

# Probabilistic Shoreline Change Modeling and Risk Estimation of Erosion

Yan Ding, Sung-Chan Kim, Ashley E. Frey,  
and Rusty L. Permenter

U.S. Army Engineer Research and Development  
Center (ERDC),  
Coastal and Hydraulics Laboratory (CHL)

ICCE 2018, 3 August 2018, Baltimore, MD



US Army Corps  
of Engineers®

**ERDC**

Engineer Research and  
Development Center



# Outline

- Introduction to GenCade
  - Discussion of new features
- Simple study case
- Monte Carlo simulation of shoreline change at Duck, NC
- Conclusions



# Objectives

- **Estimation of probability of shoreline change for risk analysis**

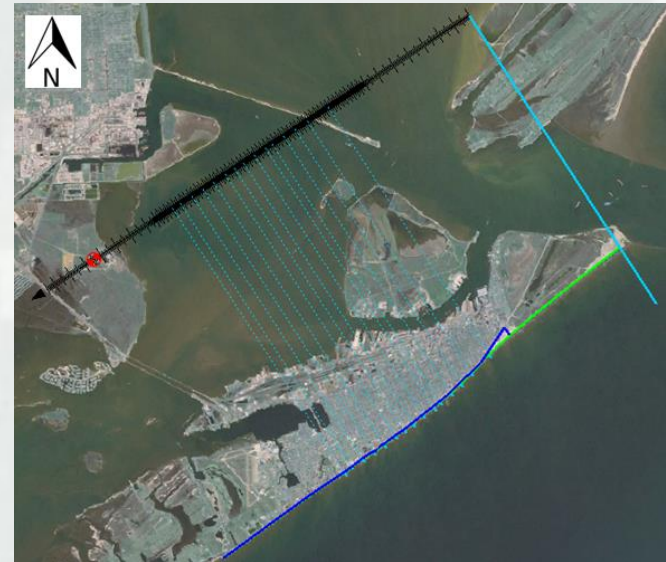
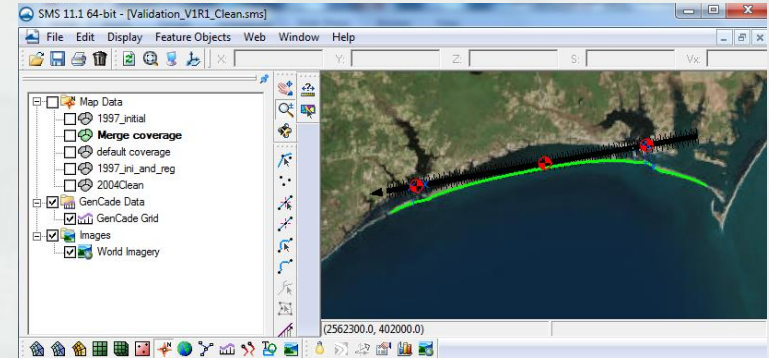
Use a stochastic method (Monte Carlo Simulation) to consider uncertainty in shoreline changes under natural wave and current conditions

- Develop probability density functions to model offshore waves under fair weather and extreme conditions
- Develop a Monte Carlo Model for USACE's shoreline evolution simulation model, GenCade
- Verify and validate this newly-developed Monte Carlo model to estimate shoreline change probabilities.



# GenCade Background

- A one-dimensional shoreline change, sand transport, and inlet-sand sharing model developed by the CIRP
- Combines the engineering power of GENESIS with the regional processes capability of the Cascade model
- Development began in 2009, GenCade Version 1 was released in 2012
- Operated within the Surface-Water Modeling System (SMS) 11.1 or higher



Top: Onslow Bay, NC application (for SAW)  
Bottom: Galveston, TX (Galv. Park Board)

**ERDC**





# Model Formulation

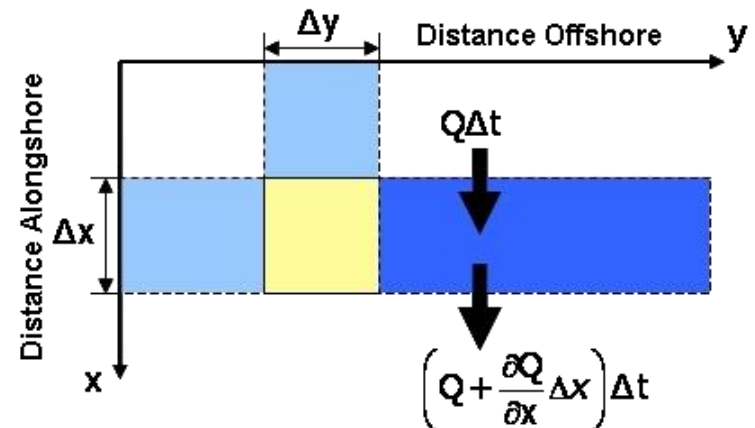
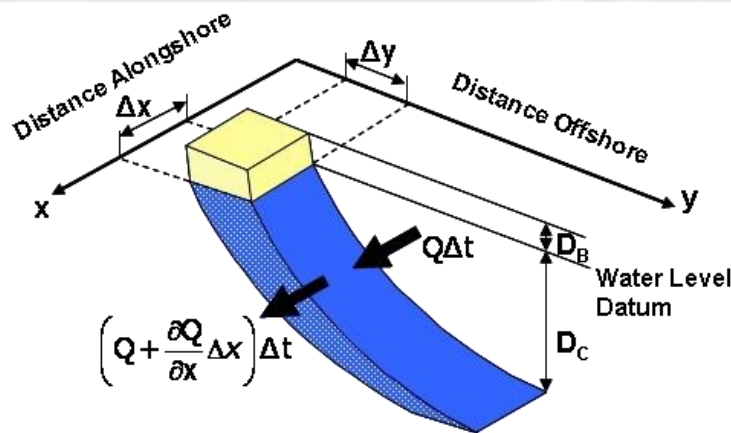
Longshore Net Volume Change:  $\frac{dQ}{dt} = \left( \frac{\partial Q}{\partial x} \right) dx dt$

Cross-shore Net Volume Change:  $dq dt$

Total Volume Change:  $dV = dx dy (D_B + D_C) = \left( \frac{\partial Q}{\partial x} \right) dx dt + q dx dt$

$\therefore$  as  $dt \rightarrow 0$ :  $\frac{\partial y}{\partial t} + \frac{1}{D_B + D_C} \cdot \left[ \frac{\partial Q}{\partial x} - q \right] = 0$

$x$  = direction alongshore;  $y$  = location of shoreline;  $Q$  = longshore sediment transport rate  
 $q$  = source or sink of sediments;  $d_c$  = offshore closure depth;  $d_b$  = berm height;  
 $D_s = d_c + d_b$  = height of sediment movement



BUILDING

# Model Formulation

Sediment transport rate  $Q$  ( $\text{m}^3/\text{s}$ ):

$$Q = \left( H^2 C_g \right)_b \left( a_1 \sin 2\alpha_{bs} - a_2 \cos \alpha_{bs} \frac{\partial H_b}{\partial x} \right)$$

Where,

$H$  = wave height (m)

$C_g$  = wave group speed (m/s)

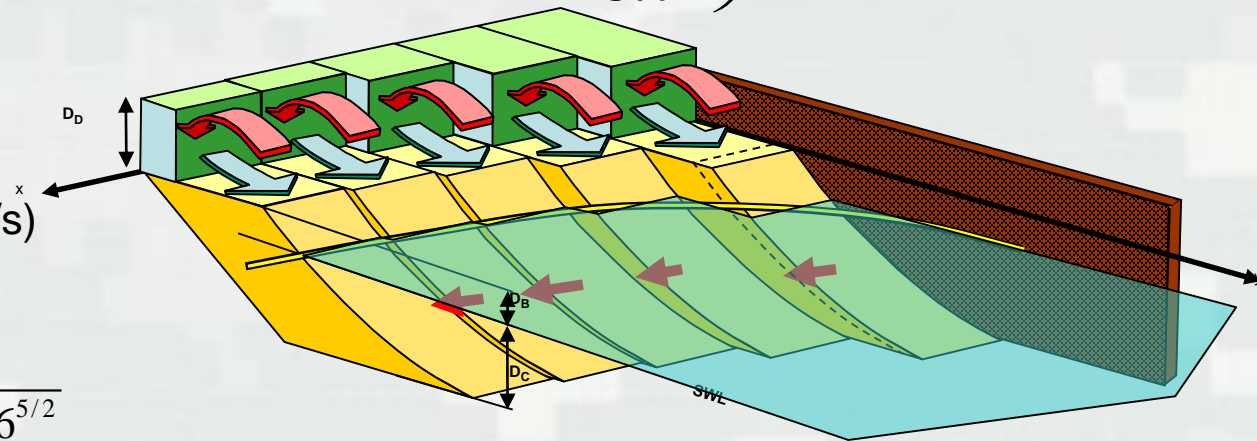
$\alpha_{bs}$  = angle of the breaking

$$a_1 = \frac{K_1}{16(\rho_s / \rho - 1)(1 - p) 1.416^{5/2}}$$

$$a_2 = \frac{K_2}{8(\rho_s / \rho - 1)(1 - p) \tan \beta 1.416^{5/2}}$$



Typically, value of  $K_2$  is:  
 $0.5K_1 < K_2 < 1.5K_1$



Where,

$K_1$  = Primary empirical transport coefficient  
 (controls magnitude of longshore transport rate)

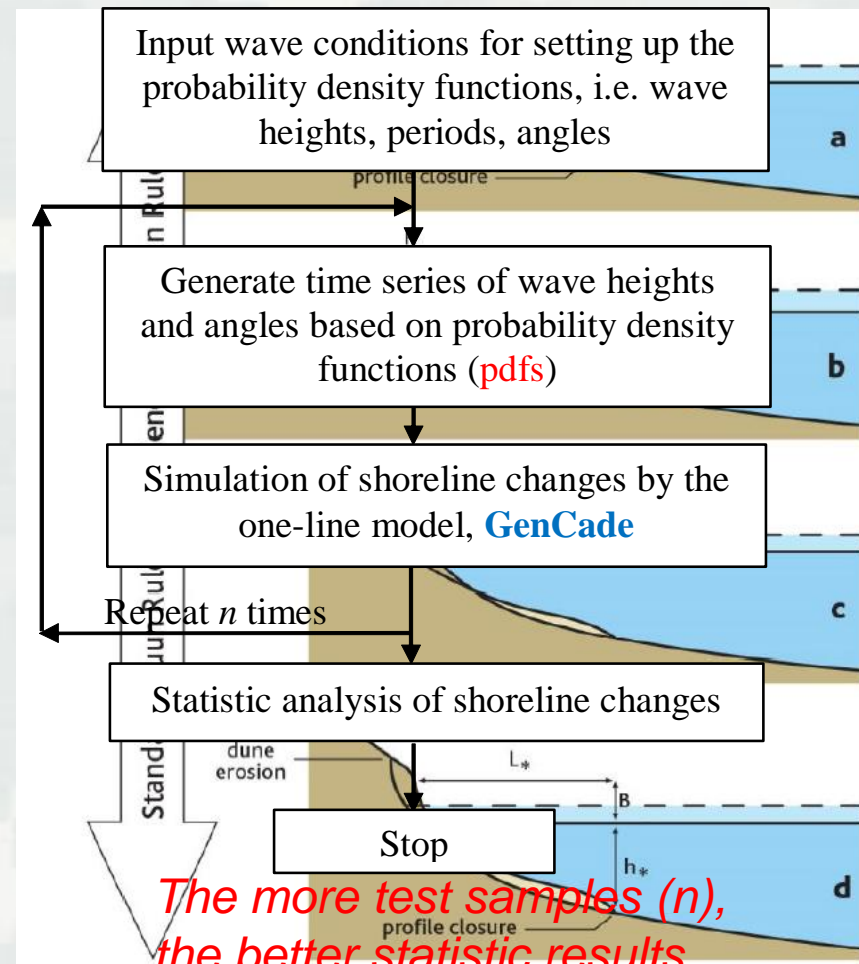
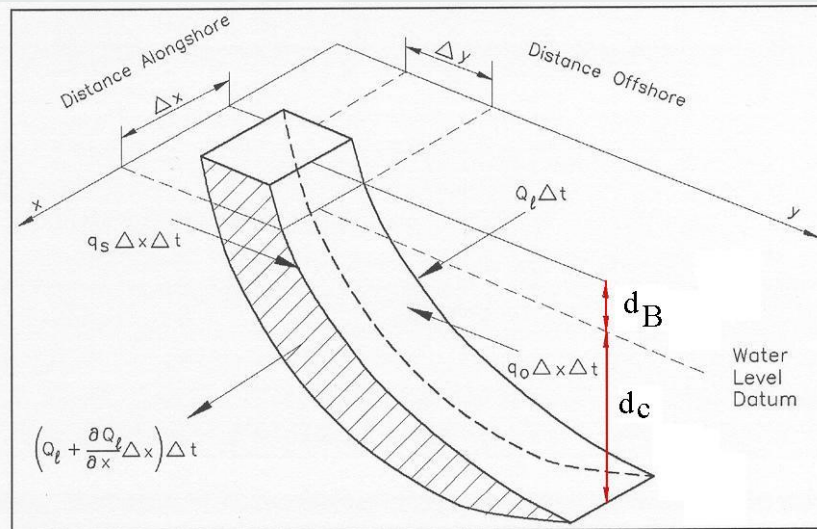
$K_2$  = Secondary empirical transport coefficient  
 (controls distribution of sand within an area; esp.  
 where large wave height gradients, e.g. salients)

