean and Dalrymple 1992),

$$r(x) = \frac{\pi}{2} x \exp\left(-\frac{\pi}{4} x^2\right) \tag{2}$$

where the non-dimensional variables $x = H / \bar{H}$, $\bar{H}$ is the mean wave height. It is known that the Rayleigh function gives a relatively narrow-band distribution, which produces a sample of wave heights too close to the frequency of the mean wave energy. Longuet-Higgins (1980) suggested that the Rayleigh distribution is still applicable with a properly chosen root-mean-square wave height by taking into account wave nonlinearity in shallow water. Because the GenCade model provides wave transformation and deformation calculation from offshore to wave breaking depth, it asks only for input incident waves located at offshore (at the edge or beyond nearshore zone). Therefore wave spectra built upon the depth wave data fit in this one-line model.

One concern about Rayleigh distribution is that during a simulation time window in a Monte Carlo experiment, this narrow-band probability distribution may not guarantee extreme events (with larger wave heights) happening in the limited number (*N*) of data samples. For example, in the case of making the daily random sample of wave heights in ten years, this function cannot ensure a ten-year wave height is included in time series by using a random number generator of a computer. Therefore, one has to modify the tail distribution to slightly increase the occurrence of extreme wave heights.

As it is well known, the Weibull density function *w(x)* can describe the extreme wave heights in a long observation period (e.g. Forristall 1978, Prevosto et al. 2000),

$$w(x) = \frac{1}{k} (\frac{x-B}{A})^{k-1} \exp\left[-(\frac{x-B}{A})^k\right] \tag{3}$$

where *A* is the scale parameter, *B* is the location parameter (or the truncated value), and *k* is the shape parameter. The three parameters are determined based on data samples of wave heights. Therefore, it is more reasonable to combine the above two distributions in order to produce a random wave sample with different characteristics in both fair and storm weather such as:

$$p(x) = \begin{cases} r(x) & x \in [0, x_0) \\ \varepsilon w(x) & x \in (x_0, +\infty) \end{cases}, \tag{4}$$

where the Rayleigh distribution *r(x)* gives the probability of a wave height when it is less than x_0 which frequently occurs in fair weather. Weibull describes the probability of the extreme wave height (larger than x_0) which typically appears during storm conditions. The coefficient ε can be determined by the condition of all probability integration, i.e. $\int_{-\infty}^{+\infty} p(x) = 1$, then

$$\varepsilon = \exp\left[(\frac{x_0 - B}{A})^k - \frac{\pi}{4} x_0^2\right] \ll 1, \quad x_0 > B \cdot$$

Figure 2 gives exceedance probabilities of large waves ($x > x_0$) produced by Rayleigh distribution $r(x)$ and the modified Weibull function $\varepsilon w(x)$ as the tail distribution. It shows that the tail distribution by the Weibull adds more frequency to the group of large wave heights.

For offshore wave directions, a reasonable assumption is that the distribution of offshore wave angles α falls into a normal or Gaussian probability density function, i.e.

$$p(\alpha) = \frac{1}{\sqrt{2\pi}\sigma_\alpha} \exp\left(-\frac{(\alpha - \mu_\alpha)^2}{2\sigma_\alpha^2}\right), \tag{5}$$

where μ_α is the mean of the angles; σ_α is the variance.

It is not clear yet if wave period can be treated as an independent stochastic variable, although several joint wave-period-height distribution have been proposed (e.g. Longuet-Higgins 1983, Goda 2000). In the present study, we assume that wave period has a strong correlation with wave height, the wave period can be obtained by a wave spectrum, e.g. the JONSWAP spectrum (Hasselman et al. 1973).

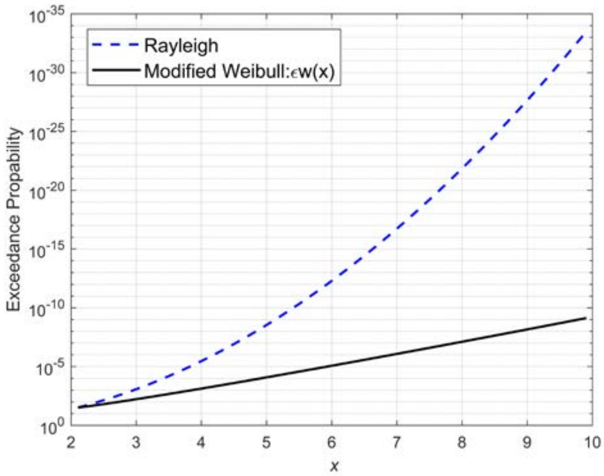

Figure 2 Comparison of exceedance probability given by Rayleigh distribution and the modified Weibull function as the tail distribution

