ANALYTICAL SOLUTION OF BEACH PROFILE RESPONSE TO SEA LEVEL RISE – APPLICATION FOR TECTONIC MOVEMENT IN SENDAI, JAPAN

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INTRODUCTION

Kriebel and Dean (1993) developed a simple approach to quantify the beach profile response to a time-varying sea level. It is based on the equilibrium concept implying that if a beach profile is exposed to a constant wave and water level climate it will attain a specific shape (*i.e.*, the equilibrium beach profile; EBP). A change in the forcing conditions will make the profile move towards a new equilibrium state, which will be attained if these conditions prevail sufficiently long. For the case of typical sea level rise (SLR), the change in the forcing conditions is slow enough so that the profile has time to adjust towards the EBP at any given time.

In this study, new analytical solutions are developed based on the convolution method to describe beach-profile response to sea water level change. Detailed, high-quality data sets obtained after the 2011 Tohoku Earthquake and Tsunami, including wave climate, beach profiles surveyed every 6 months (Fig. 1), and bed level change due to tectonic movement (subsidence followed by rather rapid recovery towards the original bed level), were used to validate the model. Bed level change is inversely related to the SLR process and the data provide a unique opportunity to evaluate the EBP concept (*i.e.*, Bruun rule, 1962). The model results are able to reproduce the observed beach retreat as a function of time using the proper SLR forcing.

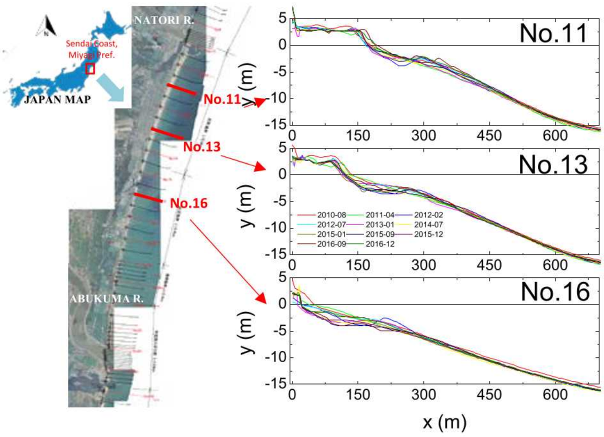

Figure 1 - Location of study area and measured beach profiles

ANALYTICAL MODEL DEVELOPMENT

The theoretical model proposed by Kriebel and Dean (1993) to describe the beach profile response to a time-varying sea level, introduced through a time function $f(t)$, may be written,

$$\frac{dy}{dt} = \alpha(y^{\infty} f(t) - y) \qquad (1)$$

where y is the response of a specific contour, y^{∞} the maximum response of this contour (at equilibrium state), t the time, and α a characteristic rate parameter that may be related to the typical response time scale of the morphological system as $\alpha = 1/T_{sl}$.

Figure 2 shows the observed bed level behavior at Sendai after the earthquake 2011 may be described as an instantaneous increase in the sea level, followed by a sea-level decrease back to pre-earthquake conditions. In terms of the model given by Eq. (1), the time function could be expressed as,

$$f(t) = -\exp(-\beta t) \qquad (2)$$

where β is another characteristic rate parameter that may be related to the typical response time scale of the bed level recovery $\beta = 1/T_m$. The time-varying land subsidence and rising function $g(t)$ is obtained by using the exponential fitting function to the measured land elevation changes after the 2011 Earthquake (Fig. 2). The result suggests that the rate parameter β in Eq. (2) equals to 7.5×10^{-4} (1/day).