$\tan \beta$  = average bottom slope



# New GenCade Features

- Cross-shore transport
- Shoreline recession due to Sea Level Rise
- Monte Carlo simulation of shoreline change



From Shand et al. (2013)



# GenCade Shoreline Evolution Model with Cross-Shore Transport and SLR

$$\frac{\partial y}{\partial t} + \frac{1}{D_s} \left( \frac{\partial Q}{\partial x} - q - \phi \right) + \left( \left( \frac{\Delta Z}{\Delta t} \right)_{SLR} - \left( \frac{\Delta Z}{\Delta t} \right)_{subsidence} \right) \frac{1}{\tan \beta} = 0$$

$\phi$  : Cross-shore sediment transport rate

$\left( \frac{\Delta Z}{\Delta t} \right)_{SLR}$  : Sea Level Change rate

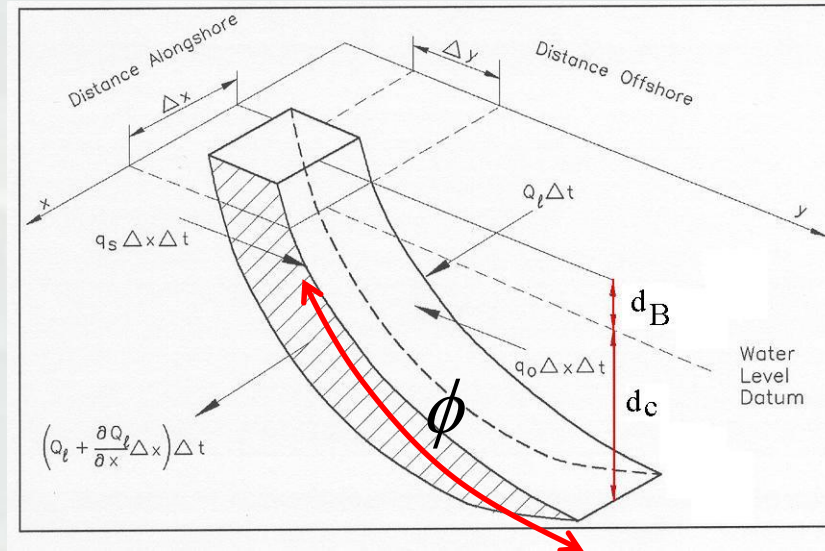
$\left( \frac{\Delta Z}{\Delta t} \right)_{subsidence}$  : Land subsidence rate

$\tan \beta$  : beach slope

$$D_s = d_c + d_b(t)$$

- Berm height varies with sea level change

$$d_b(t) = d_{b0} - \left( \left( \frac{\Delta Z}{\Delta t} \right)_{SLR} - \left( \frac{\Delta Z}{\Delta t} \right)_{subsidence} \right) t$$



**ERDC**



# Probabilistic Distributions of Wave Heights

According to observations of random wave heights in deepwater, the stochastic features of wave heights can be approximately described by Rayleigh function

$$R(x) = -\frac{\pi}{2} x \exp\left(-\frac{\pi}{4} x^2\right) \quad x=H/H_{\text{mean}}, H_{\text{mean}} \text{ is the mean value of wave height}$$

- *May miss extreme waves in a limited discrete series of the wave samples*

Weibull density function can describe the extreme random wave heights in a long observation period

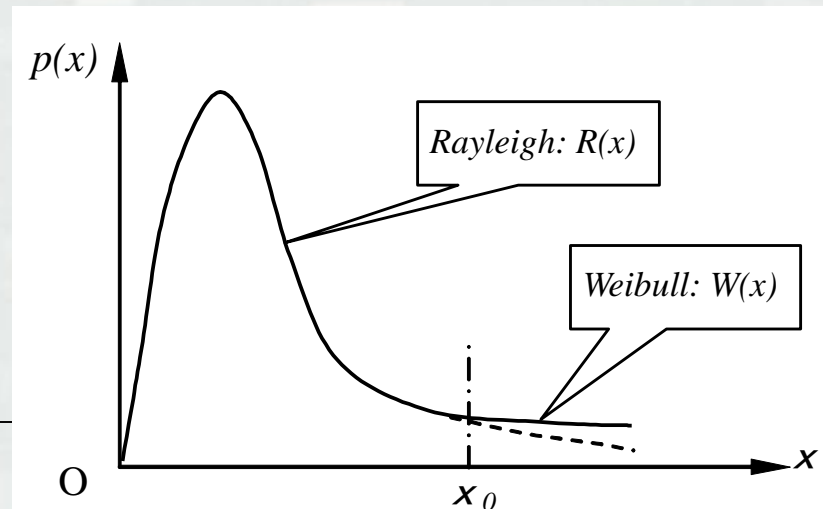
$$W(x) = \frac{1}{k} \left(\frac{x-B}{A}\right)^{k-1} \exp\left(-\left(\frac{x-B}{A}\right)^k\right)$$

$A$ ,  $B$ , and  $k$  : Weibull's parameters

$$p(x) = \begin{cases} R(x) & x \in [0, x_0) \\ \varepsilon W(x) & x \in (x_0, +\infty) \end{cases}$$

$\varepsilon$ : parameter

$x_0$ : an extreme value of wave height



# Wave Direction and Period

Incident Wave Angles: Gaussian Distribution

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$

$\sigma$ : Standard deviation of wave direction

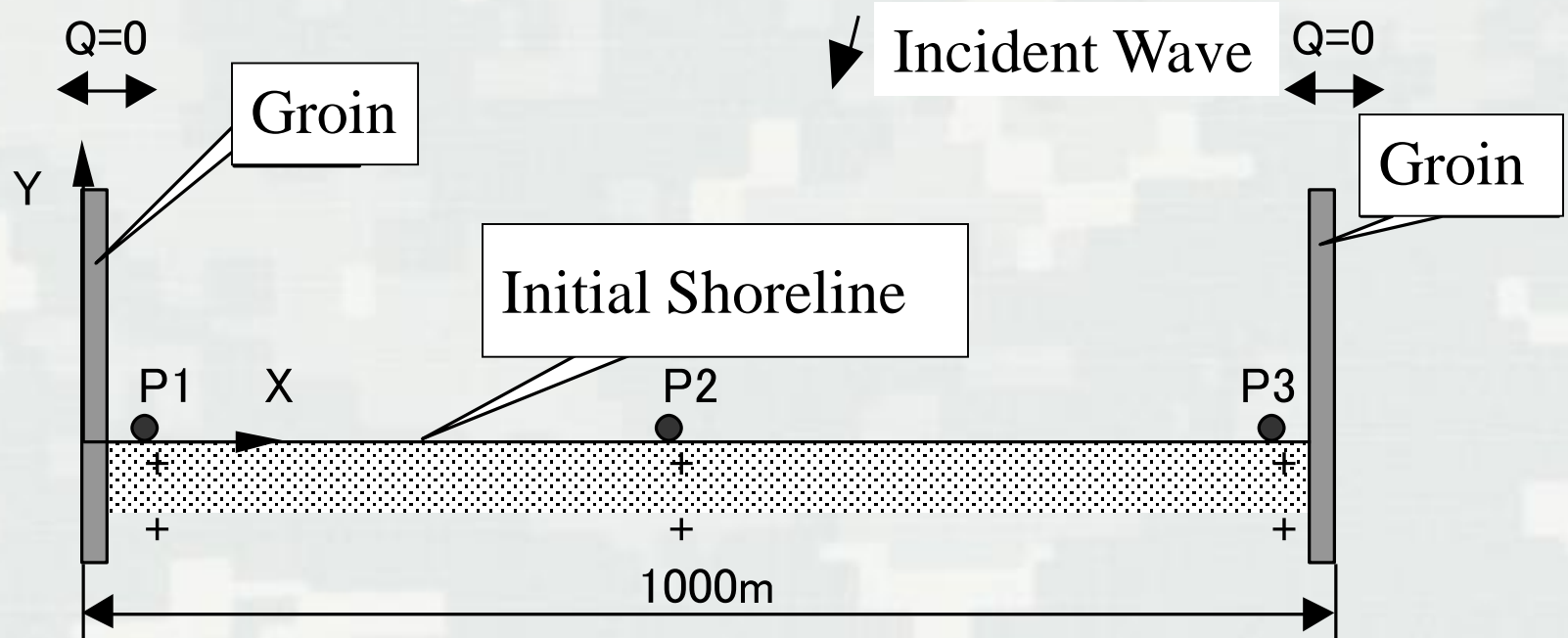
$\mu$ : Mean value of direction

Significant Wave Period: based on Pierson-Moskowitz Spectrum

$$T_s = 5\sqrt{H_s}$$



# Study Case



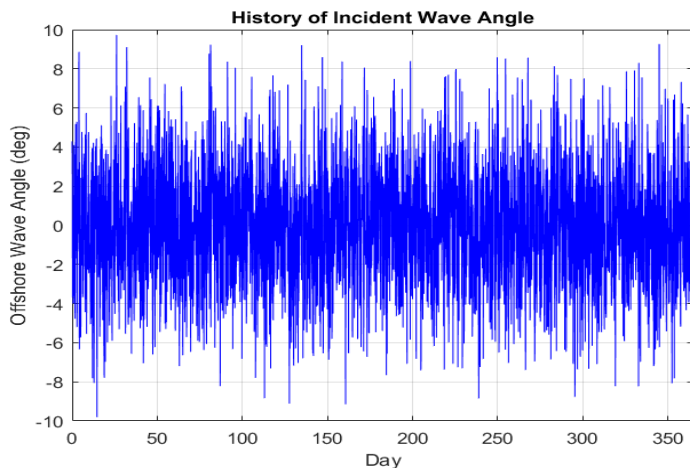
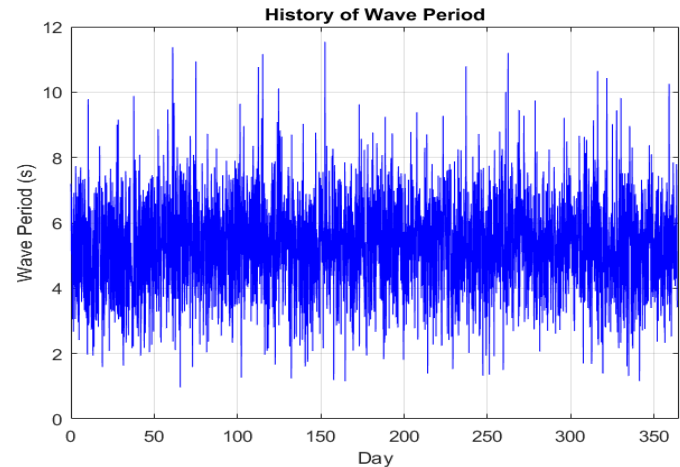
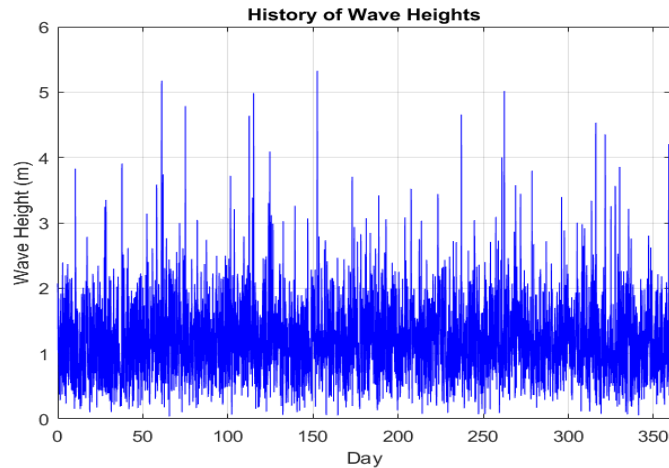
Stochastic variables: wave height and direction

Computational period: 10 years

Numerical experiments: 256



# Wave Parameters Generated by Wide Band Spectrum



- $H_{\text{mean}} = 1.19\text{m}$
- Mean Angle = 0.0 with  $\sigma^2=10$
- Data Interval = 3.0 hours

$$p(x) = \begin{cases} R(x) & x \in [0, x_0) \\ \varepsilon W(x) & x \in (x_0, +\infty) \end{cases}$$