Maximum Likelihood Method for Estimation of Extreme Shoreline Changes

By the means of maximum likelihood estimation approach, the extreme (landward-most or seaward-most) shoreline changes in return interval (year) are calculated by fitting parametric distribution functions such as the Gumbel or Weibull distribution functions with the extreme values computed by the Monte Carlo simulations. The Gumbel cumulative distribution function is given as follow:

$$F(x:A,B) = \exp\left[-\exp\left(\frac{x-B}{A}\right)\right], \tag{6}$$

where x is the extreme value. Similar to Eq. (3), A is the scale parameter, and B is the location parameter (or the truncated value). The Weibull cumulative distribution function is based on the Weibull density function $w(x)$ in Eq. (8), namely,

$$F(x:A,B,k) = 1-\exp\left\{\left(\frac{x-B}{A}\right)^k\right\}, \tag{7}$$

where k is the shape parameter, which is usually greater than unity. The Goda method (Goda 1988) for estimating extreme wave heights is used here for the best fitting of those distribution function families. The best-fitted function will be identified based on the minimized error.

CASE STUDY FOR UNCERTAINTY AND RISK ESTIMATION OF SHORELINE CHANGE USING PROBABILISTIC SHORELINE CHANGE MODELING

To compute probability of shoreline changes in response to stochastic wave conditions, Monte Carlo simulations were completed by considering a hypothetical beach with two groins installed on the two sides of the beach. An initial straight beach is assumed to be 1.0 km long and have a beach slope of 0.01. In the shoreline change simulations, the shoreline was divided into 50 sections with uniform grid size Δx=20 m; the total closure depth D_s=11.0 m (the closure depth d_C=10m, and the berm height d_B=1 m); the empirical coefficients K_1 and K_2 in the CERC sediment transport rate formulae are given as 0.9 and 0.3, respectively; the median grain size (d_{50}) of beach sand is 0.19 mm; the simulation duration is 10 years. Due to numerical stability of an explicit solution scheme, a time step size (Δt) of 54.0 s was given for running all the simulations.

A 17-year wave dataset observed at the Naka Port, Japan, from 1980 to 1996 (Tori et al. 2001) was utilized for constructing the significant wave height distribution given by Eqs. (2)-(4). This observation dataset gives the following parameters for defining the distribution functions: the mean wave height $\bar{H}$ =1.19, the truncated wave height =2.50 m, and then x_0=2.10. The best-fitted Weibull distribution for the wave heights greater than the truncated wave height is determined by the Weibull parameters: k=1.1, A=0.5792, and B=2.0554. The tail distribution of wave heights defined by the Weibull parameters is plotted in Figure 2, by which the occurrence of large wave heights will increase in the time series of wave input data.

The 256 sets of Monte Carlo simulations for predicting the 10-year shoreline change were executed. One set of wave data (H, T, and α) was generated every three hours. This number of numerical samples gives sufficient data samples to produce a desirable randomness for statistical analysis of the shoreline changes. One solution of the 10-year shoreline change simulation typically takes the GenCade model less than 2.0 minutes of CPU time on a 3.60GHz Intel Xeon processor.

Two test cases are designed for examining the sensitivity of the GenCade Monte Carlo simulation model with respect to wave conditions. Therefore cross-shore sediment transport and the effect of sea level rise and subsidence were not considered in the two cases. As shown in Table 1, the combined wave height distribution Eq. (4) and the Gaussian (normal) distribution of wave angles were used to generate the offshore wave parameters. The first test case has a mean wave angle perpendicular to the initial shoreline; the second one gives a mean oblique wave angle of 5.0 degree.

