NUMERICAL MODELING OF A TURBULENT BOTTOM BOUNDARY LAYER UNDER SOLITARY WAVES ON A SMOOTH SURFACE

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INTRODUCTION

A number of studies on bottom boundary layers under sinusoidal and cnoidal waves were carried out in the past owing to the role of bottom shear stress on coastal sediment movement. In recent years, the bottom boundary layers under long waves have attracted considerable attention due to the occurrence of huge tsunamis and corresponding sediment movement. In the present study two-equation turbulent models proposed by Menter(1994) have been applied to a bottom boundary layer under solitary waves. A comparison has been made for cross-stream velocity profile and other turbulence properties in x-direction.

THEORETICAL BACKGROUND

The free-stream velocity under solitary wave condition is given as:

$$U = U_c \mathrm{sech}^2(\alpha t) \qquad (1)$$

Where,

$$\alpha = \sqrt{\frac{3H}{4h^3}}\sqrt{g(h+H)} \qquad (2)$$

Where, U=free-stream velocity, U_c= maximum velocity under wave crest, t=time, h=water depth, H=wave height and g=gravitational acceleration. The free-stream velocity and the pressure gradient in dimensionless form is shown in Fig. 1.

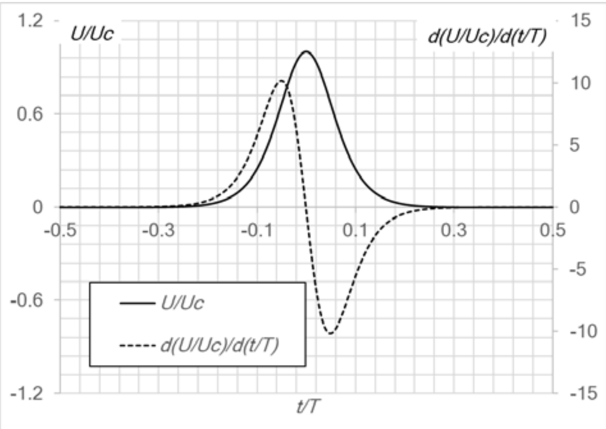

Figure 1. Free-stream velocity and pressure gradient

SOLITARY WAVE BL EXPERIMENTS

The generation of solitary waves in a laboratory is difficult and measurement of boundary layer properties under such waves is extremely challenging, .therefore experimental data in this regard is scarce. For regular waves in wave flumes or conduits it is possible to collect the instantaneous velocity data for a number of wave cycles within a reasonable time span and ensemble averaging can be done to separate mean and fluctuating velocity components. Whereas using the same approach for solitary wave is considerably difficult because of the requirement of maintaining tranquil period between every two waves. Tanaka et al (2011) proposed a new solitary wave generation system in a conduit and reported the results of boundary layer cases covering laminar and transitional regions. Here Case 2-2 (U_c = 78.7 cm/s, T=16.9 s, ν = 0.011 cm^2/s) data has been used for comparison with the model results.

TWO-EQUATION TURBULENCE MODEL

Menter(1994) proposed composite two-equation models based on k-ε and k-ω models. utilizing latter model near the wall and the former one in the rest of region. The use of k-ω model near the wall is beneficial because of its simplicity in defining the bed roughness. Menter(1994) named the two-layer k-ω models as Baseline (BSL) model and Shear Stress Transport (SST) model. The BSL model was devised considering the relative performance of k-ε and k-ω models in viscous sublayer, logarithmic region and wake region of a steady flow. In the viscous sublayer and logarithmic region k-ω and in the wake region k-ε model, transformed in terms of k and ω, has been utilized. A blending function has been used to set the demarcation level between the two layers at y^+=70.

Governing equations were expressed in dimensionless form using the crest velocity; Uc, distance from the bed to the axis of symmetry of the conduit (representing free-stream), α from Eq. (2), and kinematic viscosity of the fluid (water in the present case). In the dimensionless form the governing equations require only Reynolds number and reciprocal of Strouhal number to find a numerical solution. The dimensionless equations were solved using a Crank-Nicolson type implicit finite difference scheme. An exponentially varying grid spacing was used to achieve better accuracy near the wall. The convergence limit was set to be 1×10^{-6}. The convergence was achieved at two-levels; at every time instant the iterations were carried out to converge the velocity, k and ω. Second level convergence was achieved by numerically running several wave cycles keeping sufficient tranquil period after every wave until maximum shear stress was converged.