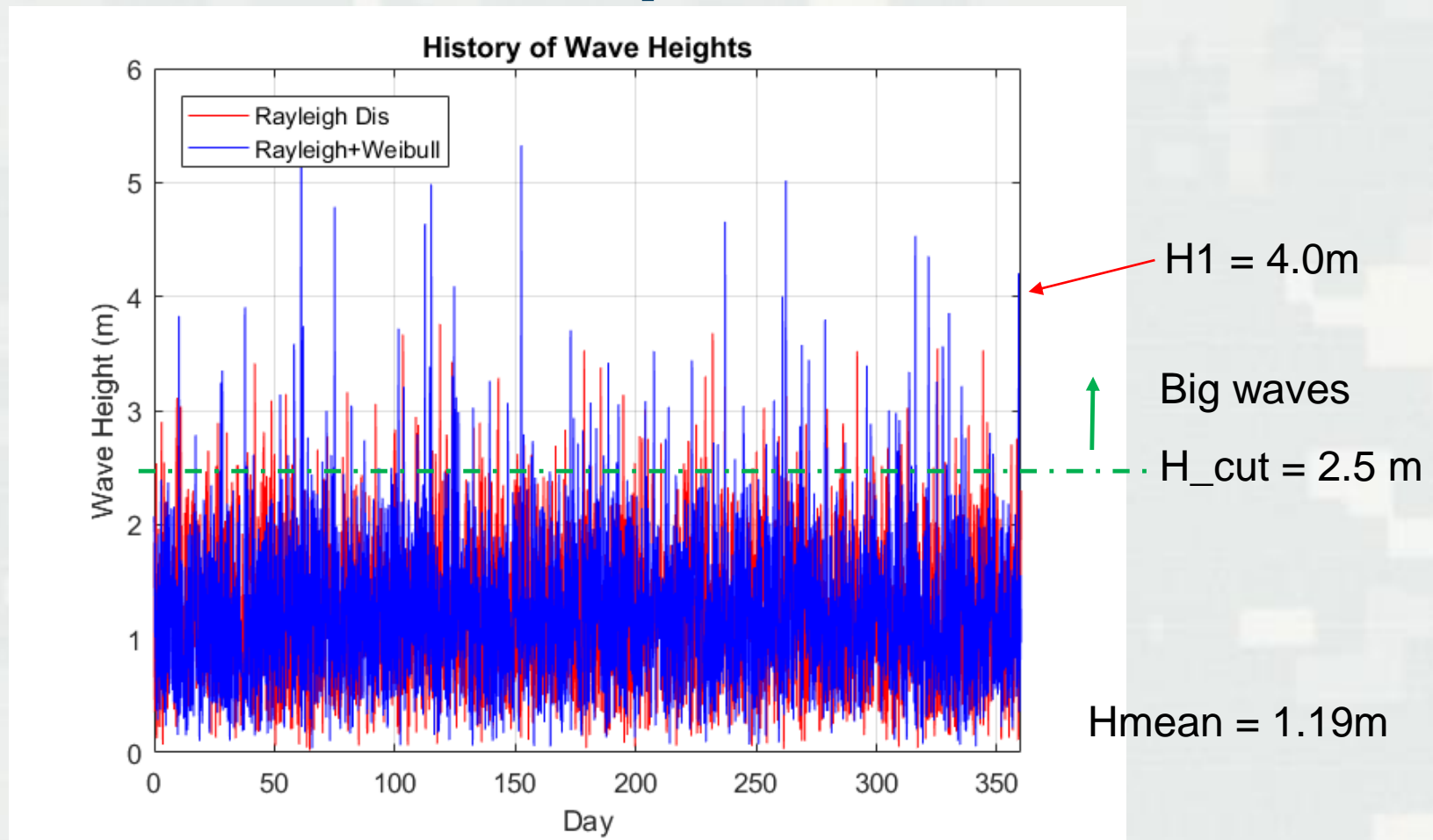
$H_{\text{cut}} = 2.5 \text{ m}$

**ERDC**





# Comparison of Wave Heights by Two Wave Spectra

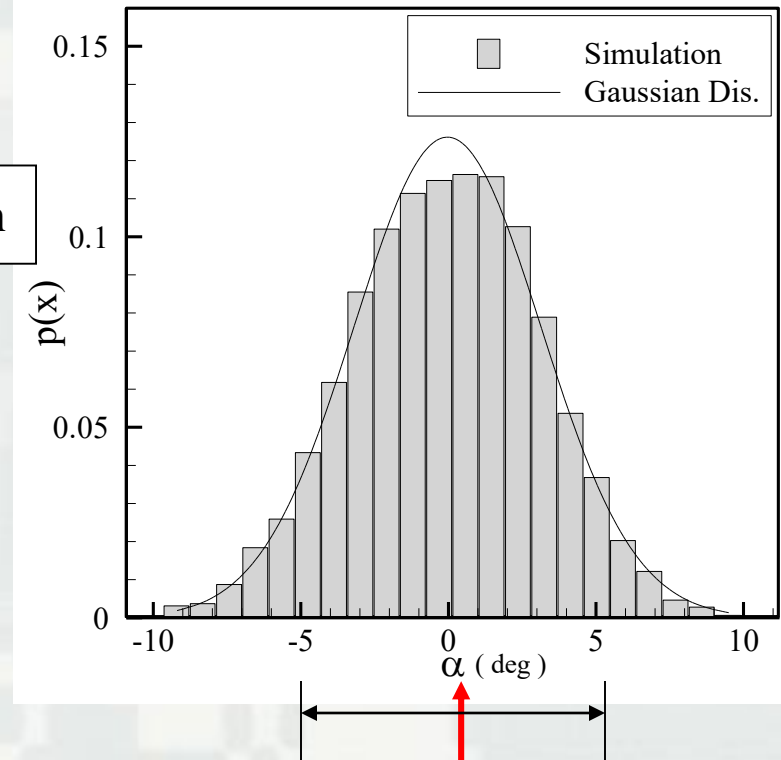
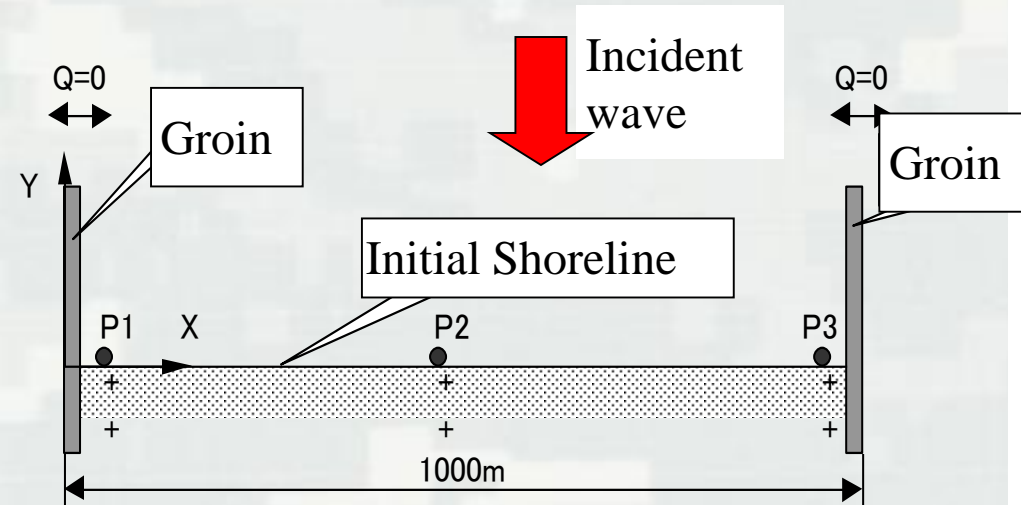