Table 1 Computational Conditions

Case No.	Wave Height (H)	Wave Angle (α)	Period (T)
1	$\bar{H}$ =1.19m; combined distribution, Eq. (4), and $\bar{H}$ =1.19, k=1.1, A=0.5792, and B=2.0554, x_0=2.10	μ_α =0.0^O; σ_α^2 =10; Normal distribution	JONSWAP Spectrum
2	$\bar{H}$ =1.19m; combined distribution, Eq. (4), the same parameters as in Case 1	μ_α =5.0^O; σ_α^2 =10; Normal distribution	JONSWAP Spectrum

One desirable capability of the GenCade-based MC simulation is to predict spatiotemporal variations of statistical parameters of shoreline changes such as mean ($\mu(x,t)$) and standard deviation ($\sigma(x,t)$). For Case 1 in which the mean wave angle is perpendicular to the shoreline, three alongshore profiles of the calculated mean shoreline changes ($\mu(\Delta y)(x,t)$) (t=87, 987, and 87600 hr), as shown in Figure 3 (a), are slightly varying around the initial shoreline (i.e., $y(x)$ = 0.0). Time histories of the mean at three locations (x = 10, 90, and 490m) shown in Figure 3 (b) behave as if the random signals of mean shoreline changes are white noise, and the amplitude of the noise increases from the central to the two sides.

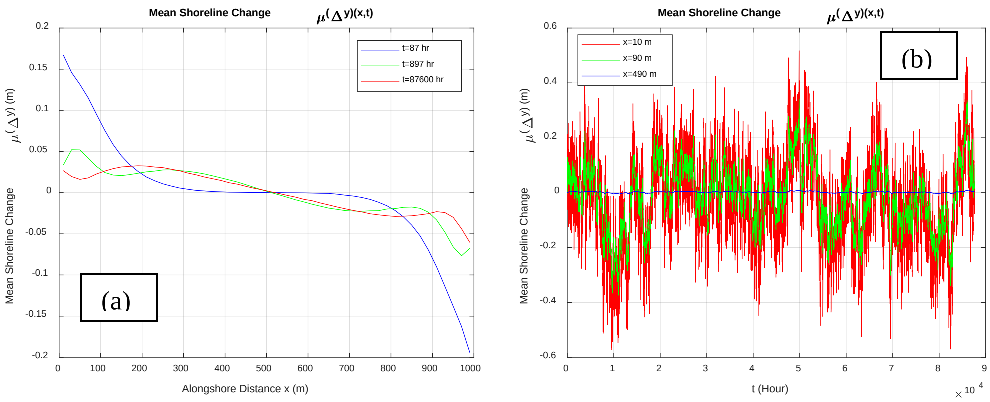

Figure 3 Mean shoreline changes (μ(Δy)(x,t)) in Case 1 (μ_σ=0^o): (a) mean shoreline change profiles at three time steps, (b) time history of mean shoreline changes at three locations

Spatiotemporal variations of variances can tell us how the errors (or deviations) are driven by the input wave data propagate in time (t) and space (x). Figure 4(a) depicts three profiles of standard deviation ($\sigma(x,t)$) at the three time steps (t=87600 hr is the end of simulation, i.e. 10 years). It is evident that the errors are propagating from the two ends near the groins toward the center of the shoreline, and the deviations quickly decay to a trivial value near zero. In other words, the maximum errors (or uncertainties) of shoreline changes along the idealized coast occur at the two ends. Time histories of the deviations at three locations (i.e. the left end, 90m, and the center) in Figure 4(b) indicate that the variances grow very quick at the first 1000 hours, and then stay fluctuating during the rest of simulation time. Interestingly, the time variations at the center are very limited with little fluctuation.

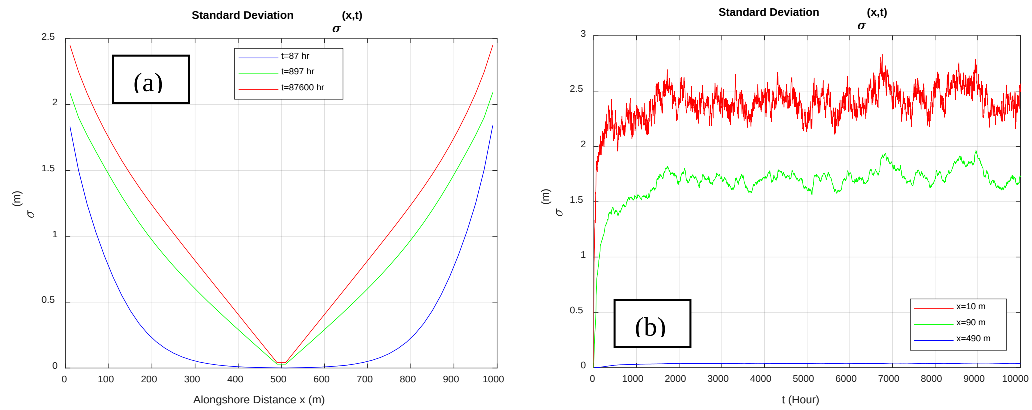

Figure 4 Standard deviations ($\sigma(x,t)$) in Case 1: (a) profiles of standard deviation at three time steps, (b) time history of σ at three locations