MODEL RESSULTS

The oscillatory Reynolds number (equivalent to wave Reynolds number) for Case 2-2 from Tanaka et al(2011) is 1436000. This value falls within transition from laminar to turbulent situation.

The velocity profiles for Case 2-2 from Tanaka et al(2011) are shown in Fig.2 (acceleration) and Fig.3 (deceleration). Here BSL model proposed by Menter(1994) was used. The model performs very well in the initial part of the acceleration phase however, during the later part of acceleration and deceleration the model could not show good agreement with the measurements. The velocity profiles close to the crest velocity (t=2.6 sec to 3.5 sec) show a well-defined logarithmic layer. In the deceleration phase the logarithmic layer remains prominent until t= 3.7 sec. Whereas, the experimental data does not show such behavior near the bed. The boundary layer thickness is also overestimated by the model, a phenomenon shown by the BSL model in sinusoidal wave boundary layer layers as well (Sana and Shuy, 2002)

Bottom shear stress calculated from laminar theory and BSL version of k-ω model shows significant difference because of transitional behavior of the wave BL. This difference conforms to the prediction of the velocity profile where the model showed turbulent behavior around the crest of the free-stream velocity. The model shows a peak almost twice as high as that calculated by laminar theory (Fig.4).

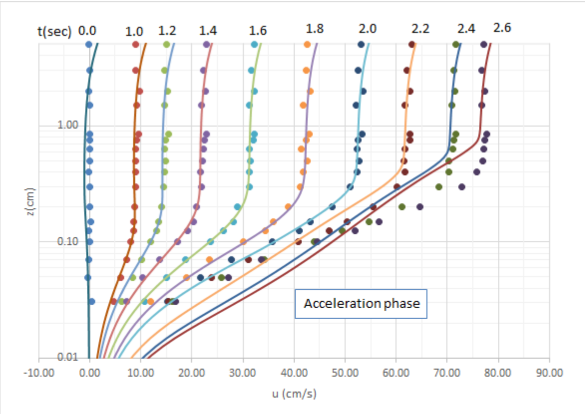

Figure 2. Velocity profile for Case 2-2 during acceleration

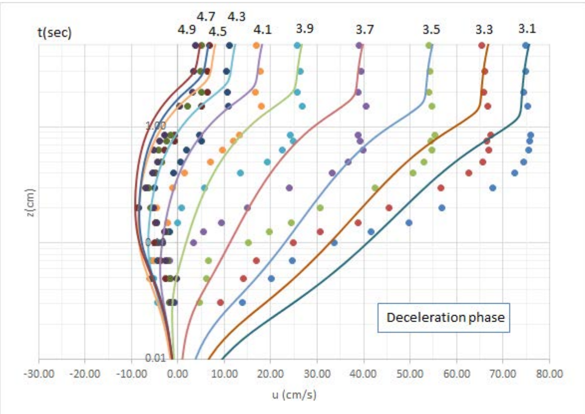

Figure 3 Velocity profile for Case 2-2 during deceleration

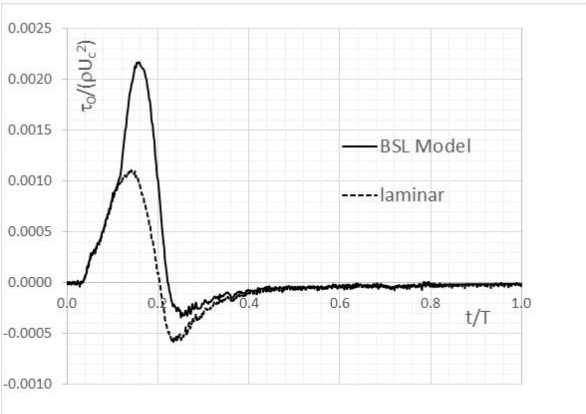

Figure 4 Bottom shear stress for Case 2-2

CONCLUSIONS

A two-layer *k-ω* model (BSL Model) was used to predict boundary layer properties under a solitary wave. The prediction for the velocity profile shows excellent agreement with the experimental data in initial part of the acceleration phase. However, before and after the free-stream crest velocity poor agreement was observed. The model showed a typical turbulent behavior around the crest depicted by well-defined logarithmic layer. The boundary layer thickness was also overestimated by the model. The bed shear stress showed good agreement with the experimental data in the initial part of the acceleration phase but the peak value was overestimated by a factor of approximately 2.

Further study is required to explore the capabilities of other two-equation models and more experimental data should be utilized for that purpose.

REFERENCES

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