Observation data: 4.0 m wave height  
for one-year return period



**ERDC**

# Probability Distribution of Wave Direction in Case 1 (Normal Wave Direction)

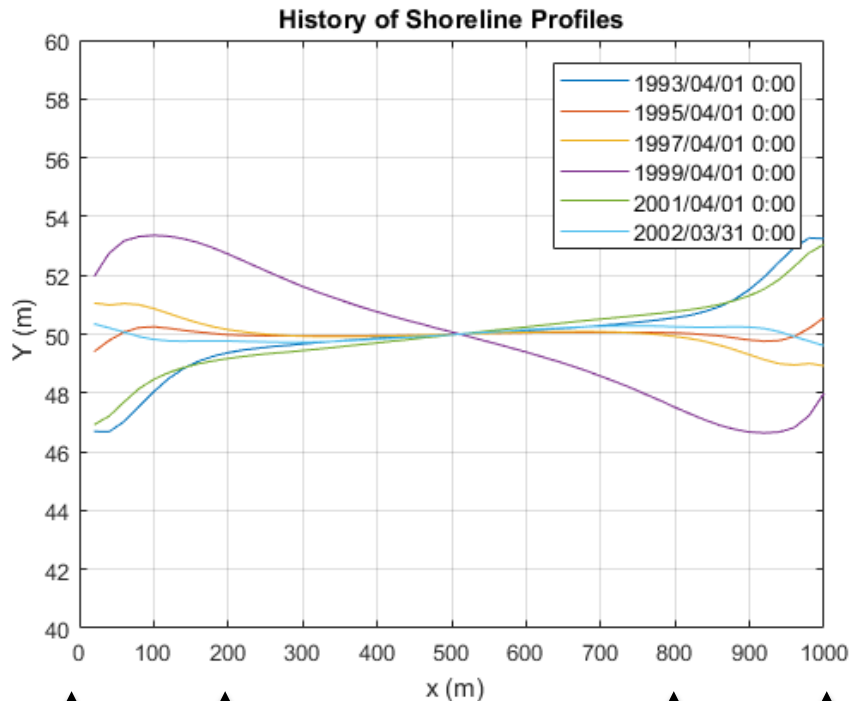


Case 1: Mean Angle  $\mu = 0$  deg,  $\sigma^2 = 10$  deg

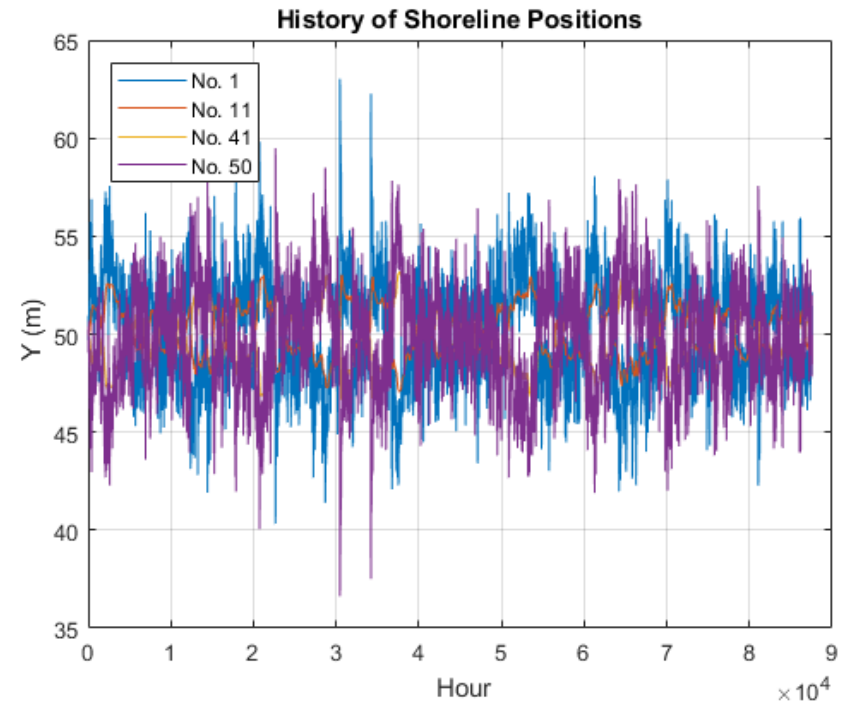


**ERDC**

# Shoreline Profiles and Changes (A 10-Year-long Simulation)



No.1      11      41      50

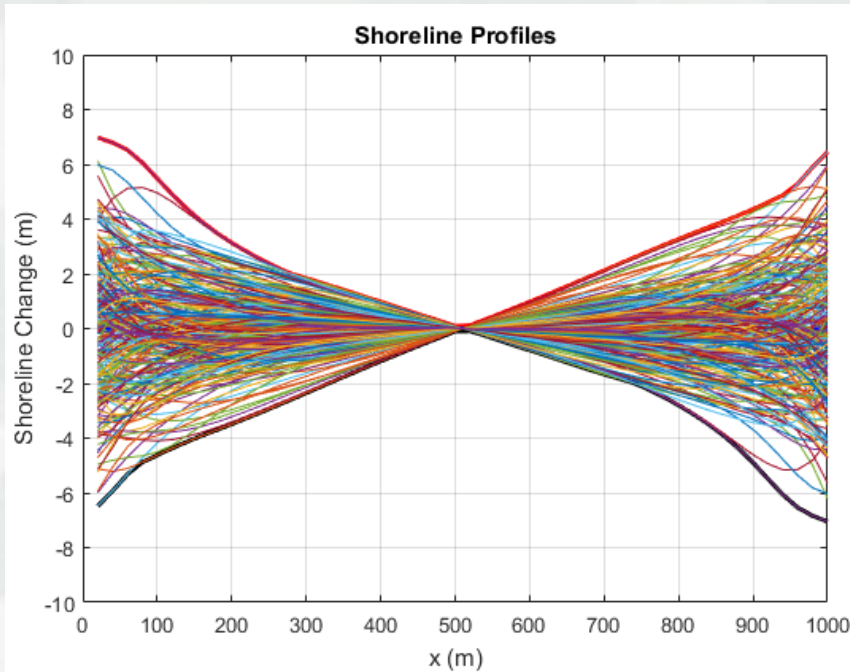


BUILDING STRONG®

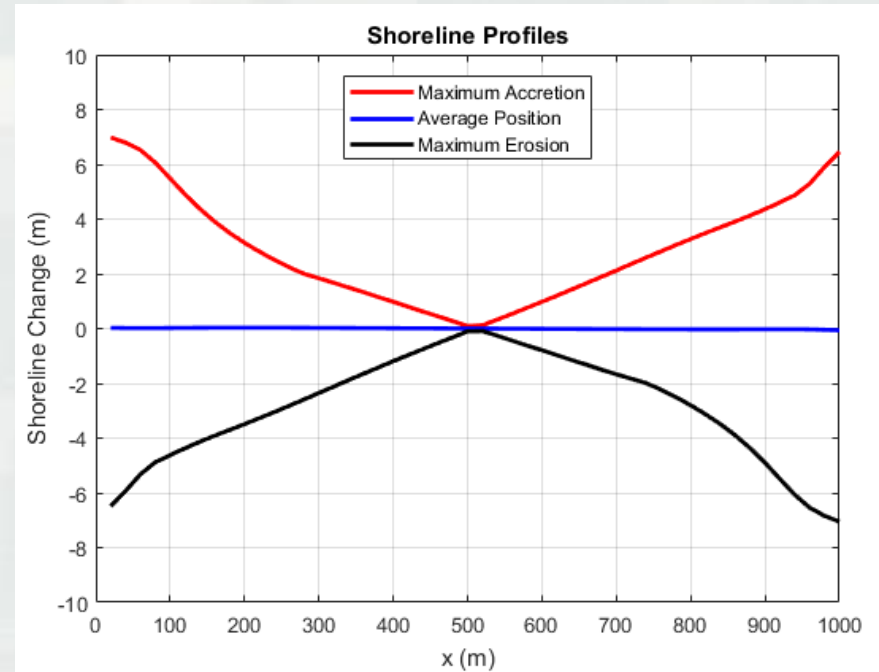
**ERDC**

*Innovative solutions for a safer, better world*

# Shoreline Profiles After 10 Years



All 256 shorelines



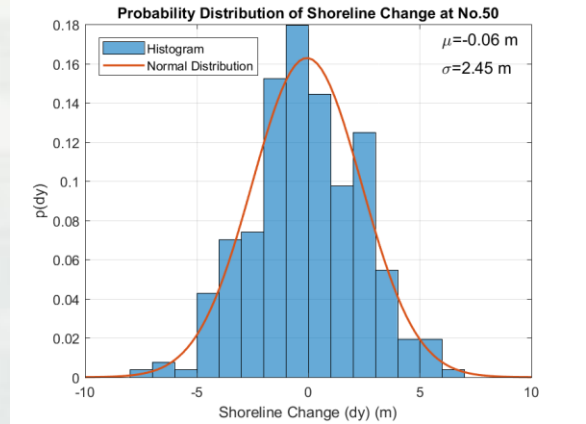
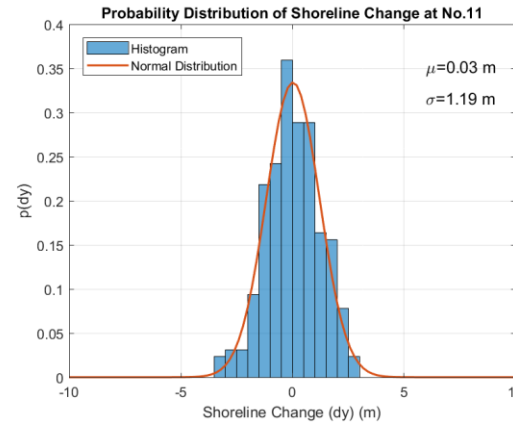
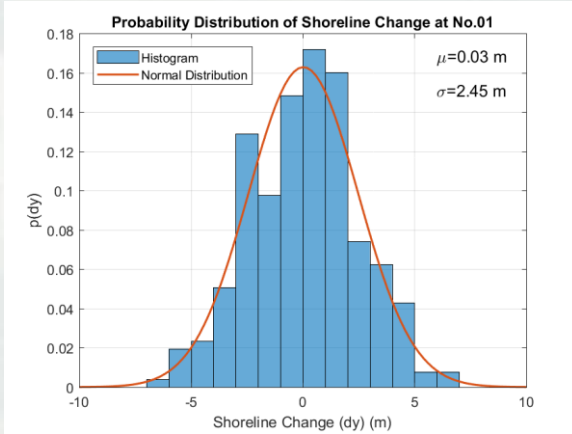
Max., average, and min positions





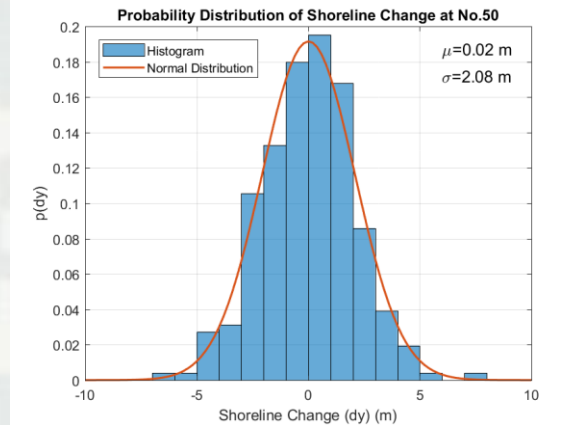
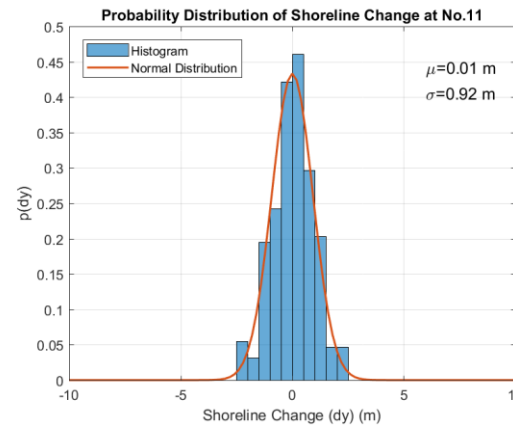
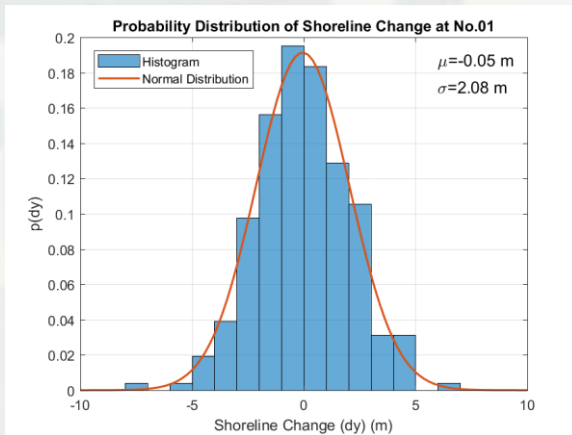
# Probability Distribution of Shoreline Changes

## (a) By Broad Wave Spectrum (Rayleigh + Weibull)

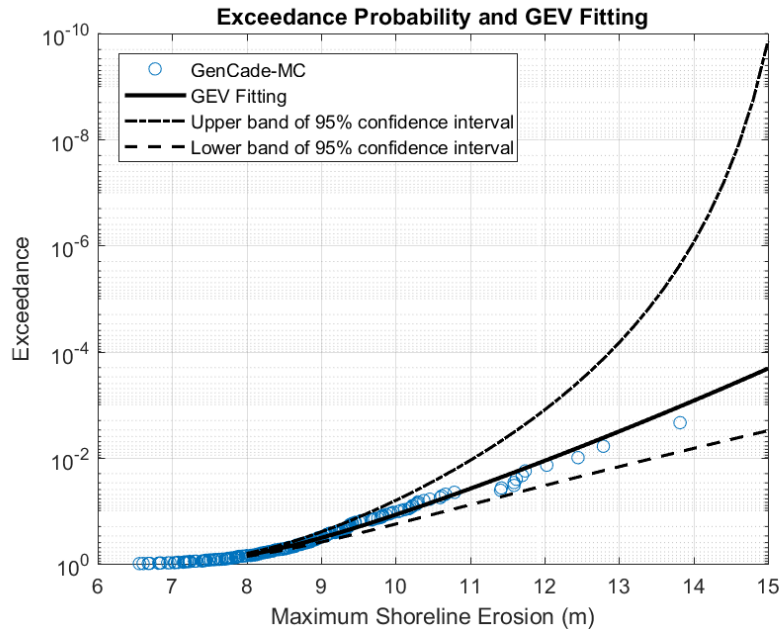


$$\mu_{\text{Broad}} \approx \mu_{\text{Narrow}}; \sigma_{\text{Broad}} > \sigma_{\text{Narrow}}$$

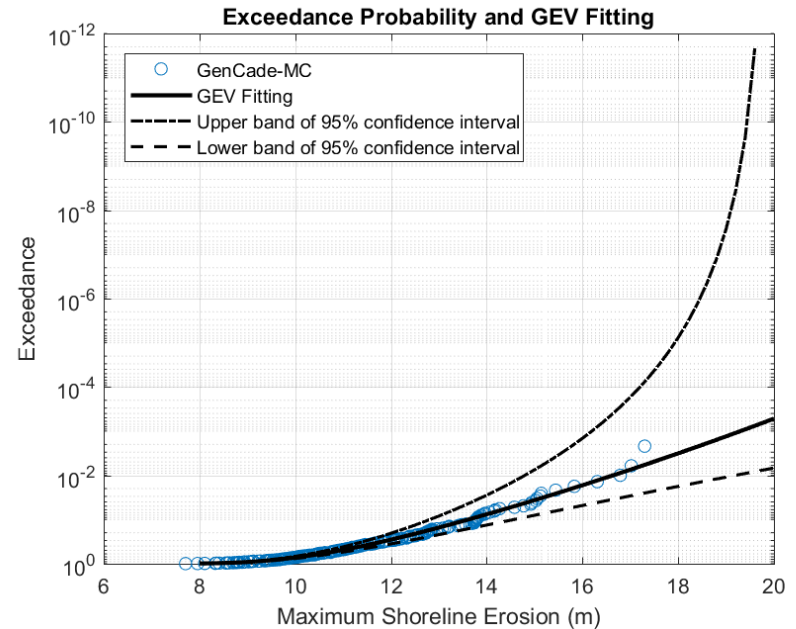
## (b) By Narrow Wave Spectrum (Rayleigh only)



# Prediction of Maximum Shoreline Erosion (Landward-most) ( $\alpha_{\text{mean}}=0.0^\circ$ )



Narrow Wave Spectrum



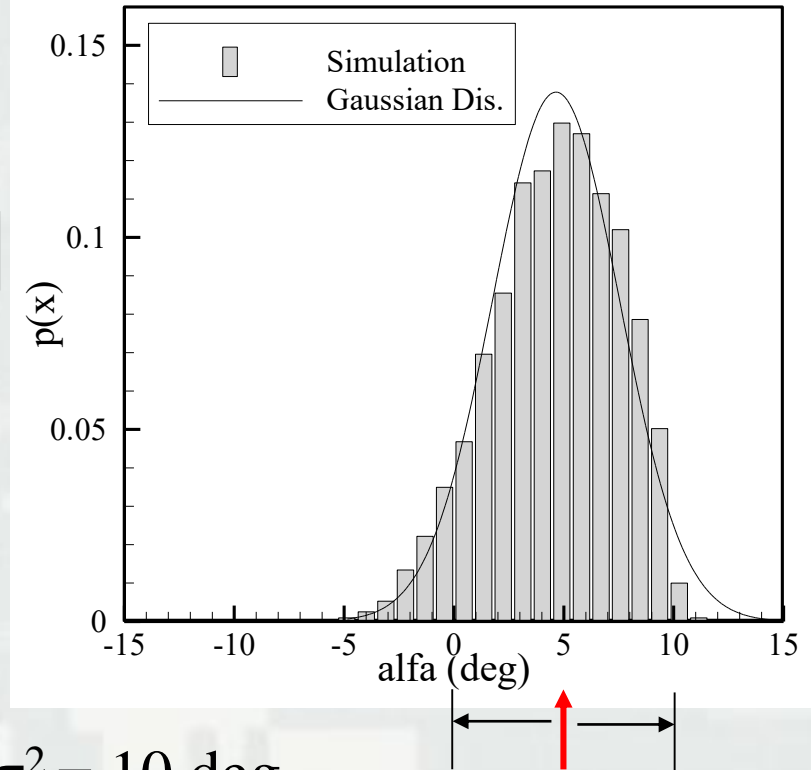
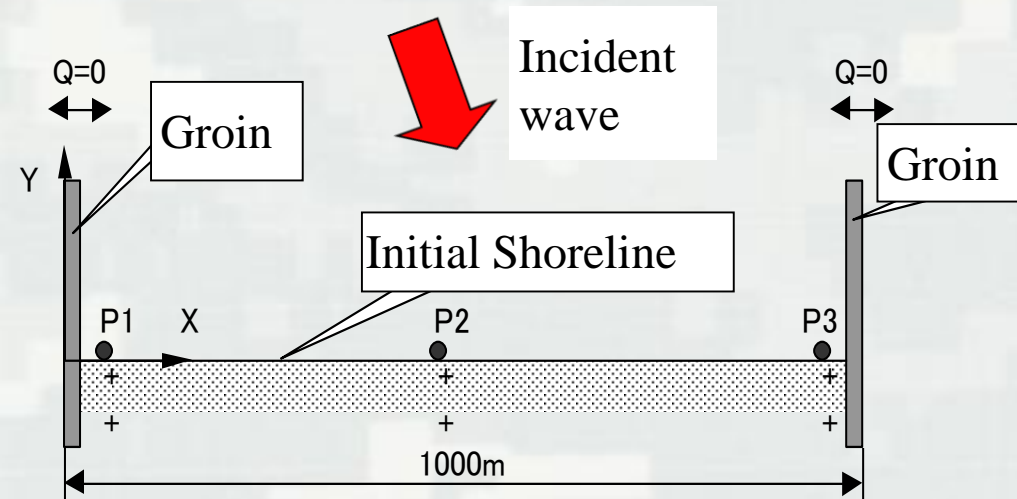
Broad Wave Spectrum