In Case 2 for which the mean wave angle μ_σ is 5º, the mean shoreline changes as shown in Figure 5(a) imply that the shoreline tilts from the initial profile (x=0) to an oblique line, as the erosion occurs at the left and the accretion at the right. Again, the center shows no changes. It means that the long-term shoreline patterns (or equilibrium beach shapes) are strongly dependent on incident wave directions, particularly the mean direction.

Figure 5(b) provides the histories of mean shoreline changes at three locations along the left hand side (the eroded coast). It takes about 6,000 hours to reach an equilibrium shoreline position with less changes in shorelines. The variances of shoreline changes ($\sigma(x,t)$) in Case 2 as shown in Figure 6 shows similar patterns for the spatiotemporal variations of the deviations, even though the mean wave angle has a five degree difference from that in Case 1.

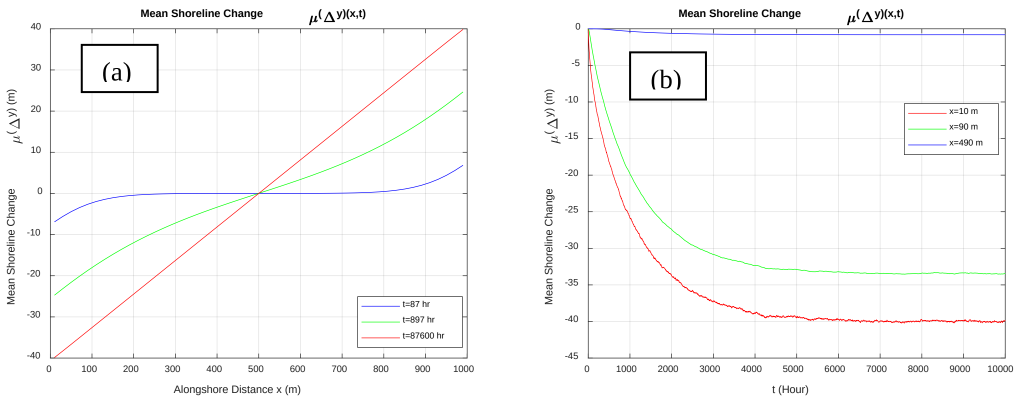

Figure 5 Mean shoreline changes ($\mu(\Delta y)(x,t)$) in Case 2 (μ_σ=5º): (a) mean shoreline change profiles at three time steps, (b) time history of mean shoreline changes at three locations

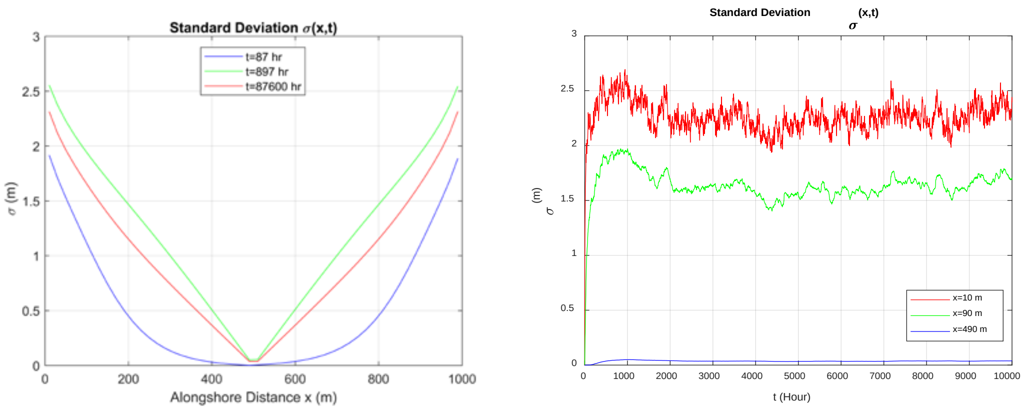

Figure 6 Standard deviations ($\sigma(x,t)$) in Case 2: (a) profiles of standard deviation at three time steps, (b) time history of σ at three locations