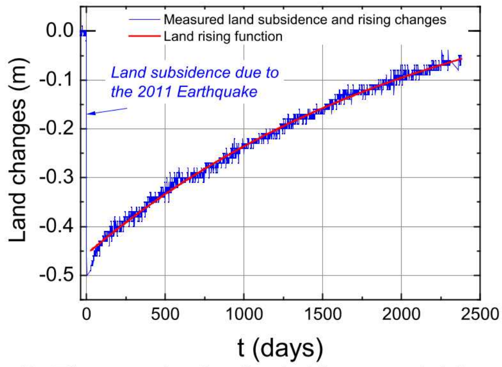

Figure 2 - Time-varying land subsidence and rising

Substituting Eq. (2) into Eq. (1) and solving by using the convolution integral method, we obtained the following non-dimensional analytical solution for shoreline response as;

$$y^* = \frac{y}{y^{\infty}} = \frac{\exp(\frac{-t^*}{\beta^*})[1-exp\{\left(1-\frac{1}{\beta^*}\right)t^*\}]}{(1-\beta^*)} \qquad (3)$$

where $t^* = \beta t$ is time constant and $\beta^* = \beta/\alpha$ is a non-dimensional parameter expressing the ratio between the morphological and bed recovery time scales.

The peak of non-dimensional shoreline response and corresponding time occurrence can be obtained from the 1st order derivative of Eq. (3) equals to zero as follows;

$$y_p^* = \beta^{*\beta^*/(1-\beta^*)} \quad \text{, and} \quad t_p^* = \frac{\beta^* ln\beta^*}{(\beta^*-1)} \qquad (4)$$

Figures 3 and 4 show the general analytical results of non-dimensional shoreline position response and the variation of the peak shoreline response as well as peak time occurrence (Eq. (4)) with different values of t^* and β^*, respectively. When β^*=0, it corresponds to the shoreline change occurs immediately in response to land rising function as in Fig. 2. After that the shoreline change over time is in exactly the same way as the external forcing function. As β^* increases, the maximum of dimensionless shoreline response, y_p^*, decreases whereas the peak occurrence time, t_p^*, increases (Fig. 4).

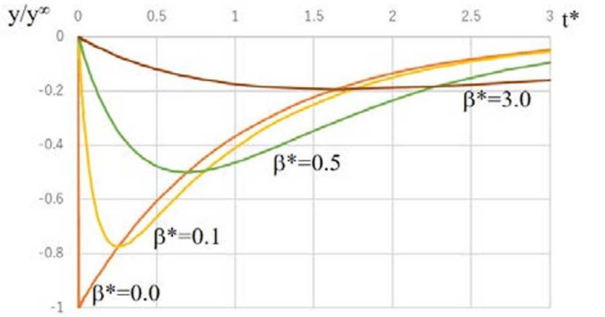

Figure 3 - Analytical results of non-dimensional shoreline position response

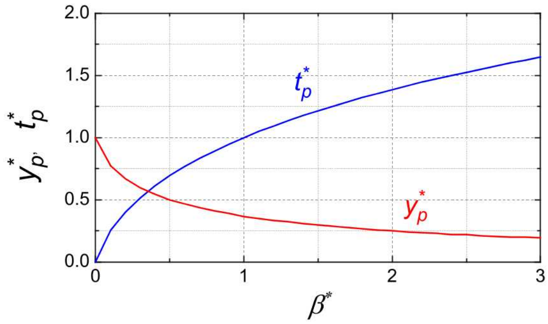

Figure 4 - Variation of peak shoreline change and time occurrence versus β^*

APPLICATION FOR SENDAI CASE

The magnitude of the erosion or deposition shoreline responses can be determined by Eq. (3) if the equilibrium or maximum potential shoreline response, y^{∞}, and typical response time scale rate of the morphological system, α, that governs the exponential rate at which the profile responds toward the new equilibrium stage are known. In this study, the detailed measurement data sets along Sendai Coast were used to calibrate the values of y^{∞} and α.

The selected measured beach profiles data used in this study are located in the center of Sendai Coast where the influence of longshore drift can be neglected. Based on the Bruun's assumption, the equilibrium beach profile form for open-coast beach profiles is expressed by a power-law curve as $h = ax^{2/3}$, where x is offshore distance from the shoreline, h is water depth at a distance x, and exponent value of $2/3$ was found when fitted to his field data.