**Table** Estimated maximum shoreline erosion at *Point No. 1* in three return years ( $\alpha_{\text{mean}}=0.0^\circ$ )

	Return Period (Year)		
	10	20	30
Broad Spectrum	18.31 m	19.30 m	19.88 m
Narrow Spectrum	13.12 m	13.74 m	14.11 m



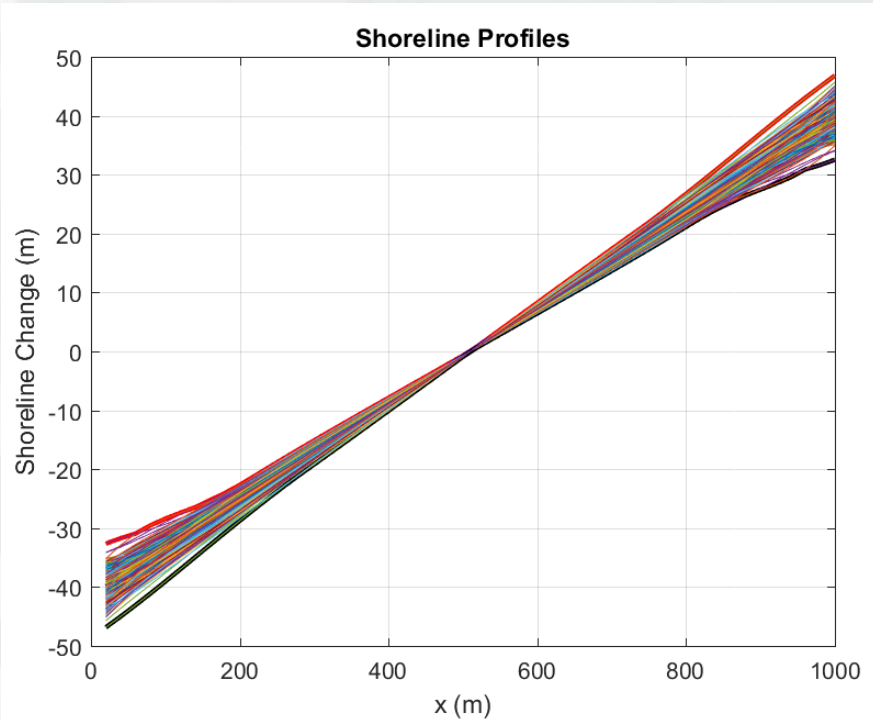
# Probability Distribution of Wave Direction in Case 2 (Oblique Wave Direction)



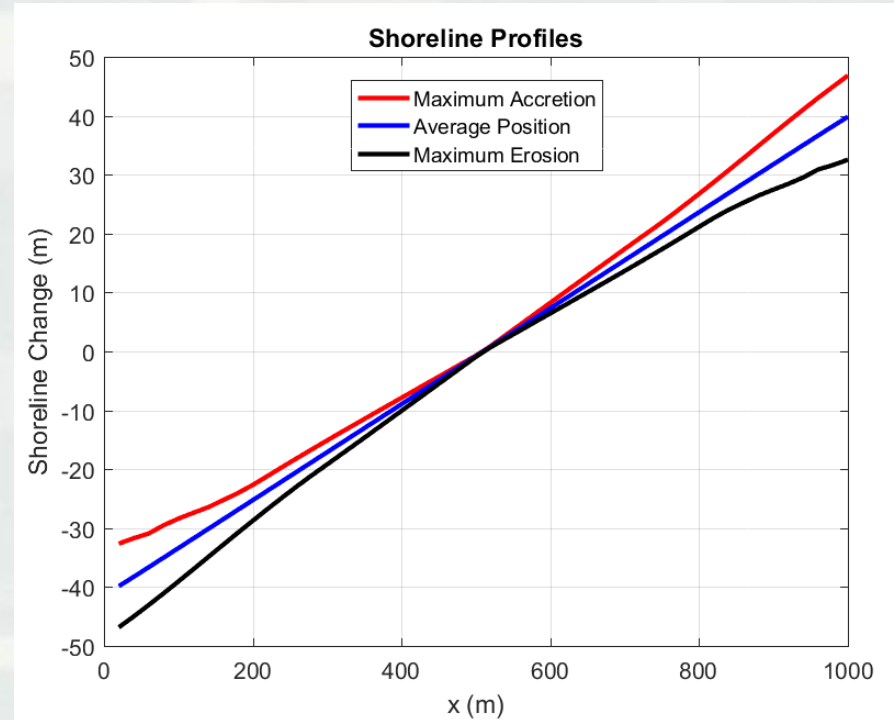
Case 2: Mean Angle  $\mu = 5$  deg,  $\sigma^2 = 10$  deg  
Dominant waves in summer in the study coast



# Shoreline Change Profiles After 10 Years



For all 256 shoreline changes



Max., average, and min positions



**ERDC**

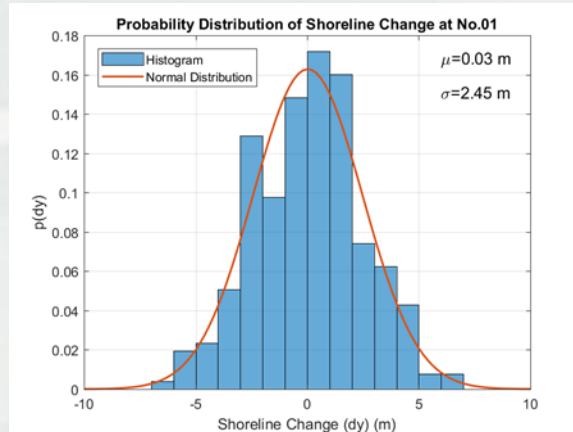
BUILDING STRONG®

*Innovative solutions for a safer, better world*

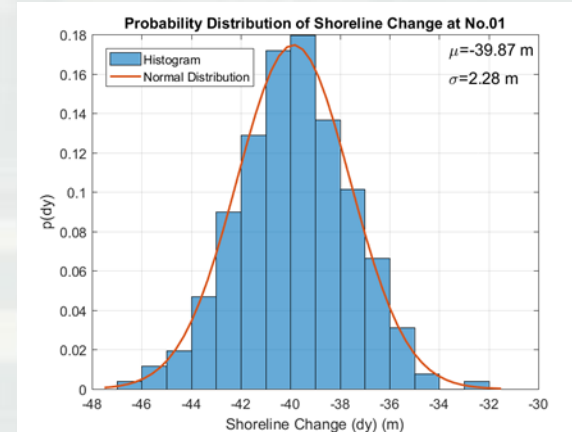


# Statistical Results of 10-year Shoreline Changes at $P_1$

CASE	$H_{mean}$	$\alpha_{mean}$	Statistical Properties at Point No.1 ( $P_1$ )		
			$\mu_p$	$\sigma_p$	$y_{max}(t=10\text{year})$
1	1.19m	0.0	0.03m	2.45m	6.48m
2	1.19m	5.0	-39.87m	2.28m	46.86m



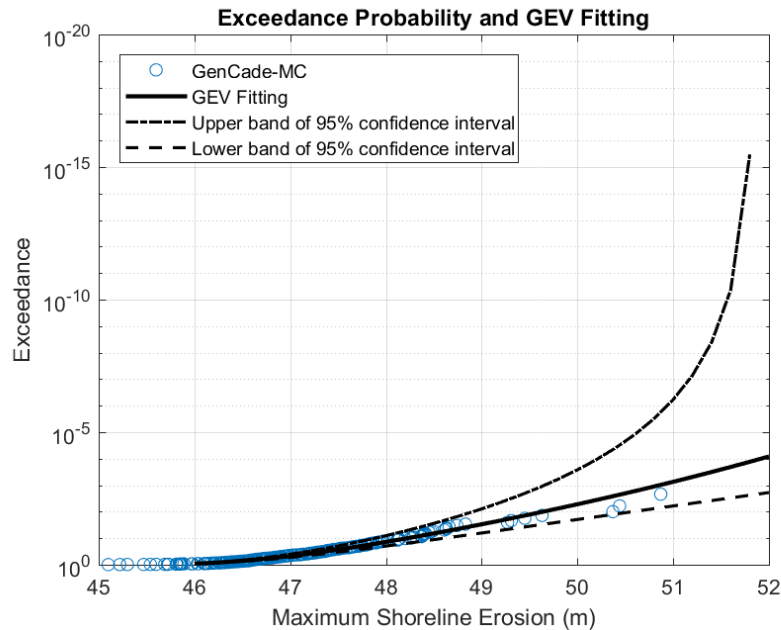
Case 1



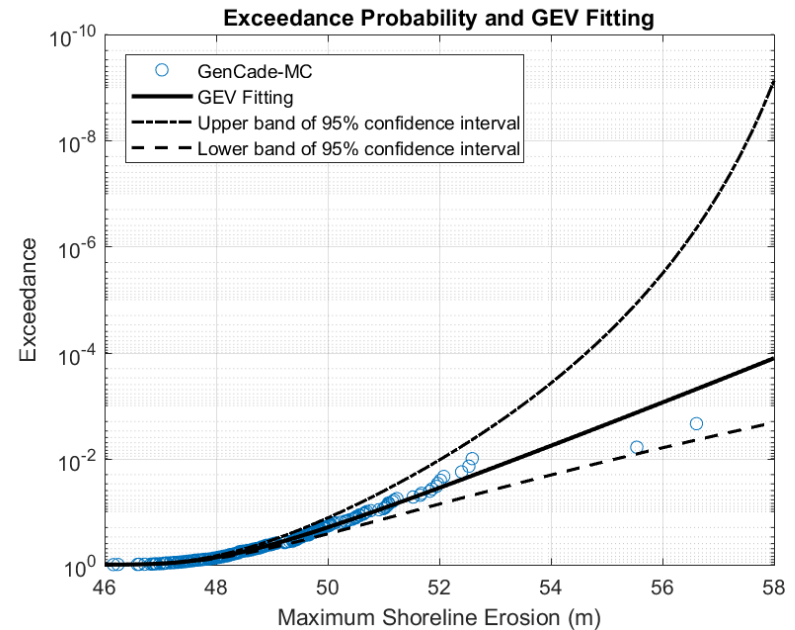
Case 2



# Prediction of Maximum Shoreline Erosion (Landward-most) ( $\alpha_{\text{mean}}=5.0^\circ$ )



Narrow Wave Spectrum



Broad Wave Spectrum

Estimated maximum shoreline erosion at *Point No. 1* in three return years ( $\alpha_{\text{mean}}=5.0^\circ$ )

	Return Period (Year)		
	10	20	30
Broad Spectrum	54.54 m	55.32 m	55.78 m
Narrow Spectrum	50.49 m	50.97 m	51.25 m



# Model Validation: Modeling of Shoreline Change in Duck, NC



Computational Period: 6 years

10/23/1999 0:00 - 10/23/2005 0:00

Time step = 3 minutes

Grain size = 0.20 mm

Berm height = 1.0 m

Closure depth = 7.0

Smooth parameter = 1 (no smoothing)

Boundary conditions: Pinned

Grid size = 20 m

Sea Level Rise rate: 4.55mm/year

Subsidence : 0.0 (N/A)

$K1 = 0.40$ ;  $K2 = 0.25$

Permeability of Pier = 0.6 (no diffracting):