Figure 7(a) plots all 256 shoreline profiles for Case 1 computed at t=10 year (the end of 10th year), which gives an envelope of all possible shoreline changes simulated by the Monte Carlo method. The shoreline positions at the two ends (two groins) of the beach vary from -7.0 (erosion) to +7.0 m (accretion). Figure 7(b) provides the mean shoreline profile, which is almost overlaid with the initial straight shoreline at y=0.0m, and the two confidence intervals defined by the standard deviation at t=10 year calculated by the 256 data samples. The shoreline changes as the two ends have the same standard deviation (σ=2.45m). However, the value of σ at the middle of the coast is very small (σ= 0.04 at x = 500 m), and it almost linearly increases to the maximum value (i.e. 2.45m) at the two ends. It indicates that the shoreline at the middle has no change, but the uncertainty of the predicted mean positions increases from a trivial value to the maximums at the two ends. In other words, the propagation of error or uncertainty due to incident wave data is from the center to the two sides. Obviously, the existence of the structures, which controls longshore sediment transport, exaggerates the uncertainty of predictions, which has been pointed out by Dong and Wu (2013) by solving a Fokker-Planck equation.

For Case 2 (Figure 8), the mean shoreline profile after 10-years of wave action is an oblique line with a 4.7 degree angle away from the initial shoreline. This mean shoreline position can be viewed as the equilibrium shoreline profile for a long-term wave action with a 5-degree mean wave angle. It indicates that the idealized shorelines in a manner of long-term change exhibit 'beach memory' as Reeve et al (2014) pointed out. The variations of the mean shoreline positions at the 10th year range from −40.0m to +40.0m from the left to the right, and it has been found that the variations of the standard deviation in Case 2 are similar to those in Case 1.

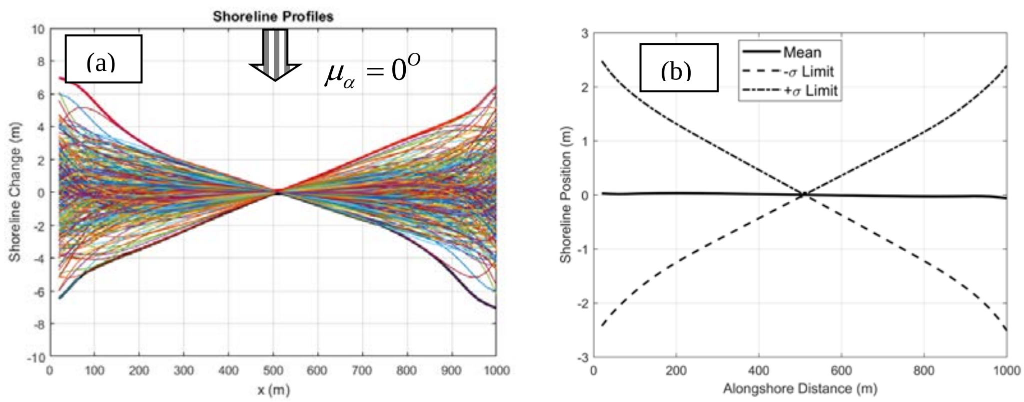

Figure 7 Shoreline profiles for Case 1 after 10 years: (a) All 256 computed shoreline profiles after 10 years, and (b) mean shoreline profile with two limits of standard deviation (σ)