Figure 5 shows the log-log plot of relationship of h and x values for all selected beach profiles. A good agreement of regression slope compared to Bruun's equilibrium profile indicates that the selected beach profiles are somehow in equilibrium state before the land subsidence event occurrence.

Figure 6 is the response of the zero contour $y(t)$ in Eq. (3) is plotted and compared to the measured shoreline positions by changing the parameter, y^{∞} and α. The best fit is obtained when y^{∞} = 20m and α = 1.47x10^{-3} (1/day). The maximum potential shoreline response y^{∞}, can also estimate by using Bruun Rule Formula such as $y^{\infty} = S * \frac{L}{(B+D_c)}$, where for the Sendai Coast the $S = 0.5m$ is the SLR height, $D_c = 10m$ is depth of closure, $L = 600m$ is a distance from shoreline to closure depth location and $B = 4m$ is berm height. Resulting y^{∞} = 21.4m which is a good agreement with the above calibrated value. The α = 1.47x10^{-3} (1/day) as equivalently to the response time constant T_{sl} = 680 days. The optimum ratio between the morphological and bed recovery time scales $\beta^* = 0.51$.

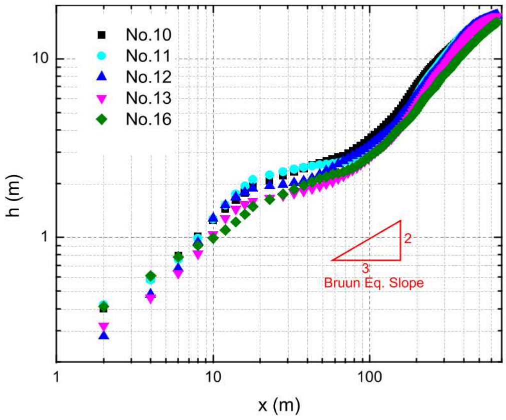

Figure 5 - Log-log plot of h - x relationship and compare to Bruun Rule's equilibrium profile for all selected measured beach profiles

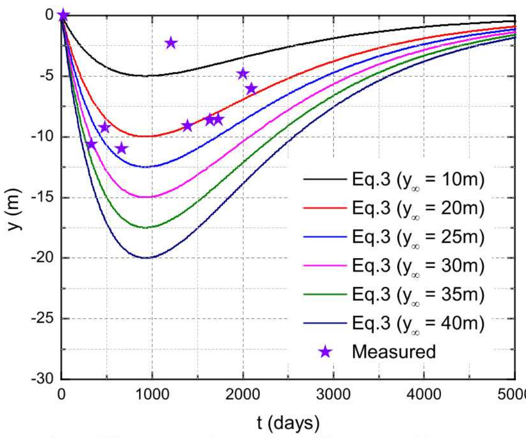

Figure 6 - Time-varying shoreline position response compared to measured shoreline data

Kriebel & Dean (1993) applied similar to the Eq. (1) to the shoreline responses over severe storm wave conditions and the time constant was obtained from several hours to several tens of hours. However, the time constant of beach response to water level rise case in Sendai was much larger.

CONCLUSIONS

New analytical solutions based on the convolution method for beach-profile response to sea water level change were developed and validated for data from the Sendai Coast obtained after the 2011 Earthquake.

The obtained time constant of the beach deformation under the sea water level rise was much larger than beach response by the storm condition.

REFERENCES

Bruun, (1962): Sea level rise as a cause of shore erosion, J. of Waterways and Harbors Division, 88 (1), pp. 117-130

Kriebel and Dean (1993): Convolution method for time-dependent beach profile response, J. of Waterway, Port, Coastal, and Ocean Engineering, 119 (2), pp. 204-226.