Scaling parameter  $\alpha_D = 0.182$

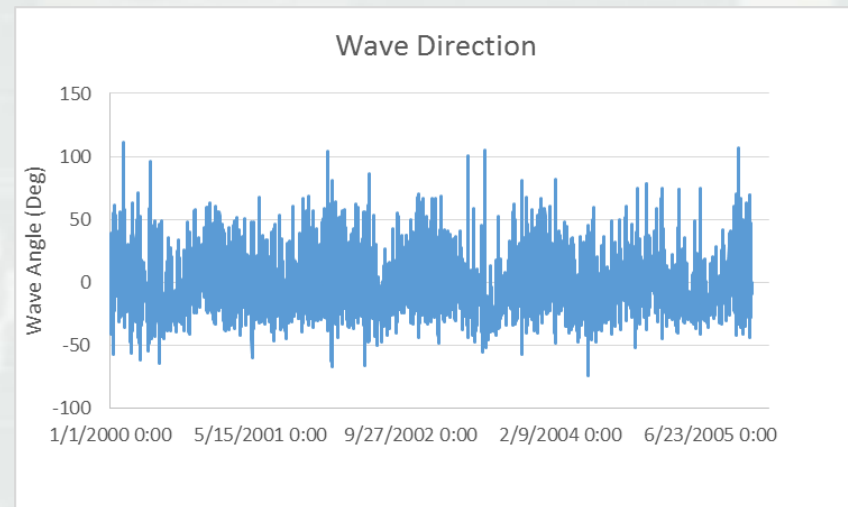
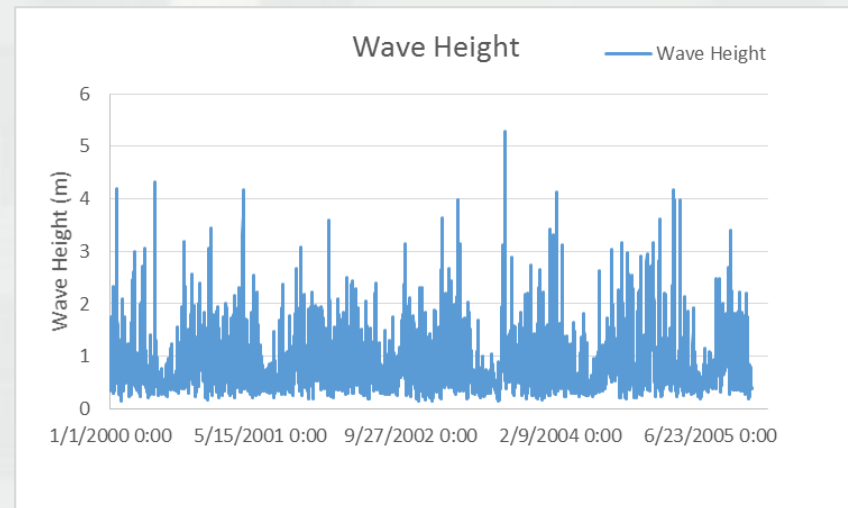
$C_w$ ,  $C_C$ ,  $\epsilon_B$ ,  $\epsilon_S$  by Fernández-Mora et al. (2015)



*Innovative solutions for a safer, better world*

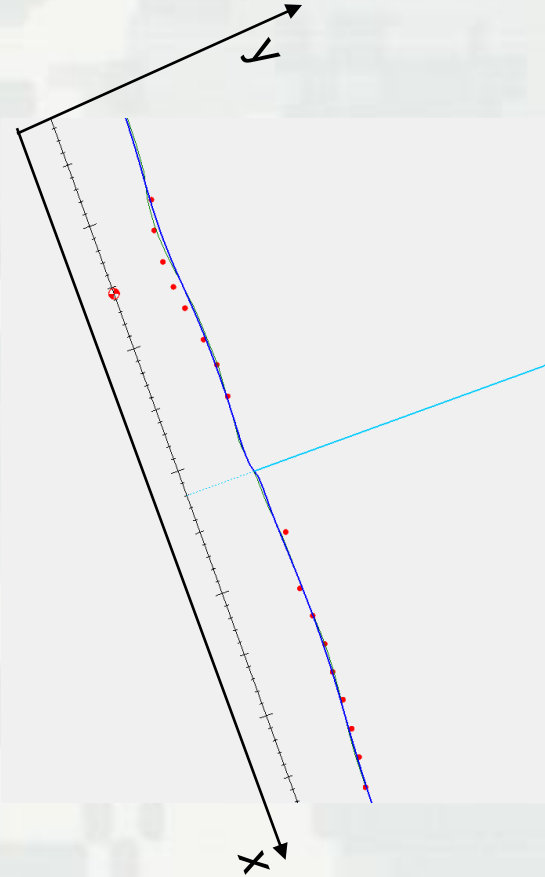
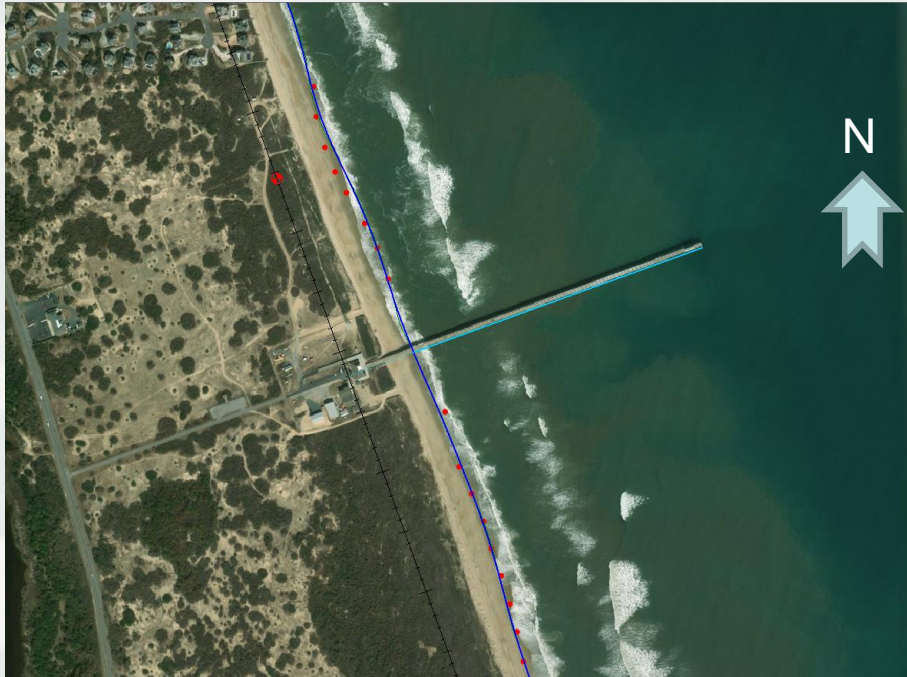
# Wave Data (1/1/2000 – 1/1/2006)

	H (m)	T (s)	alpha (deg)
Average	0.82	9.18	-5.06
Min	0.14	3.09	-74.62
Max	5.28	18.96	111.32
$\sigma$	0.53	2.68	18.52





# Model Validation: Shoreline on 11/1/1999 (after 1 week)

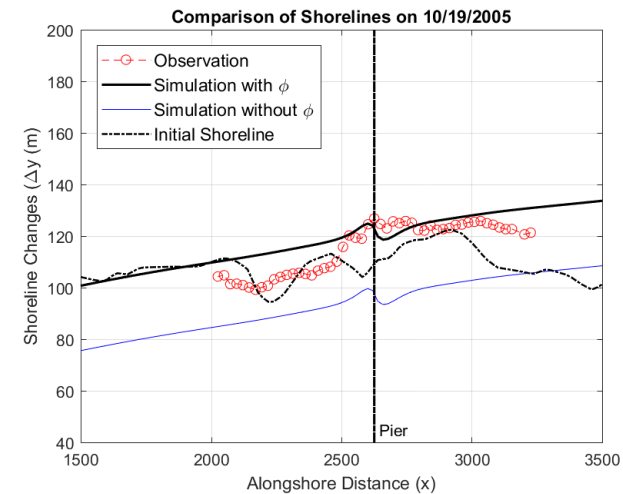
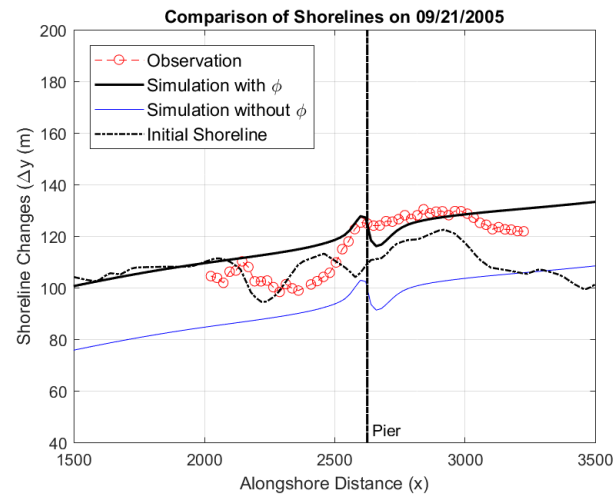
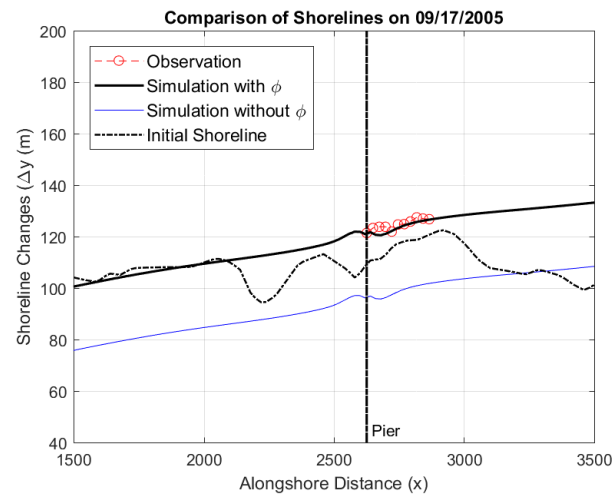
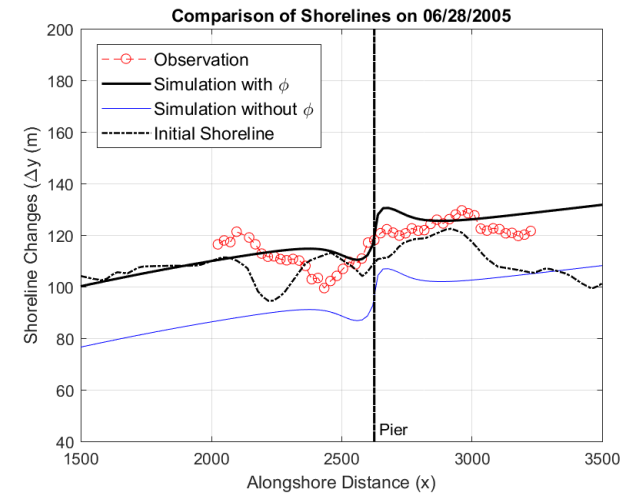
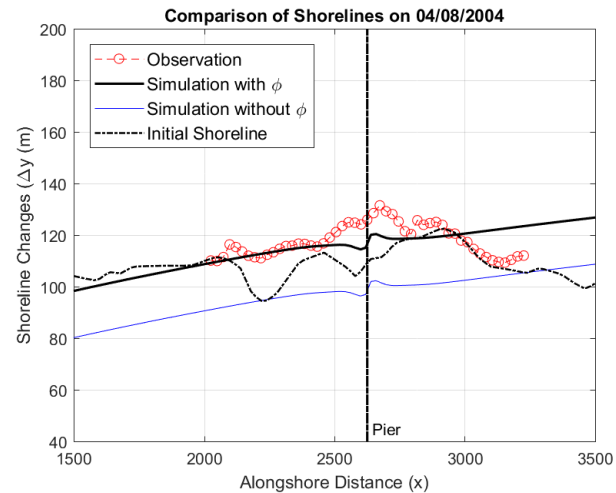
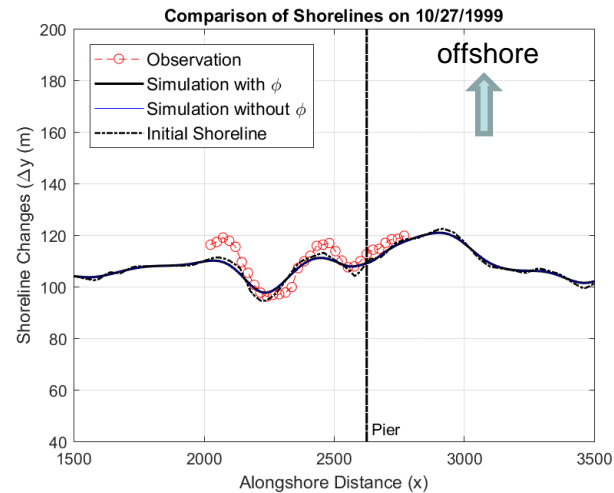