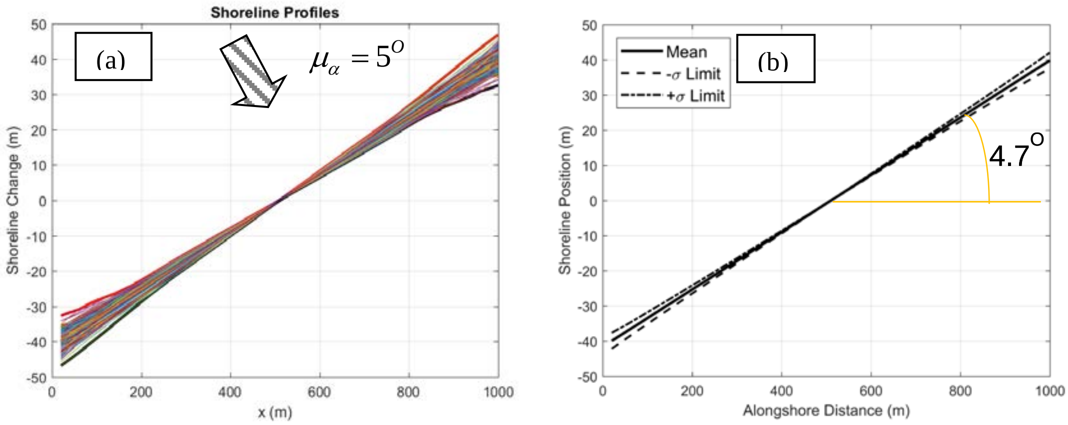

Figure 8 Shoreline profiles for Case 2 after 10 years: (a) All 256 computed shoreline profiles after 10 years, and (b) mean shoreline profile with two limits of standard deviation (σ)

Based on the 256 Monte Carlo simulation results for the two test cases, we are able to develop probabilistic distributions of shoreline changes in each transect of the model at any simulation time step, which will form a spatiotemporal distribution of shoreline changes for the computed coast. The statistical parameters such as mean, standard deviation, and extreme values of shoreline erosion or accretion can be obtained readily. For Case 1 (the mean wave angle is zero degree), Figure 9 presents the two probability density functions of shoreline changes after 10 years (a) at the left groin and (b) at the right groin. The normal distribution functions based on the mean values (μ) and standard deviations (σ) of the shoreline changes are also added in the figure. It implies that the histograms calculated by the Monte Carlo simulations are similar to the normal distributions.

For Case 2 (the mean wave angle is 5 degree), as shown in Figure 10, the probability density functions obtained from the numerical experiments are similar to those in Case 1. However, the absolute mean shoreline changes at the two ends increase up to 40m, the mean erosion occurs at the left, and mean deposition with the amount at the right side. Interestingly, the maximum standard deviation (=2.29) at the end of the coast is close to the maximum σ value in Case 1. Presumably, the deviation of shoreline change may change with the variations of incidence wave angle deviation (here the covariance of the angle is 10 for both cases).

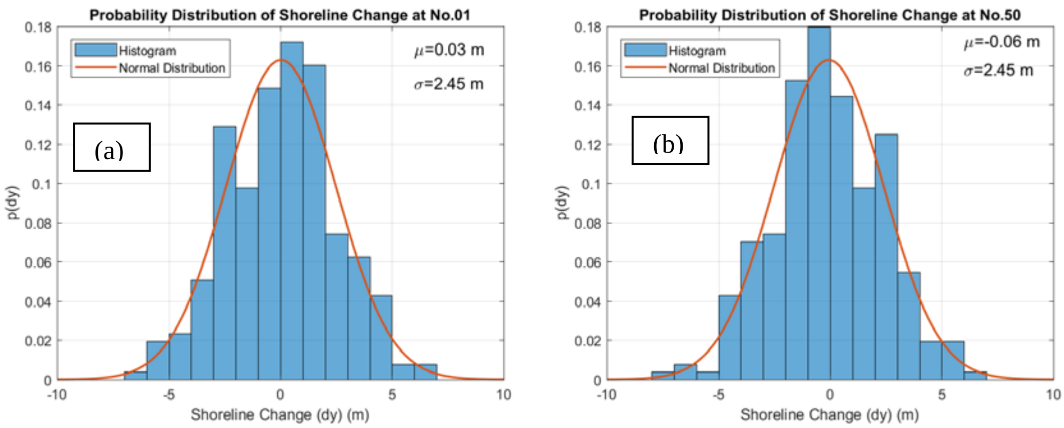

Figure 9 Probability density functions of shoreline changes after 10 years for Case 1, (a) at the left groin, and (b) at the right groin (μ is the mean shoreline change; σ is the standard deviation)

