A REVISIT ON THE LEADING WAVEFORM DUE TO A TRANSIENT DISTURBANCE

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INTRODUCTION

A tsunami is a series of transient waves often triggered by a submarine earthquake over a sufficiently large area. The horizontal scale of the wave motion of tsunamis is considerably large, which can be a few hundreds of kilometers in the deep ocean. While a tsunami travels across an open ocean as a packet of very long waves, we have learned from the past tsunami events that the knowledge of the first few leading waves is of particular importance since it provides necessary information for tsunami scientists to predict the arrival time and runup height of the tsunami waves. Based on the theoretical arguments, experimental studies, or field observations, a number of simple wave models, namely sinusoidal waves, Gaussian profiles, solitary waves, N-waves among others, have been proposed in literature as the model waveforms for the leading tsunami waves. Undoubtedly, each model has achieved considerable success in representing leading waves when comparing to certain sets of field or laboratory data. However, a universal model for describing the waveform of leading tsunami waves is seemly still unavailable. In this paper, we will study theoretically the evolution of leading waves due to a sudden disturbance. We will first revisit the asymptotic solutions of linear dispersive equations for the leading wave train caused by a surface disturbance in a constant depth (Kajiura 1963). The applicability of these theoretical predictions will then be examined. Finally, we attempt to explore the connection between the profiles of the asymptotic solutions and the familiar sinusoidal waves and solitary waves.

THEORETICAL ANAYSIS

To facilitate the understanding of the waveform of leading tsunami waves, we consider a simple two-dimensional model, where the waves are evolved from a symmetrical surface impulse in a constant water depth. As we are interested in the formation of the leading waves, it is generally recognized that the linear KdV equation is a reasonable model (Mei, Stiassnie & Yue 2005). In fact, the problem has been studied extensively. For instance, Kajura (1963) and Whitham (1974) reported two notable analytical results. If we consider a rectangular hump of height A initially spanning the region $-B < x < 0$ in a constant depth h, the surface profile ζ of the resulting positive-going wave is (Whitham 1974)

$$\zeta(x,t) = \frac{A}{2}\int_{Z}^{Z+\delta} \mathrm{Ai}(s)ds \qquad (1)$$

where $\delta = (B/h)(2/\tau)^{1/3}$, $Z = (x/h - \tau)(2/\tau)^{1/3}$, $\tau = t/\sqrt{h/g}$, g the gravitational acceleration, t the dimensional time, and $\mathrm{Ai}(\cdot)$ the Airy function. At a large time or distance, i.e. $\tau \geq (16B/h)^3$, the integral in (1) can be approximated to yield an asymptotic solution, $\zeta = \frac{AB}{2h}(2/\tau)^{1/3}\mathrm{Ai}(Z)$. Hence, the leading waves is a wave train described by the Airy function. However, Madsen, Fuhrman & Schäffer (2008) argued that while the solution is mathematically correct, it is unrealistic to apply it to geophysical tsunamis in the ocean. Using a set of field-scale data, they have shown that the direct application of the asymptotic solution is only valid after a traveling time of a few hundred days. This suggests that the applicability of the solution is very limited.

However, we realize that the solution can still be useful for the field-scale tsunamis if an additional simple decomposition is applied beforehand. Moreover, the Airy function can be further approximated in terms of a series of cosine waves or $\mathrm{sech}^2(\cdot)$ profiles, suggesting that there might be a connection between the leading wave profiles and these canonical wave models. Figure 1 demonstrates the idea using the numerical example illustrated by Madsen, Fuhrman & Schäffer (2008), where the initial surface disturbance is an alternative positive-negative step given by $\zeta(x,0_+) = A_1$, $-B_1 < x < 0$; A_2 for $-(B_1+B_2) < x < -B_1$; 0 elsewhere, with field-scale parameters $A_1 = -1$ m, $A_2 = 3$ m, $B_1 = 150$ km, and $B_2 = 250$ km. The preliminary result looks encouraging. Further analysis is still ongoing.

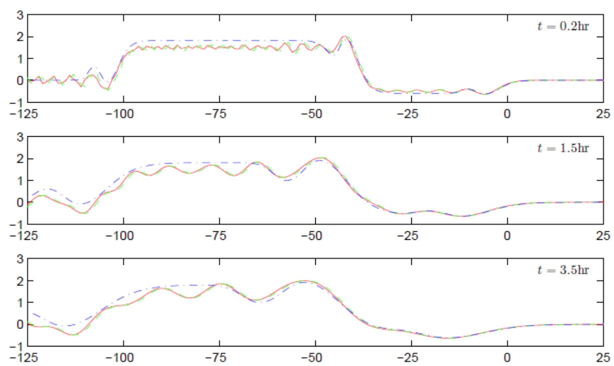

Figure 1: Snapshots of leading waves at select time instants. Vertical axis: dimensionless surface elevation, $\zeta/|A_1|$; horizontal axis: dimensionless distance, $(x - t\sqrt{gh})/h$. Solid: integral solution, (1); Dashed: asymptotic solution with decomposition treatment; Dash-dotted: $\mathrm{sech}^2(\cdot)$-profile approximation.

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