**ERDC**

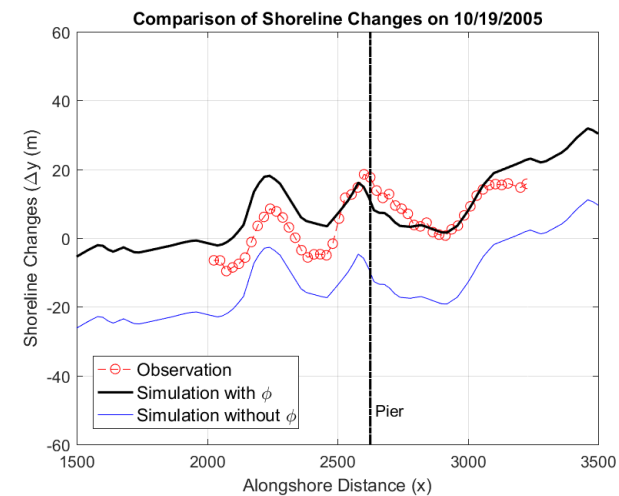
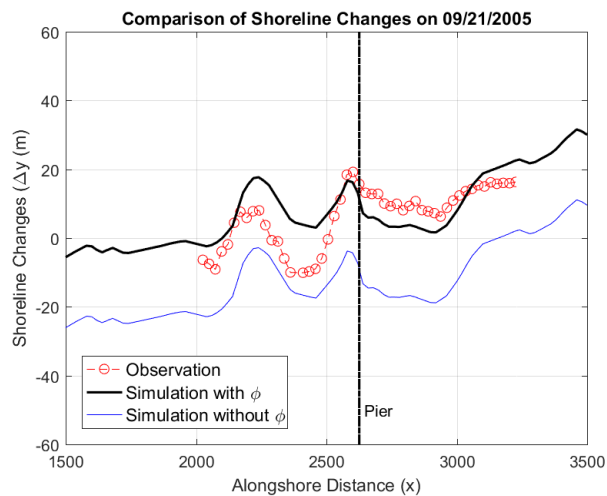
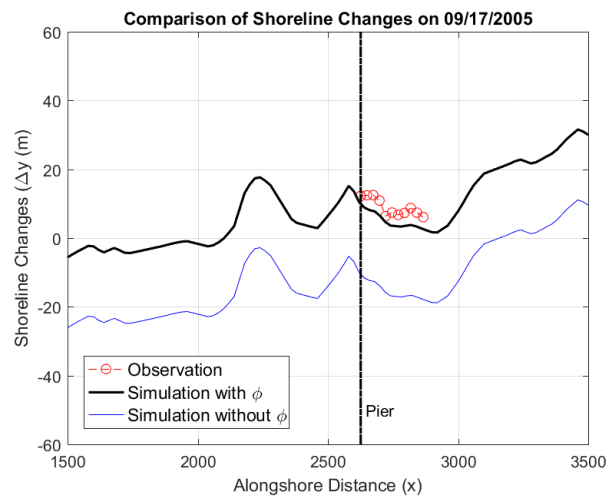
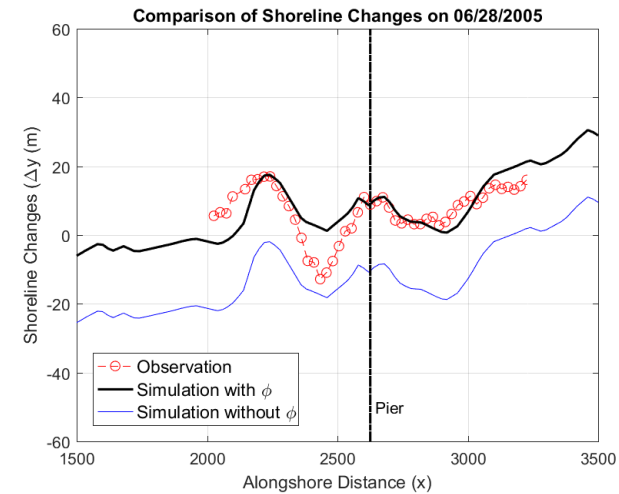
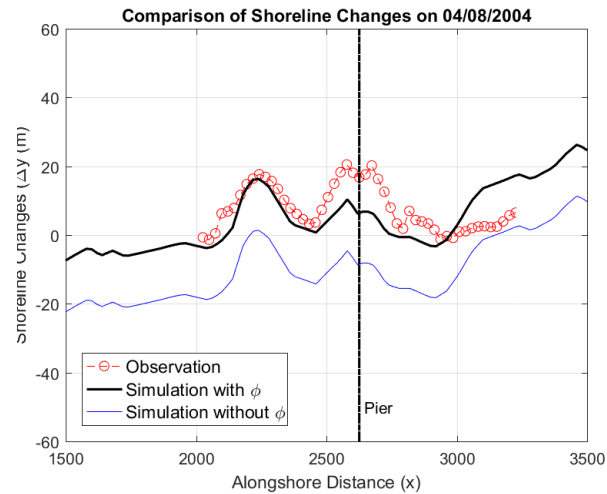
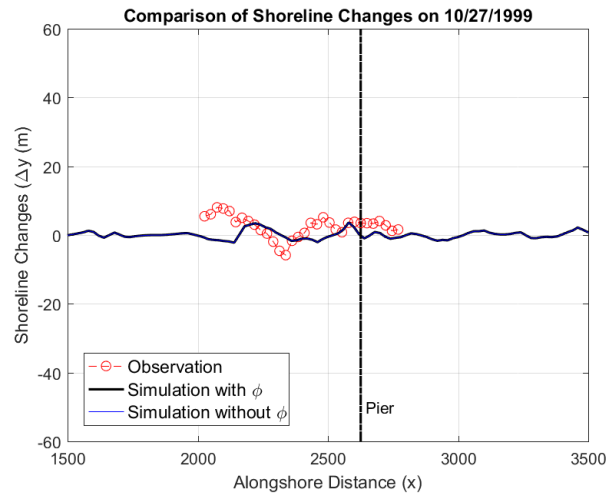
BUILDING STRONG®

*Innovative solutions for a safer, better world*

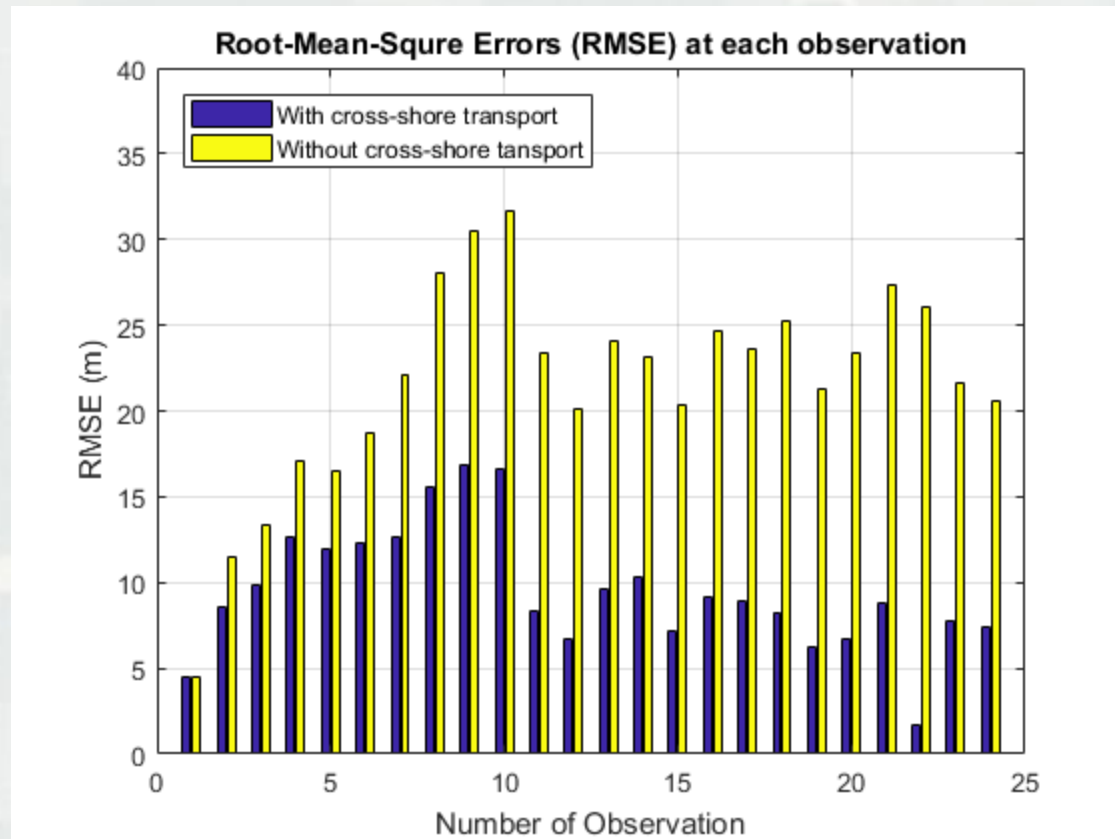
# Model Validation: (With and Without Cross-Shore Transport)



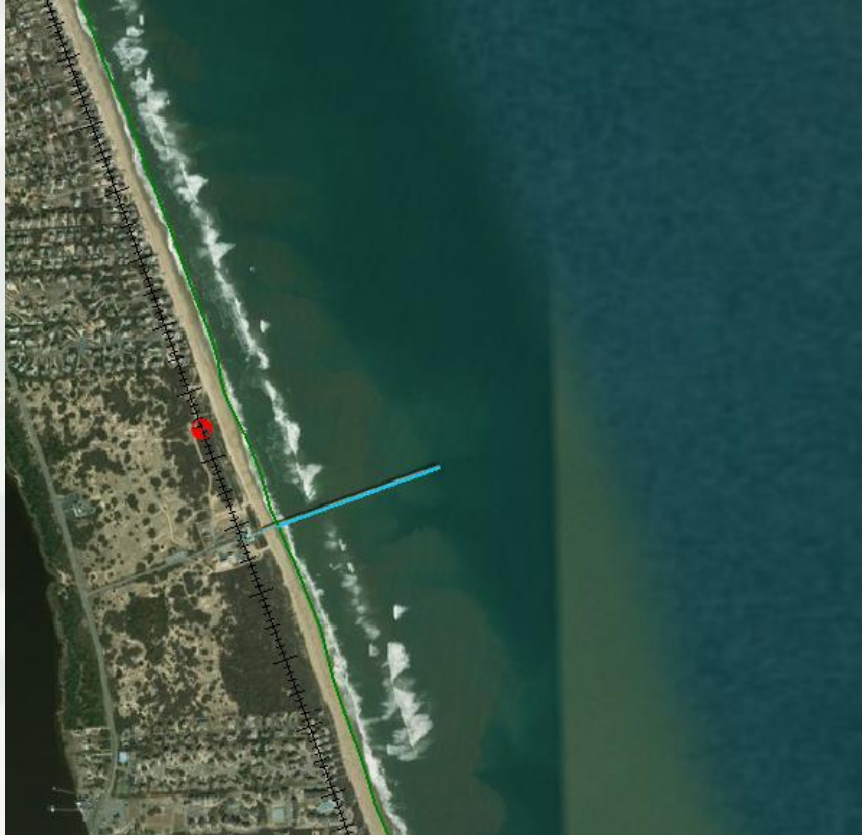
# Model Validation: (With and Without Cross-Shore Transport)



# Model Skill Assessment: Root-Mean-Square Errors at Observation Times (1999-2005)



# Monte Carlo Simulation of Shoreline Change in Duck, NC



Number of Monte Carlo = **128**

## **Wave Conditions:**

**Wave Height: Rayleigh+Weibull**

**Direction: Gaussian**

**Period: PM Spectrum**

**Truncated Wave Height: 2.0 m**

Computational Period: 6 years

10/23/1999 0:00 - 10/23/2005 0:00

Time step = 3 minutes

K1 = 0.40; K2 = 0.25

Grain size = 0.20 mm

Berm height = 1.0 m

Closure depth = 7.0

Sea Level Rise Rate = 4.55 mm/year

Smooth parameter = 1 (no smoothing)

Boundary conditions: Pibned

Grid size = 20 m

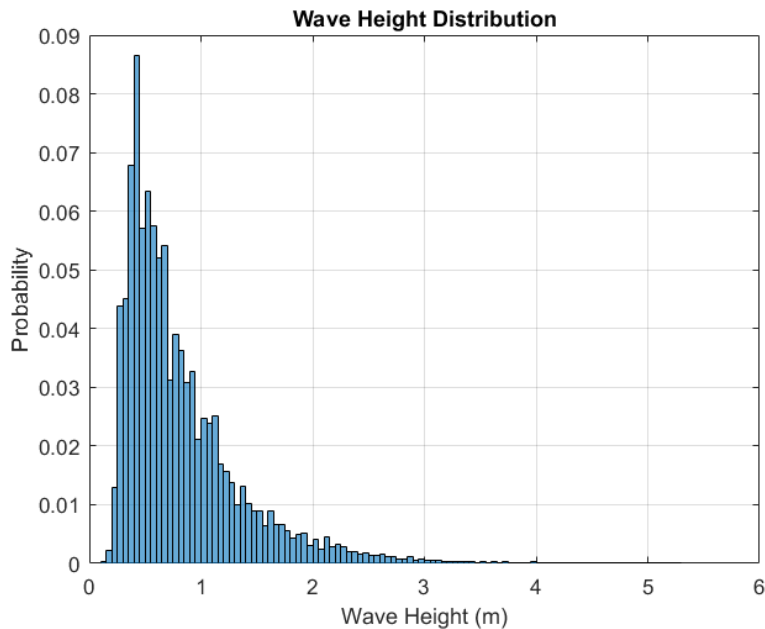
Permeability of Pier = 0.6 (no diffracting)

