Lagrangian breaker characteristics for nonlinear water waves propagating on sloping bottoms

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INTRODUCTION

In this paper, a new third-order Lagrangian asymptotic solution describing nonlinear wave propagation on the surface of a uniform sloping bottom is presented. The model is formulated in the Lagrangian variables and we use a two parameter perturbation method to develop a new mathematical derivation. The analytical solution in Lagrangian form satisfies the normal pressure at the free surface. The condition of the free surface is satisfied in order to get the free surface profile. The breaking wave indices obtained from the present theory show some differences from those obtained from some previous theories.

WAVE BREAKING

Because of the change of water depth, the water shoals and is refracted in the propagation process from deep to shallow water. The energy is reduced; hence, the particle velocity of the wave crest is faster than the energy and the wave breaks. In order to describe the breaking wave mechanism, the determination of wave breaking is applied and the breaking criterion is \( U/C = \theta \) where \( U \) is the energy and \( C \) is the horizontal velocity of particle at the wave crest. According to the determination of wave breaking two functions could be obtained as follow:

\[
\begin{align*}
\alpha = x = c = f_n + f_m + f_{2,0} + f_{1,0}' + f_{2,1}' &= 1 \\
\frac{dy}{dx} = g_{1,0} + g_{1,1} + g_{2,0,1} &= 0
\end{align*}
\]

BREAKING CRITERIA

The limit wave steepness for breaking waves on a uniform water depth was first presented in Miche(1951) based on the theoretical analysis. Goda(2004) expressed the breaking criterion graphically according to many experimental works and presented an approximate expression for the curves. This empirical formula has been applied practically and widely in design of most maritime structures. Being united with Eq(1) by Eq(2) can determine the breaking point. So our theoretical solution can discuss a succession of breaking indices. Compare our solution from Fig.3a-b with empirical formula of Goda(2004), we found that when the slope is 1/10, result of Goda(2004) and our theoretical results are consistent, and when slope is 1/5, the consistency of our results between theoretical solution and experimental data is better than the empirical formula of Goda(2004).

CONCLUSIONS

The breaking wave indices obtained from the present theory shows similar tendency to the experimental data and the theoretical curves and the experimental data are more consistent than other’s empirical formula when the bottom slope steep. While the purpose of the present study is to establish a rational mathematical model, which satisfies the bottom boundary condition, its accuracy needs further improvement to include the effects of higher order wave steepness on a sloping bottom and the effect of bottom friction in shallow water.

REFERENCES