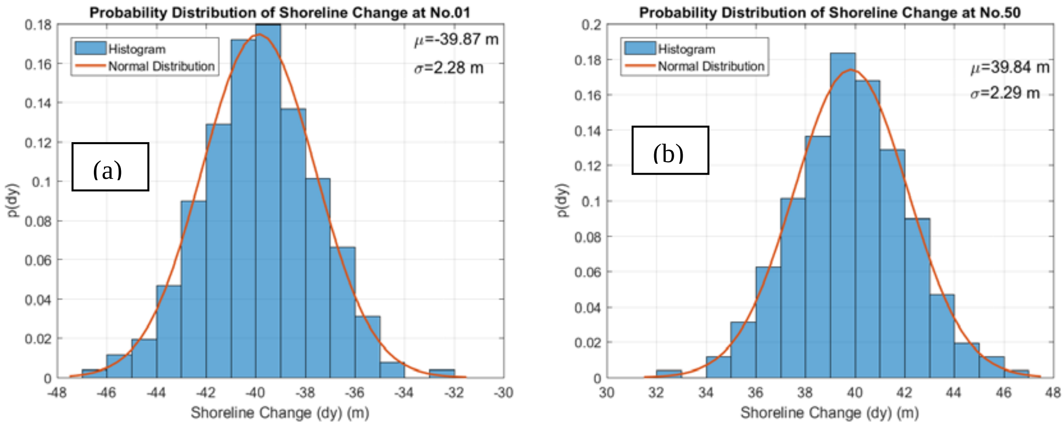

Figure 10 Probability density functions of shoreline changes after 10 years for Case 2, (a) at the left groin, and (b) at the right groin

Furthermore, to extend the prediction of the extreme shoreline change beyond the simulation period (i.e. ten years), the maximum likelihood method is used. Taking the maximum erosion after 10 years at the left groin for example, by best-fitting the distribution functions from the 256 data samples, it has been found that the best distribution function is a type of Weibull distribution. Figure 11 plots the exceedance probability of the computed maximum erosion and the best Weibull distributions for (a) Case 1 and (b) Case 2. The identified distribution parameters (i.e. *A*, *B*, and *k* in Eq. (7)) and the values of R_2 (coefficient of determination) are also shown in the figures.

Based on the best-fitted Weibull distribution by means of the maximum likelihood, the maximum erosions are predicted in return years (*Ry*), i.e.

$$R_y = \frac{1}{\lambda(1 - F(\Delta y : A, B, k))}, \tag{8}$$

where Δy is the maximum erosion at the return year *Ry*, $\lambda = N/T_{sim}$ is the mean of data sample (Goda 1988), N is the number of Monte Carlo simulation (i.e. 256), and T_{sim} is the simulation period (i.e. 10 years in the two cases). Figure 12 presents the mean maximum shoreline erosions in return years from 1 to 100 years and their two confidence intervals (i.e. ±σ) at the end of 10 years for the two test cases. It is found the mean maximum erosion for Case 1 varies from 15.0 m (1-year return) to 21.5 m (100-year return), and 52.0 m to 57.1 m for Case 2. The maximum width of confidence interval (2σ) is about 0.5 m.

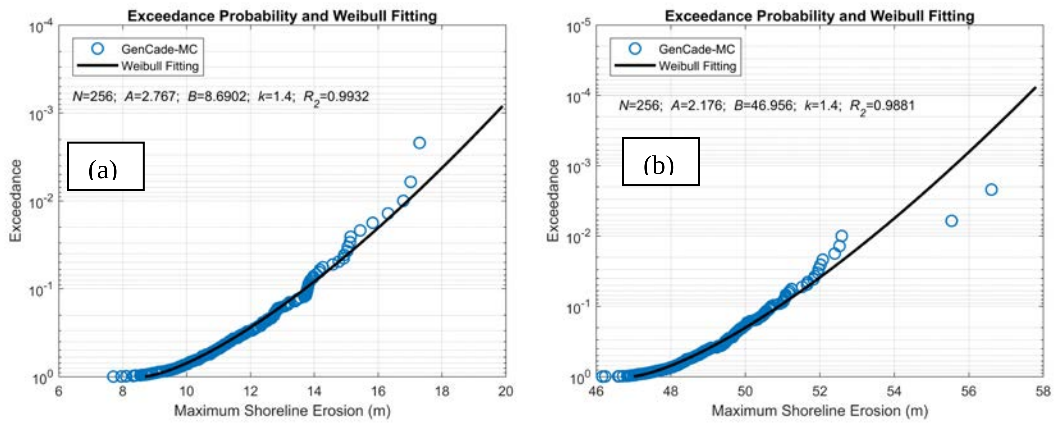

Figure 11 Estimations of maximum shoreline erosion at the left groin for (a) Case 1 and (b) Case 2