Scaling parameter of cross-shore transport: 0.182

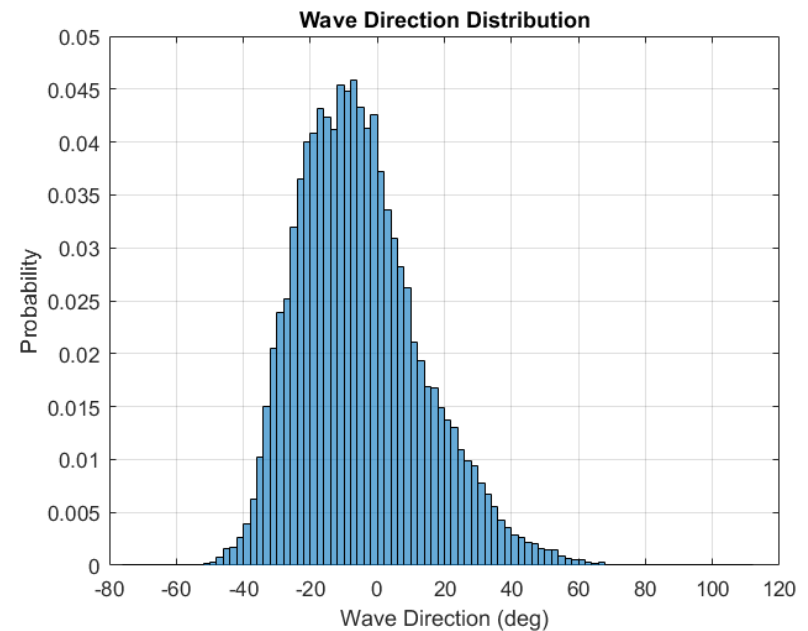


**ERDC**

# Monte Carlo Simulation for Shoreline Change in Duck, NC (1999-2005)



Mean  $H_s = 0.82$  m



Mean wave angle =  $-5.06^\circ$

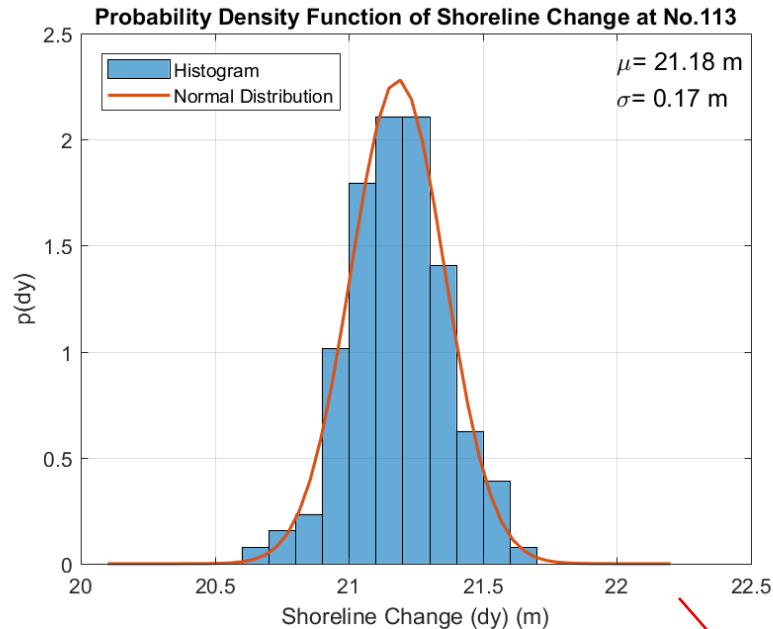


**ERDC**

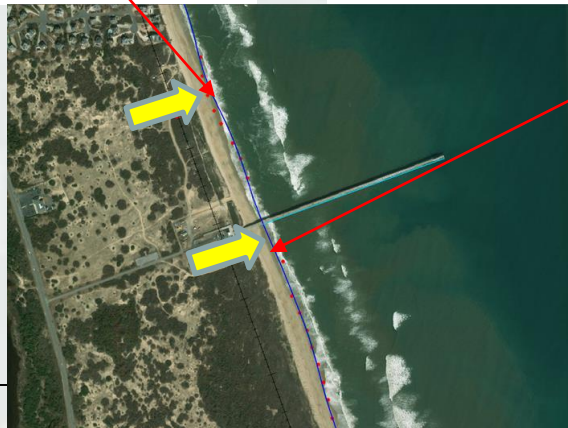
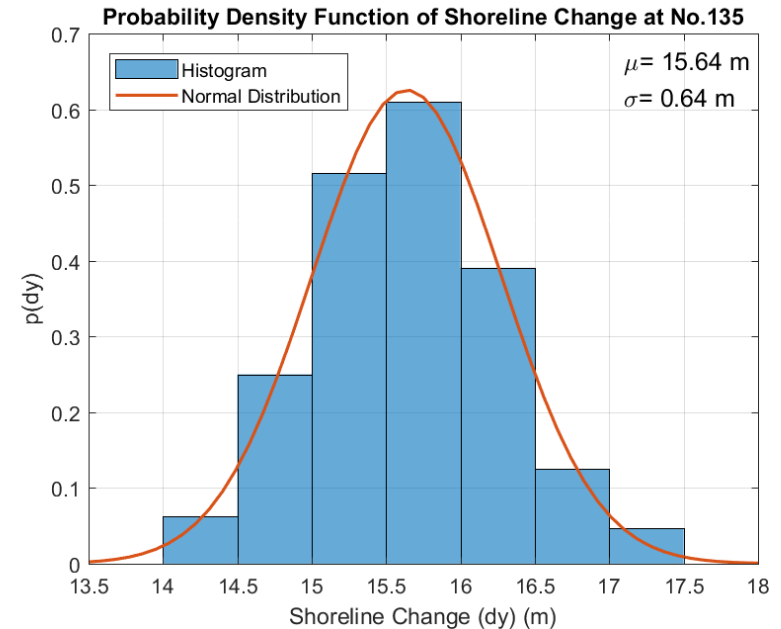


# Probability Density Functions: 6-Years Shoreline Change

At 400-m north of the Pier



At 40-m south of the Pier

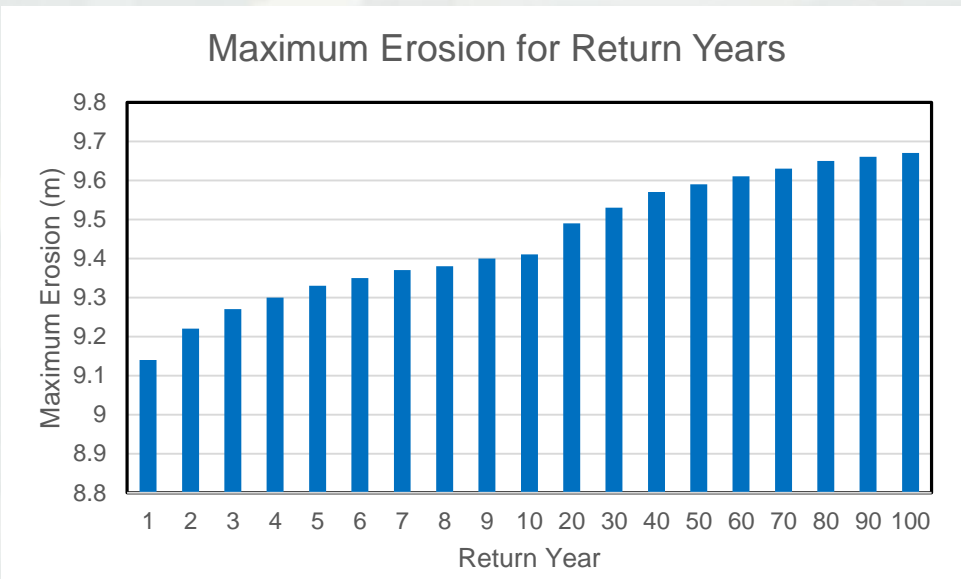
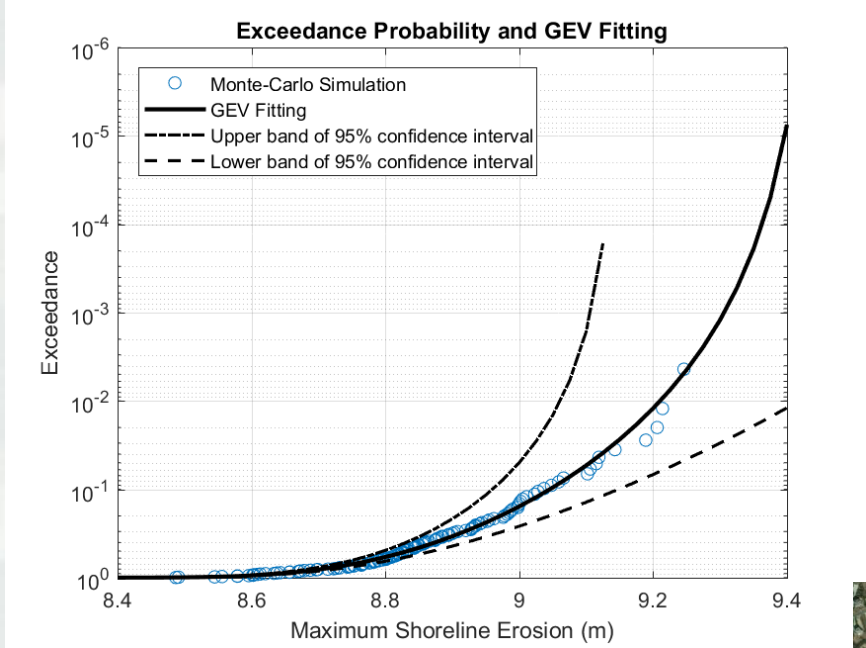


BUILDING STRONG®

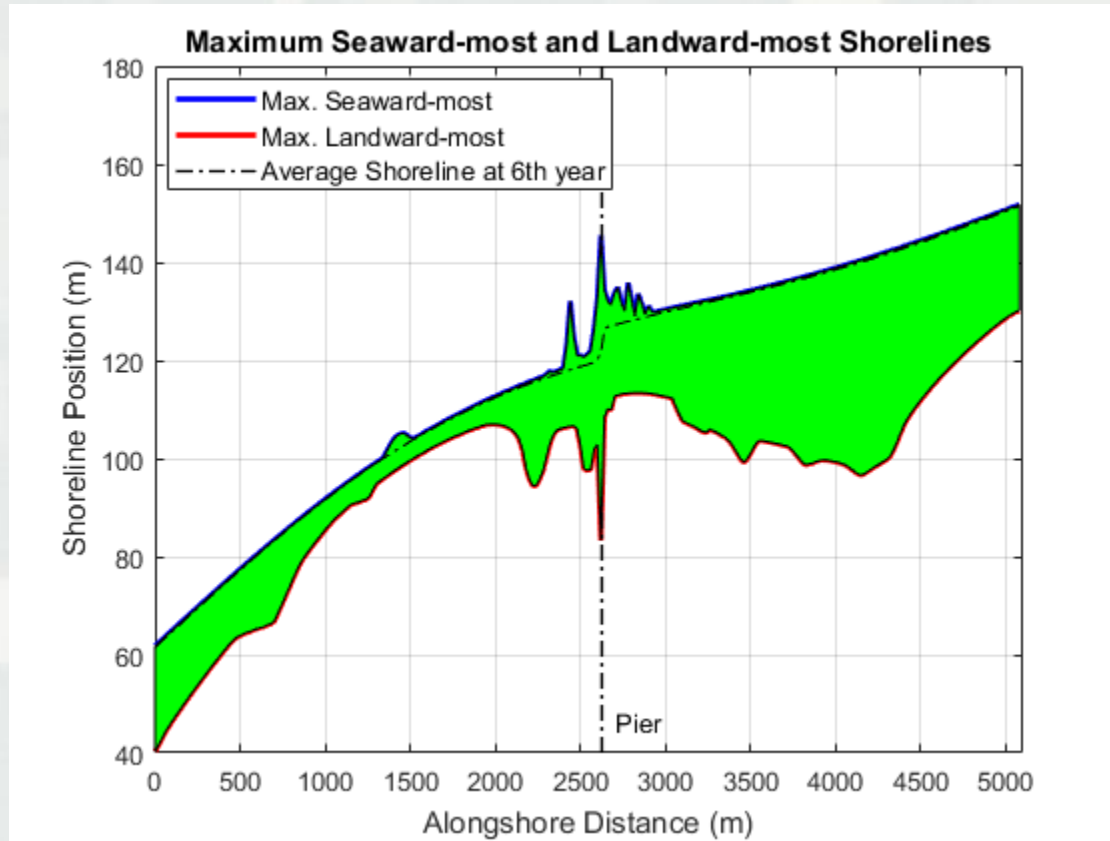
ERDC

innovative solutions for a safer, better world

# Estimation of Maximum Erosion at 300-m South of Pier



# Maximum Seaward-most and Landward-most Shoreline Positions



The filled area is a spatial range of shoreline variations (from maximum landward-most position to maximum seaward-most position) during the simulation period of 6 years.



# Conclusions

- This presentation presents a stochastic approach to simulate probabilistic shoreline change by using Monte Carlo numerical experiments.
- A combined probability density function is proposed to combine large wave and small wave heights in limited simulation duration.
- The probabilities of the maximum shoreline erosion on a hypothetical coast were analyzed by using the extreme probability distribution model (Weibull function). The risks of maximum shoreline erosion at different return periods (years) are quantified.
- Cross-shore sediment transport is important in simulating shoreline evolution in Duck, NC. With cross-shore transport, validation of GenCade is successful.
- Preliminary results of probabilistic shoreline change at Duck, NC, are reasonable. Estimation of extreme shoreline change provides maximum erosion risk in a return-interval manner.



# Thank you for your attention!

## Questions?

[Ashley.E.Frey@usace.army.mil](mailto:Ashley.E.Frey@usace.army.mil)



**ERDC**