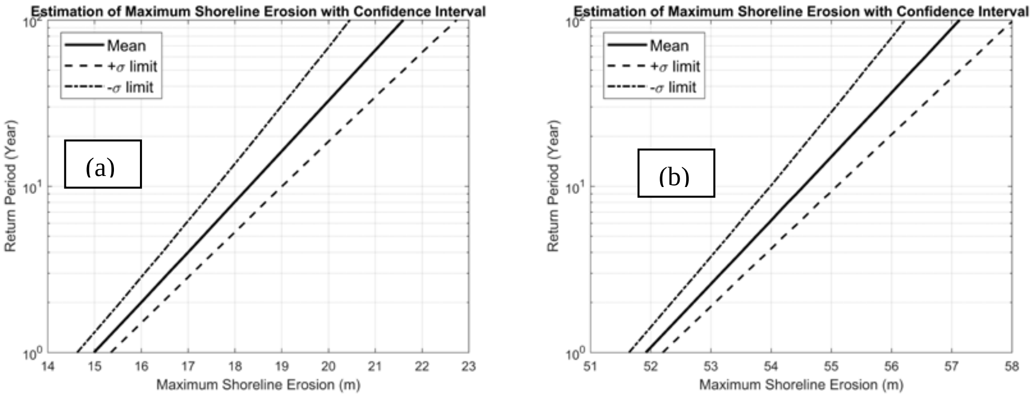

Figure 12 Estimations of maximum shoreline erosion at the left groin for (a) Case 1 and (b) Case 2

CONCLUSIONS

Due to random features of offshore waves and sediment transport, quantifying uncertainties of shoreline evolution and risks of extreme shoreline changes (erosion and accretion) is a key for practicing uncertainty- or risk-based management of shoreline protection. This paper presents a probabilistic simulation model to quantify uncertainties of shoreline changes by using numerical-model-based Monte Carlo simulations. The USACE's shoreline evolution model, GenCade, is used to simulate shoreline changes driven by random waves from offshore. A wide-band probability density function is developed to capture stochastic features of wave heights under both fair weather and storm conditions by modifying tail distribution to increase the occurrence of extreme wave heights. This produces time series of wave heights including small and extreme waves based on their probabilities (or frequencies of appearance).

Probabilistic shoreline change modeling is demonstrated by computing spatiotemporal variations of statistical parameters such as mean and variance of shoreline changes along an idealized coast with two groins bounded. By assuming wave height and wave direction are stochastic variables in the simulation model, and period is dependent on the JONSWAP wave spectrum, this model-based MC simulation produced a desirable randomness of incident wave conditions to compute shoreline changes. Two test cases with two mean wave angles exhibit spatiotemporal distributions of mean and standard deviation of shoreline changes. It was found that the mean of shoreline positions is a good representative for the equilibrium shoreline shape, and the uncertainty of shoreline changes vary and propagate from the structures (groins) to the center of the beach. Based on long-term and large MC simulation samples, the computed statistical features reveal long-term characteristics of shoreline variations with desirable measures of uncertainty.

Maximum shoreline changes in return years are also estimated by means of maximum likelihood method by fitting exceedance probability of extreme values with the Weibull functions. The Goda method also gives confidence intervals of mean maximum shoreline changes. Reasonable results of probabilistic shoreline changes indicate that this model-based MC simulation and uncertainty estimation approach reproduces a desirable randomness of wave forcing conditions, and effectively predicts long-term shoreline changes and their errors.

The GenCade model simulates shoreline changes driven by coastal processes and assesses effects of engineering design and planning (e.g. coastal structures, beach nourishment, inlet sediment transport, etc.). Therefore, this GenCade-based MC simulation is expected to be an engineering application tool for assessing uncertainty and risk of shoreline changes and facilitating uncertainty-based coastal design.

Assessment of shoreline change uncertainties along natural coasts is ongoing and will be reported in the near future, including assessment of model errors. Uncertainties from the GenCade model parameters (e.g. the closure depth, the coefficients K_1 and K_2 in CERC formula, sea level rise, and cross-shore sediment transport) and boundary conditions will be investigated.

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